



EL-MONASSER

# In Mathematics

The Main Book

For **1<sup>st</sup>** Prep.  
Second Term



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By  
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### Distribution of Maths Syllabus First year preparatory - Second term

Month	Algebra and statistics	Geometry and measurement
	«One period weekly»	«One and half periods weekly»
February	<b>Numbers and algebra :</b> <ul style="list-style-type: none"> <li>Repeated multiplication in <math>\mathbb{Q}</math></li> <li>Non-negative integer powers.</li> <li>Negative integer powers.</li> </ul>	<ul style="list-style-type: none"> <li>Deductive proof.</li> <li>The polygon :               <ul style="list-style-type: none"> <li>(convex - concave - regular)</li> </ul> </li> <li>The sum of measures of the interior angles of a polygon.</li> <li>The sum of measures of the exterior angles of a polygon.</li> <li>Parallelogram and its properties.</li> <li>Parallelogram and its special cases.</li> </ul>
March	<ul style="list-style-type: none"> <li>Scientific notation of the rational number.</li> <li>Order of mathematical operations.</li> <li>The square root of a perfect square rational number.</li> <li>Solving equations in <math>\mathbb{Q}</math></li> <li>Solving inequalities in <math>\mathbb{Q}</math></li> </ul>	<ul style="list-style-type: none"> <li>The triangle :               <ul style="list-style-type: none"> <li><b>Theorem (1) :</b> The sum of measures of the interior angles of a triangle is <math>180^\circ</math></li> <li>The exterior angles of a triangle.</li> <li><b>Theorem (2) :</b> The ray drawn from the midpoint of a side of a triangle parallel to ...</li> <li><b>Corollary :</b> The line segment joining the midpoints of two sides ..</li> <li><b>Theorem (3) :</b> The length of the line segment joining the midpoints of two sides ...</li> </ul> </li> <li>Pythagoras' theorem</li> </ul>
April	<b>Statistics and probability :</b> <ul style="list-style-type: none"> <li>Samples</li> <li>Probability</li> </ul>	<ul style="list-style-type: none"> <li>Geometric transformations :               <ul style="list-style-type: none"> <li>Reflection.</li> <li>Translation.</li> <li>Rotation.</li> </ul> </li> </ul>
May	General Exercises	



## The used symbols in this book

$=$	is equal to
$\neq$	is not equal to
$\in$	belongs to
$\notin$	does not belong to
$\subset$	is a subset of
$\not\subset$	is not a subset of
$\cap$	intersection
$\cup$	union
$\emptyset$ or $\{ \}$	empty set (null set)
$\mathbb{N}$	the set of natural numbers
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$<$	is less than
$>$	is more than
$\leq$	is less than or equal to
$\geq$	is more than or equal to
$ a $	the absolute value of a
$a^n$	a to the power n (the $n^{\text{th}}$ power of a)
$\sqrt{a}$	the positive square root of the rational number a
$\therefore$	since
$\therefore$	then , hence , therefore
$\overline{AB}$	the straight line AB
$\overrightarrow{AB}$	the ray AB
$\overline{AB}$	the line segment AB
$AB$	the length of the line segment AB
$\angle B$	the angle B
$m(\angle B)$	the measure of the angle B
$//$	is parallel to
$\perp$	is perpendicular to
$\rightangle$	a right angle
$\triangle$	triangle
$\cong$	is congruent to

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## First Algebra and Statistics

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## Algebra and Statistics



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UNIT

1

## Numbers and Algebra



هذا العمل خاص بموقع ذاكرولي التعليمي ولا يسمح بتداوله على مواقع أخرى

كتاب المعاصر

موقع ذاكرولي التعليمي

الصف الاول الاعدادي



### ► Lessons of the unit :

1. Repeated multiplication.
2. Non-negative integer powers.
3. Negative integer powers.
4. Scientific notation of the rational number.
5. Order of mathematical operations.
6. The square root of a perfect square rational number.
7. Solving equations in  $\mathbb{Q}$ .
8. Solving inequalities in  $\mathbb{Q}$ .

### ► Unit Objectives :

By the end of this unit, student should be able to :

- remember what have been studied on repeated multiplication in  $\mathbb{Z}$
- multiply repeated multiplication of the rational numbers.
- recognize the laws of powers in  $\mathbb{Q}$
- recognize the negative power of the non-zero rational number.
- recognize the standard scientific notation of the rational number.
- write a rational number in the standard form.
- perform the mathematical operations according to the priority of their performances.
- recognize the square root of a perfect square rational number.
- find the square root of a perfect square rational number.
- solve an equation of the first degree in one unknown in  $\mathbb{Q}$
- use the equation to solve word problems.
- solve an inequality of the first degree in one unknown in  $\mathbb{Q}$



**Ghiyath Al-Din Ibn  
Masoud Al-Kashi**  
(1380 A.D. - 1438 A.D.)

He was an Arab scientist who had many investigations in mathematics :

- He had invented the decimal fraction.
- He put a theory concerning the sum of the natural numbers that are raised to the fourth power.
- He reached a very accurate rate for the approximate ratio ( $\pi$ ) that nearly equates the accuracy of the calculators.





# Lesson 1

## Repeated multiplication

We had known before in the set of integers that :  $3^4 = 3 \times 3 \times 3 \times 3$  where we found that the number 3 has repeated 4 times in the multiplication operation and we read it as :

«3 to the power 4»

Also , we can apply the previous on normal fractions :

For example:  $\left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

From multiplying normal fractions , we find that :

$$\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3} = \frac{2^4}{3^4} \quad \text{i.e.} \quad \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$$

### Generally

If  $\frac{a}{b}$  is a rational number and  $n$  is a positive integer , then :  $\left(\frac{a}{b}\right)^n = \frac{a}{b} \times \frac{a}{b} \times \frac{a}{b} \dots$  to  $n$  times

It is read as «  $\frac{a}{b}$  to the power  $n$  » or « the  $n^{\text{th}}$  power of the number  $\frac{a}{b}$  » i.e.  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

For example:  $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3} = \frac{8}{125}$

$(0.7)^2 = \left(\frac{7}{10}\right)^2 = \frac{7^2}{10^2} = \frac{49}{100}$

### Remark

If  $\frac{a}{b}$  is a rational number , then :  $\left(\frac{a}{b}\right)^0 = 1$  where  $a \neq 0$

For example:  $\left(\frac{1}{3}\right)^0 = 1$

$\left(-\frac{3}{7}\right)^0 = 1$



## Remark

If  $a$  is a rational number and  $m$  is a positive integer

then

$$(-a)^m = (a)^m$$

when  $m$  is an even number.

For example:

$$\left(-\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$(-a)^m = -(a)^m$$

when  $m$  is an odd number.

For example:

$$\left(-\frac{1}{2}\right)^3 = -\left(\frac{1}{2}\right)^3 = -\frac{1}{8}$$

**Example 1** Find each of the following in the simplest form :

1  $\left(\frac{2}{3}\right)^2 \times \frac{9}{4}$

2  $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{3}\right)^4$

3  $\left(3\frac{1}{2}\right)^2 + \left(-10\frac{1}{2}\right)$

4  $\left(-\frac{2}{3}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0$

**Solution**

1  $\left(\frac{2}{3}\right)^2 \times \frac{9}{4} = \frac{2^2}{3^2} \times \frac{9}{4} = \frac{4}{9} \times \frac{9}{4} = 1$

2  $\left(-\frac{5}{4}\right)^2 \times \left(\frac{2}{3}\right)^4 = \frac{5^2}{4^2} \times \frac{2^4}{3^4} = \frac{25}{16} \times \frac{16}{625} = \frac{1}{25}$

3  $\left(3\frac{1}{2}\right)^2 + \left(-10\frac{1}{2}\right) = \left(\frac{7}{2}\right)^2 + \left(-\frac{21}{2}\right) = \frac{7^2}{2^2} \times \left(-\frac{21}{2}\right)$   
 $= \frac{49}{4} \times \left(-\frac{21}{2}\right) = -\frac{7}{6}$

4  $\left(-\frac{2}{3}\right)^2 \times \left(-\frac{5}{2}\right)^3 \times \left(\frac{1}{5}\right)^0 = \frac{2^2}{3^2} \times \left(-\frac{5^3}{2^3}\right) \times 1 = \frac{4}{25} \times \left(-\frac{125}{8}\right) = -\frac{5}{2}$

## TRY 1

by yourself

Find each of the following in its simplest form :

1  $\left(\frac{1}{3}\right)^2$

2  $\left(-\frac{2}{3}\right)^3$

3  $\left(-\frac{4}{5}\right)^4$

4  $\left(1\frac{1}{2}\right)^4$

5  $\left(-\frac{3}{9}\right)^2 \times \left(\frac{9}{4}\right)^2 \times \left(\frac{81}{16}\right)^0$

**Final answers**

of try by yourself questions are at the end of each lesson to check your answer.



## Unit 1

**Example 2** If  $x = -\frac{1}{2}$ ,  $y = \frac{1}{4}$  and  $z = 4$

Find the value of :  $(x + y)^3 \times z^3$

**Solution**

$$\begin{aligned}(x + y)^3 \times z^3 &= \left(-\frac{1}{2} + \frac{1}{4}\right)^3 \times 4^3 = \left(-\frac{2}{4} + \frac{1}{4}\right)^3 \times 4^3 \\ &= \left(-\frac{1}{4}\right)^3 \times 4^3 = -\frac{1^3}{4^3} \times 4^3 = -1\end{aligned}$$

## TRY 2

by yourself

If  $x = -\frac{2}{3}$ ,  $y = \frac{1}{2}$  and  $z = -\frac{4}{3}$

find the value of :  $x^2 - y^2 z$

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At the end

of each lesson, you will find the final answers of try by yourself questions in the same form.

3  $\frac{529}{256}$

2  $-\frac{47}{8}$

5  $\frac{16}{9}$

1  $\frac{25}{16}$

4  $\frac{16}{81}$

2  $x^2 - y^2 z = \frac{6}{7}$

Answers of try by yourself





## 2

## Non-negative integer powers

You have studied in the primary stage the laws of non-negative integer powers in  $\mathbb{Z}$ . In this lesson we will illustrate that these laws are also applicable for the rational numbers.

## Definition

From the definition of repeated multiplication, you know that :

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}, \quad \left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$$

$$\text{i.e. } \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^7$$

## Generally

If  $\frac{a}{b}$  is a rational number,  $n$  and  $m$  are non-negative integers

$$\text{, then : } \left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$$

i.e. When multiplying the like bases, we add their powers (indices).

For example:

$$\bullet \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right)^{3+2} = \left(\frac{2}{3}\right)^5$$

$$\bullet \left(-\frac{1}{2}\right)^4 \times \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^{4+3} = \left(-\frac{1}{2}\right)^7$$

## Unit 1

**Example 1** Calculate each of the following , then put the result in its simplest form :

$$1. \frac{2}{3} \times (\frac{2}{3})^2 \times (\frac{2}{3})^3 \quad | \quad 2. (-\frac{1}{3})^3 \times (\frac{1}{3})^2$$

$$3. \frac{3}{4} \times (-\frac{3}{4})^2$$

**Solution**

$$1. \frac{2}{3} \times (\frac{2}{3})^2 \times (\frac{2}{3})^3 = (\frac{2}{3})^{1+2+3} = (\frac{2}{3})^6 = \frac{2^6}{3^6} = \frac{64}{729}$$

$$2. (-\frac{1}{3})^3 \times (\frac{1}{3})^2 = -(\frac{1}{3})^3 \times (\frac{1}{3})^2 \\ = -(\frac{1}{3})^5 = -\frac{1^5}{3^5} = -\frac{1}{243}$$

**Notice that :**

$(-\frac{1}{3})^3 = -(\frac{1}{3})^3$   
because the index is  
an odd number.

$$3. \frac{3}{4} \times (-\frac{3}{4})^2 = \frac{3}{4} \times (\frac{3}{4})^2 \\ = (\frac{3}{4})^3 = \frac{3^3}{4^3} = \frac{27}{64}$$

**Notice that :**

$(-\frac{3}{4})^2 = (\frac{3}{4})^2$   
because the index is  
an even number.

### The second law

According to the first law , you know that ,  $a^6 = a^2 \times a^4$

therefore :  $a^6 \div a^2 = a^4$  ,  $a^6 \div a^4 = a^2$

#### Generally

If  $\frac{a}{b}$  is a rational number , where  $\frac{a}{b} \neq 0$  , n and m are non-negative integers ,  $n \geq m$

$$\text{, then : } \left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$$

i.e. When dividing like bases , we subtract their powers (indices).

For example:

$$\bullet \left(\frac{3}{8}\right)^5 \div \left(\frac{3}{8}\right)^2 = \left(\frac{3}{8}\right)^{5-2} = \left(\frac{3}{8}\right)^3$$

$$\bullet (-\frac{7}{9})^4 \div (-\frac{7}{9})^2 = (-\frac{7}{9})^{4-2} = (-\frac{7}{9})^2$$



**Example 2** Calculate each of the following , then put the result in the simplest form :

1  $(\frac{4}{3})^2 \times (\frac{4}{3})^5 + (\frac{4}{3})^7$

2  $\frac{2^5 \times 2^4}{2^6}$

**Solution**

$$\begin{aligned} 1 \quad & \left[ \left( \frac{4}{3} \right)^2 \times \left( \frac{4}{3} \right)^5 \right] + \left( \frac{4}{3} \right)^7 = \left( \frac{4}{3} \right)^{2+5} + \left( \frac{4}{3} \right)^7 \\ & = \left( \frac{4}{3} \right)^7 + \left( \frac{4}{3} \right)^7 \\ & = \left( \frac{4}{3} \right)^{7+7} = \left( \frac{4}{3} \right)^{14} = 1 \end{aligned}$$

$$2 \quad \frac{2^5 \times 2^4}{2^6} = \frac{2^{5+4}}{2^6} = \frac{2^9}{2^6} = 2^{9-6} = 2^3 = 8$$



Find each of the following in the simplest form :

1  $(\frac{1}{3})^2 \times (\frac{1}{3})^2$

2  $(\frac{3}{7})^8 + (\frac{3}{7})^6$

3  $(-\frac{2}{3})^5 \times (-\frac{2}{3})^2 + (-\frac{2}{3})^6$

4  $(\frac{1}{4})^7 + (\frac{1}{4})^6 \times \frac{1}{4}$

**Remark 1**

From the repeated multiplication , notice that :

$$\begin{aligned} \left( \frac{3}{4} \times \frac{5}{7} \right)^3 &= \left( \frac{3}{4} \times \frac{5}{7} \right) \times \left( \frac{3}{4} \times \frac{5}{7} \right) \times \left( \frac{3}{4} \times \frac{5}{7} \right) \\ &= \left( \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \right) \times \left( \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \right) \\ &= \left( \frac{3}{4} \right)^3 \times \left( \frac{5}{7} \right)^3 \end{aligned}$$

**Generally:**

If  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers , n is a non-negative integer ,

then :  $\left( \frac{a}{b} \times \frac{c}{d} \right)^n = \left( \frac{a}{b} \right)^n \times \left( \frac{c}{d} \right)^n$



## Unit 1

## Remark 2

• From the repeated multiplication, notice that :

$$\begin{aligned}\left(\frac{2}{3} + \frac{5}{11}\right)^4 &= \left(\frac{\frac{2}{3} + \frac{5}{11}}{1}\right)^4 = \frac{\frac{2}{3} + \frac{5}{11}}{1} \times \frac{\frac{2}{3} + \frac{5}{11}}{1} \times \frac{\frac{2}{3} + \frac{5}{11}}{1} \times \frac{\frac{2}{3} + \frac{5}{11}}{1} \\ &= \frac{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} + \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11} \times \frac{5}{11}}{\frac{1}{1} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}} \\ &= \left(\frac{2}{3}\right)^4 + \left(\frac{5}{11}\right)^4\end{aligned}$$

Generally:

If  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers,  $\frac{c}{d} \neq 0$ ,  $n$  is a non-negative integer

, then :  $\left(\frac{a}{b} + \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n + \left(\frac{c}{d}\right)^n$  (where  $\frac{c}{d} \neq 0$ )

**Example 3** Find the result of each of the following in its simplest form :

1  $\left(\frac{xy}{z}\right)^2$

2  $\left(\frac{2x}{3y}\right)^4$

**Solution**

1  $\left(\frac{xy}{z}\right)^2 = \frac{(xy)^2}{z^2} = \frac{x^2 y^2}{z^2}$

2  $\left(\frac{2x}{3y}\right)^4 = \frac{2^4 x^4}{3^4 y^4} = \frac{16x^4}{81y^4}$

## The third law

You know that  $(a^2)^3 = a^2 \times a^2 \times a^2$  and according to the first law :  $a^2 \times a^2 \times a^2 = a^6$

i.e.  $(a^2)^3 = a^6$

Generally

If  $\frac{a}{b}$  is a rational number,  $n$  and  $m$  are non-negative integers

, then :  $\left[\left(\frac{a}{b}\right)^n\right]^m = \left(\frac{a}{b}\right)^{n \times m}$

For example:

•  $\left[\left(\frac{3}{5}\right)^3\right]^2 = \left(\frac{3}{5}\right)^{3 \times 2} = \left(\frac{3}{5}\right)^6$

•  $\left[\left(-\frac{1}{2}\right)^4\right]^2 = \left(-\frac{1}{2}\right)^{4 \times 2} = \left(-\frac{1}{2}\right)^8$



## Lesson Two

**Example 4** Find each of the following , in the simplest form :

1  $\left[(-2\frac{1}{2})^2\right]^2$

2  $\left(\frac{x^2}{y^3}\right)^3$

3  $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

**Solution**

1  $\left[(-2\frac{1}{2})^2\right]^2 = (-2\frac{1}{2})^{2 \times 2} = (-2\frac{1}{2})^4 = (2\frac{1}{2})^4 = (\frac{5}{2})^4 = \frac{5^4}{2^4} = \frac{625}{16}$

2  $\left(\frac{x^2}{y^3}\right)^3 = \frac{(x^2)^3}{(y^3)^3} = \frac{x^{2 \times 3}}{y^{3 \times 3}} = \frac{x^6}{y^9}$

3  $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^{3 \times 2} \times y^{4 \times 2}}{(-2)^4 \times x^4 \times y^{2 \times 4}} = \frac{16x^6y^8}{16x^4y^8} = x^{6-4} = x^2$

**Example 5** If  $x = \frac{1}{2}$  ,  $y = -\frac{3}{4}$  and  $z = \frac{3}{2}$  , find the numerical value of each of the following in the simplest form :

1  $\left(\frac{x^2}{z}\right)^3$

2  $\left(\frac{x^2z}{y}\right)^2$

**Solution**

1  $\left(\frac{x^2}{z}\right)^3 = \left[\left(\frac{1}{2}\right)^2 \div \frac{3}{2}\right]^3 = \left(\frac{1^2}{2^2} \times \frac{2}{3}\right)^3$   
 $= \left(\frac{1}{4} \times \frac{2}{3}\right)^3 = \left(\frac{1}{6}\right)^3 = \frac{1^3}{6^3} = \frac{1}{216}$

Notice that :

$$\frac{x^2}{z} = x^2 \div z$$

2  $\left(\frac{x^2z}{y}\right)^2 = \frac{x^{2 \times 2} z^2}{y^2} = \frac{x^4 z^2}{y^2}$   
 $= \frac{\left(\frac{1}{2}\right)^4 \times \left(\frac{3}{2}\right)^2}{\left(-\frac{3}{4}\right)^2} = \frac{\frac{1^4}{2^4} \times \frac{3^2}{2^2} \times \frac{4^2}{3^2}}{\frac{3^2}{4^2}} = \frac{1}{16} \times \frac{1}{4} \times 16 = \frac{1}{4}$

**TRY 2**  
by yourself

Calculate each of the following , then put the result in the simplest form :

1  $\left[\left(\frac{1}{2}\right)^2\right]^3$

2  $\left(\frac{a^2b^2}{c^3d^4}\right)^2$

3  $\left(\frac{5^2 \times 5^4}{5^3}\right)^2$

4  $-\frac{1}{16}$

3  $-\frac{3}{2}$

2  $\frac{2}{9}$

1  $\frac{625}{16}$

Answers of try by yourself



3

## Negative integer powers

You know that, if  $a$  is a rational number,  $a \neq 0$ , then  $a^0 = 1$

therefore,  $\frac{1}{a^n} = \frac{a^0}{a^n} = a^{0-n} = a^{-n}$  i.e.  $a^{-n} = \frac{1}{a^n}$

### Definition ~

If  $a$  is a rational number,  $a \neq 0$  and  $n$  is a positive integer,

then  $a^{-n} = \frac{1}{a^n}$  and  $a^n = \frac{1}{a^{-n}}$

For example:

$$3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

$$3 \times 5^{-1} = 3 \times \frac{1}{5} = \frac{3}{5}$$

$$2 \times 5^2 = 2 \times 5^2 = 2 \times 25 = 50$$

$$0.1 = \frac{1}{10} = 10^{-1}, \quad 0.01 = \frac{1}{100} = \frac{1}{10^2} = 10^{-2} \quad \dots \text{and so on.}$$

### Remarks

① If  $a$  is a rational number,  $a \neq 0$  and  $n$  is a positive integer,

then  $a^n \times a^{-n} = a^n \times \frac{1}{a^n} = 1$  (the multiplicative neutral)

i.e. each of  $a^n$  and  $a^{-n}$  is the multiplicative inverse of the other

② If  $\frac{a}{b}$  is a rational number not equal to zero and  $n$  is a positive integer

, then:  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

For example:  $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$



**Example 1** Find the value of each of the following in the simplest form :

1  $2^4 \times 2^{-2}$

2  $\frac{5^{-2}}{5^{-3}}$

3  $(3^2)^{-2}$

4  $\frac{6^{-3} \times 6^5}{6^2}$

5  $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2}$

6  $(7^3)^2 \times (7^{-2})^2$

7  $\left(\frac{3}{5}\right)^{-3} + \left(\frac{4}{3}\right)^{-3}$

**Solution**

1  $2^4 \times 2^{-2} = 2^4 \times \frac{1}{2^2} = \frac{2^4}{2^2} = 2^{4-2} = 2^2 = 4$

2  $\frac{5^{-2}}{5^{-3}} = \frac{5^{-2}}{5^{-3}} = 5^{-2-(-3)} = 5$

3  $(3^2)^{-2} = \frac{1}{(3^2)^2} = \frac{1}{3^4} = \frac{1}{81}$

4  $\frac{6^{-3} \times 6^5}{6^2} = \frac{6^5}{6^3 \times 6^2} = \frac{6^5}{6^5} = 1$

5  $\left(\frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4}\right)^{-2} = \left(\frac{5^3 \times 5}{5^2 \times 5^4}\right)^{-2} = \left(\frac{5^4}{5^6}\right)^{-2}$   
 $= \left(\frac{5^6}{5^4}\right)^2 = (5^{6-4})^2 = (5^2)^2 = 5^4 = 625$

6  $(7^3)^2 \times (7^{-2})^2 = (7^3)^2 \times \left(\frac{1}{7^2}\right)^2 = 7^6 \times \frac{1}{7^4} = 7^{6-4} = 7^2 = 49$

7  $\left(\frac{3}{5}\right)^{-3} + \left(\frac{4}{3}\right)^{-3} = \left(\frac{5}{3}\right)^3 + \left(\frac{3}{4}\right)^3 = \left(\frac{5}{3} + \frac{3}{4}\right)^3$   
 $= \left(\frac{20}{12} + \frac{9}{12}\right)^3 = \left(\frac{29}{12}\right)^3 = \frac{29^3}{12^3} = \frac{24389}{1728}$

**Remark**

All laws of powers that we have studied in the previous lesson are correct in the case of the negative powers. So , the previous example can be solved by using laws of powers as follows :

1  $2^4 \times 2^{-2} = 2^{4+(-2)} = 2^2 = 4$

2  $\frac{5^{-2}}{5^{-3}} = 5^{-2-(-3)} = 5^{-2+3} = 5$

3  $(3^2)^{-2} = 3^{2 \times (-2)} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

4  $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$

## Unit 1

$$5 \left( \frac{5^3 \times 5^{-2}}{5^{-1} \times 5^4} \right)^{-2} = (5^{3+(-2)-(-1)-4})^{-2} = (5^{3-2+1-4})^{-2} \\ = (5^{-2})^{-2} = 5^{(-2) \times (-2)} = 5^4 = 625$$

$$6 (7^3)^2 \times (7^{-2})^2 = (7^3 \times 7^{-2})^2 = (7^{3+(-2)})^2 = 7^2 = 49$$

$$7 \left( \frac{3}{5} \right)^3 \div \left( \frac{4}{5} \right)^3 = \left( \frac{3}{5} \div \frac{4}{5} \right)^3 = \left( \frac{3}{5} \times \frac{5}{4} \right)^3 = \left( \frac{3}{4} \right)^3 = \left( \frac{4}{3} \right)^3 = \frac{4^3}{3^3} = \frac{64}{27}$$

## TRY 1

Find the value of each of the following in the simplest form :

$$1) 5^{-3}$$

$$2) \left( \frac{3}{7} \right)^{-2}$$

$$3) (2^{-3})^2$$

$$4) \left( \frac{2^{-2} \times 2^6}{2^3} \right)^{-3}$$

Example 2 Simplify each of the following to the simplest form where  $x \neq 0$  :

$$1) x^5 \times x^{-2} \times x^{-3}$$

$$2) (x^2)^{-3} + (x^{-1})^2$$

$$3) \left( \frac{x^4 \times x^{-3}}{x^{-4} \times x} \right)^{-2}$$

## Solution

$$1) x^5 \times x^{-2} \times x^{-3} = x^{5+(-2)+(-3)} = x^{5-2-3} = x^0 = 1$$

$$2) (x^2)^{-3} + (x^{-1})^2 = x^{-6} + x^{-2} = x^{-6+(-2)} = x^{-6-2} = x^{-8} = \frac{1}{x^8}$$

$$3) \left( \frac{x^4 \times x^{-3}}{x^{-4} \times x} \right)^{-2} = (x^{4+(-3)-(-4)-1})^{-2} = (x^{4-3+4-1})^{-2} \\ = (x^4)^{-2} = x^{-8} = \frac{1}{x^8}$$

## TRY 2

Simplify each of the following to the simplest form putting the result in positive integer power where the denominator doesn't equal zero :

$$1) (x^{-2})^5$$

$$2) \left( \frac{a^4}{a^3} \right)^2$$

$$3) (y^3 \times y^{-2})^3$$





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## Scientific notation of the rational number

- The scientific notation of the number is a useful method to deal with the very large numbers or the very small numbers like the numbers in the following examples



Neptune planet is far from the sun  
by 2 800 000 000 miles.  
(the mile = 1.6 km approximately)



The diameter length  
of a virus = 0.00000000025 cm.

- Before explaining how to write the numbers in their scientific notation , we should notice the following :

1  $10 = 10^1$  ,  $100 = 10 \times 10 = 10^2$  ,  $1000 = 10 \times 10 \times 10 = 10^3$  and so on

Hence we find that :

$$2000 = 2 \times 1000 = 2 \times 10^3 \quad , \quad 50\,000 = 5 \times 10\,000 = 5 \times 10^4$$

## Unit 1

2  $0.1 = \frac{1}{10} = 10^{-1}$  ,  $0.01 = \frac{1}{100} = \frac{1}{10 \times 10} = 10^{-2}$  ,

$0.001 = \frac{1}{1000} = \frac{1}{10 \times 10 \times 10} = 10^{-3}$  and so on

Hence we find that :

•  $0.03 = \frac{3}{100} = \frac{3}{10 \times 10} = 3 \times 10^{-2}$

•  $0.0007 = \frac{7}{10\,000} = \frac{7}{10 \times 10 \times 10 \times 10} = 7 \times 10^{-4}$

### The standard scientific notation of a number

The number is written in the standard form as :  $a \times 10^n$  where  $1 \leq |a| < 10$  and  $n \in \mathbb{Z}$

There are examples for some numbers written in its standard form :

•  $4.6 \times 10^8$

•  $5.236 \times 10^{-6}$

•  $-9.6 \times 10^{10}$

•  $-1.001 \times 10^{-3}$

•  $-3 \times 10^{12}$

•  $1 \times 10^{-7}$

Each of the previous numbers is the product of two numbers :

- The first number could be positive or negative and its absolute value must be greater than or equal 1 and less than 10
- The second number expresses the powers of the number 10  
(These powers could be positive or negative)

• Examples for some numbers not in the standard form :

• 8 200 000 000

• 0.000 000 135

•  $45 \times 10^8$  (because  $45 > 10$ )

•  $706.4 \times 10^5$  (because  $706.4 > 10$ )

•  $0.248 \times 10^{-7}$  (because  $0.248 < 1$ )

•  $-0.0015 \times 10^{-9}$  (because  $|-0.0015| < 1$ )

and the next example will show how to write these numbers in the standard form.

### Writing the number in the standard form

**Example 1** Write each of the following numbers in the standard form :

1 8 200 000 000

2 0.000 000 135

3  $45 \times 10^8$

4  $706.4 \times 10^5$

5  $0.248 \times 10^{-7}$

6  $-0.0015 \times 10^{-9}$



## Lesson Four

## Solution

$$1 \quad 8\,200\,000\,000.0 = 8.2 \times 10^9$$

Moving the decimal  
point **9** places  
towards **left**

Using the power  
**9** of the  
number 10

$$2 \quad 0.000\,000\,135 = 1.35 \times 10^{-7}$$

Moving the decimal  
point **7** places  
towards **right**

Using the power  
**7** of the  
number 10

- 3 To put  $45.0 \times 10^8$  in the standard form we move the decimal point one place to the left , then we multiply by 10

$$\therefore 45.0 \times 10^8 = 4.5 \times 10^8 \times 10 = 4.5 \times 10^9$$

- 4 To put  $706.4 \times 10^5$  in the standard form we move the decimal point two places to the left , then we multiply by  $10^2$

$$\therefore 706.4 \times 10^5 = 7.064 \times 10^5 \times 10^2 = 7.064 \times 10^7$$

- 5 To put  $0.248 \times 10^{-7}$  in the standard form we move the decimal point one place to the right , then we multiply by  $10^{-1}$

$$\therefore 0.248 \times 10^{-7} = 2.48 \times 10^{-7} \times 10^{-1} = 2.48 \times 10^{-8}$$

- 6 To put  $-0.0015 \times 10^{-9}$  in the standard form we move the decimal point three places to the right , then we multiply by  $10^{-3}$

$$\therefore -0.0015 \times 10^{-9} = -1.5 \times 10^{-9} \times 10^{-3} = -1.5 \times 10^{-12}$$

## Remark

The standard form for 1 is  $1 \times 10^{\text{zero}}$

, So is the standard form for 2 is  $2 \times 10^{\text{zero}}$  , etc ...

Unit 1

## TRY 1

In the following, determine the numbers that are not in the standard form, then write them in the standard form:

1  $8.5 \times 10^{-4}$

2  $17 \times 10^8$

3  $0.5 \times 10^{-7}$

4  $530.5 \times 10^9$

5  $-0.999 \times 10^{-5}$

6  $6 \times 10^6$

7  $650\,000\,000$

8  $0.000\,001\,02$

9  $-2.5 \times 10^8$

## Operations on the numbers in the standard form

Example 2 Write the result of each of the following in the standard form:

1  $(1.2 \times 10^5) \times (4 \times 10^3)$

2  $(6.5 \times 10^4) \times (8 \times 10^2)$

3  $(2.4 \times 10^{11}) + (1.2 \times 10^{-4})$

4  $(6.6 \times 10^7) \times (3 \times 10)^4$

5  $(2.3 \times 10^6) + (3.7 \times 10^5)$

Solution

1  $(1.2 \times 10^5) \times (4 \times 10^3) = (1.2 \times 4) \times (10^5 \times 10^3) = 4.8 \times 10^8$

2  $(6.5 \times 10^4) \times (8 \times 10^2) = (6.5 \times 8) \times (10^4 \times 10^2)$

$= 52 \times 10^6$

$= 5.2 \times 10^7$

Notice that:

$52 \times 10^6$  is not in the standard form, then we should put it in the standard form.

3  $(2.4 \times 10^{11}) + (1.2 \times 10^{-4}) = \frac{2.4}{1.2} \times \frac{10^{11}}{10^{-4}} = 2 \times 10^{15}$

4  $(6.6 \times 10^7) \times (3 \times 10)^4 = (6.6 \times 10^7) \times (3^4 \times 10^4)$

$= (6.6 \times 3^4) \times (10^7 \times 10^4)$

$= 534.6 \times 10^{11} = 5.346 \times 10^{13}$

5  $(2.3 \times 10^6) + (3.7 \times 10^5) = 10^5 (2.3 \times 10 + 3.7)$

$= 10^5 (23 + 3.7) = 10^5 \times 26.7 = 2.67 \times 10^6$



## Lesson Four

**Example 3** Write the result of each of the following in the standard form :

1  $30\,000 \times 400\,000$

2  $140\,000 \times 0.005$

3  $0.000\,015 \div 30$

4  $(50\,000)^3$

5  $(0.000\,3)^5$

6  $(-0.001)^6$

**Solution**

$$1 \quad 30\,000 \times 400\,000 = (3 \times 10^4) \times (4 \times 10^5) = (3 \times 4) \times (10^4 \times 10^5) \\ = 12 \times 10^9 = 1.2 \times 10^{10}$$

$$2 \quad 140\,000 \times 0.005 = (1.4 \times 10^5) \times (5 \times 10^{-3}) \\ = (1.4 \times 5) \times (10^5 \times 10^{-3}) = 7 \times 10^2$$

$$3 \quad 0.000\,015 \div 30 = (1.5 \times 10^{-5}) \div 3 \times 10 \\ = \frac{1.5}{3} \times \frac{10^{-5}}{10} = 0.5 \times 10^{-6} = 5 \times 10^{-7}$$

$$4 \quad (50\,000)^3 = (5 \times 10^4)^3 = 5^3 \times 10^{12} = 125 \times 10^{12} = 1.25 \times 10^{14}$$

$$5 \quad (0.000\,3)^5 = (3 \times 10^{-4})^5 = 3^5 \times 10^{-20} = 243 \times 10^{-20} = 2.43 \times 10^{-18}$$

$$6 \quad (-0.001)^6 = (0.001)^6 = (1 \times 10^{-3})^6 = 1^6 \times 10^{-18} = 10^{-18}$$

**TRY 2**

Write the result of each of the following in the standard form :

1  $(5.3 \times 10^7) \times (3 \times 10^3)$

2  $0.000\,6 \div 20$

3  $(400\,000)^2$

4  $(3.2 \times 10^3) - (0.2 \times 10^3)$

1  $3.18 \times 10^4$

2  $1.6 \times 10^{11}$

3  $3 \times 10^5$

4  $1.59 \times 10^{11}$

5  $1.02 \times 10^{-6}$

6  $6.5 \times 10^8$

7  $9.99 \times 10^4$

8  $5.305 \times 10^{11}$

9  $5 \times 10^6$

10  $1.7 \times 10^8$

Answers of try by yourself

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## Order of mathematical operations

We know that : addition , subtraction , multiplication and division are the basic mathematical operations which are performed on the numbers

Sometimes one problem contains the four operations or some of them , so it is necessary to be agreed on rules which determine the priority of performance of these operations.

The following situation shows the Importance of that :

The following problem was given to each of Heba and Ahmed.

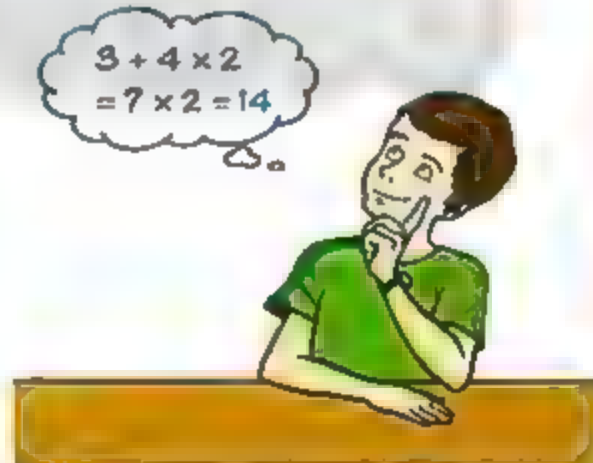
$$\text{Calculate : } 3 + 4 \times 2$$

Their answers were as follows :



Heba multiplied at first , then she added.

She got : 11



Ahmed added at first , then he multiplied.

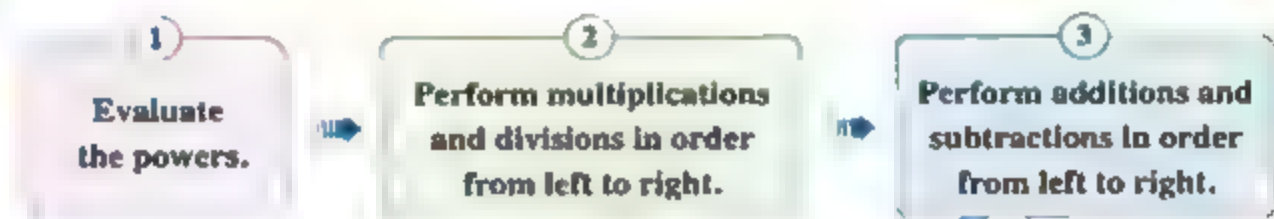
He got : 14

For the results are different , then it is necessary to be agreed on some rules that determine the order of performing the mathematical operations , that are :

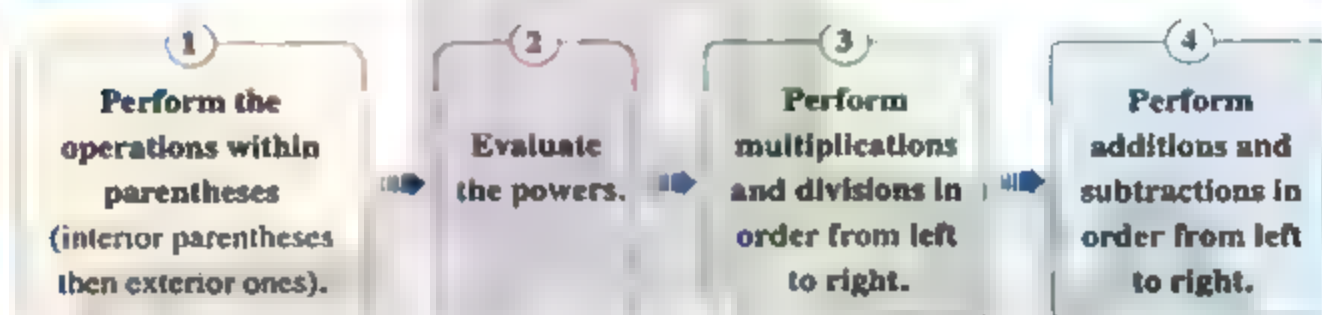


## Lesson Five

**First** Order of performing the mathematical operations in an expression has no parentheses



**Second** Order of performing the mathematical operations in an expression has parentheses



- According to these rules, we can determine that Heba held the correct answer because she performed the multiplication operation first, then the addition operation

**Notice that :**

The scientific calculators and computers follow the same rules of ordering the operations.

**Example 1** Calculate the value of each of the following :

1  $3 + 6 \times (5 + 4) + 3 - 7$

2  $9 - 5 + (8 - 3) \times 2 + 6$

**Solution**

1  $3 + 6 \times (5 + 4) + 3 - 7 = 3 + 6 \times 9 + 3 - 7$

(parentheses)

$= 3 + 54 + 3 - 7$

(multiplication)

$= 3 + 18 - 7$

(division)

$= 21 - 7$

(addition)

$= 14$

(subtraction)

## Unit 1

$$\begin{aligned}
 2 \quad 9 - 5 + (8 - 3) \times 2 + 6 &= 9 - 5 + 5 \times 2 + 6 && \text{(parentheses)} \\
 &= 9 - 1 \times 2 + 6 && \text{(division)} \\
 &= 9 - 2 + 6 && \text{(multiplication)} \\
 &= 7 + 6 && \text{(subtraction)} \\
 &= 13 && \text{(addition)}
 \end{aligned}$$

**Example 2** Calculate the value of each of the following :

$$1 \quad 4 - 3 [4 - 2 (6 - 3)] + 2 \qquad 2 \quad 16 + [8 - 3 (4 - 2)] + 1$$

**Solution**

$$\begin{aligned}
 1 \quad 4 - 3 [4 - 2 (6 - 3)] + 2 &= 4 - 3 [4 - 2 \times 3] + 2 && \text{(the interior parentheses)} \\
 &= 4 - 3 [4 - 6] + 2 && \text{(multiplication inside parentheses)} \\
 &= 4 - 3 [-2] + 2 && \text{(subtraction inside parentheses)} \\
 &= 4 + 6 + 2 && \text{(multiplication by parentheses)} \\
 &= 4 + 3 && \text{(division)} \\
 &= 7 && \text{(addition)}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad 16 + [8 - 3 (4 - 2)] + 1 &= 16 + [8 - 3 \times 2] + 1 && \text{(the interior parentheses)} \\
 &= 16 + [8 - 6] + 1 && \text{(multiplication inside parentheses)} \\
 &= 16 + 2 + 1 && \text{(subtraction inside the parentheses)} \\
 &= 8 + 1 && \text{(division)} \\
 &= 9 && \text{(addition)}
 \end{aligned}$$

**Example 3** Calculate the value of each of the following :

$$\begin{aligned}
 1 \quad 8 \times 2^2 - 7 \times (4 + 1) & \qquad 2 \quad 2 + 3 [5 + (4 - 1)^2] \\
 3 \quad 3 [(3^2 + 1) - (2^3 - 2)] &
 \end{aligned}$$

**Solution**

$$\begin{aligned}
 1 \quad 8 \times 2^2 - 7 \times (4 + 1) &= 8 \times 2^2 - 7 \times 5 && \text{(addition inside parentheses)} \\
 &= 8 \times 4 - 7 \times 5 && \text{(powers)} \\
 &= 32 - 35 && \text{(multiplication)} \\
 &= -3 && \text{(subtraction)}
 \end{aligned}$$



## Lesson Five

$$\begin{aligned}
 2 \quad 2 + 3 [5 + (4 - 1)^2] &= 2 + 3 [5 + 3^2] && \text{(subtraction inside interior parentheses)} \\
 &= 2 + 3 [5 + 9] && \text{(powers inside parentheses)} \\
 &= 2 + 3 \times 14 && \text{(addition inside parentheses)} \\
 &= 2 + 42 && \text{(multiplication)} \\
 &= 44 && \text{(addition)}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad 3 [(3^2 + 1) - (2^3 - 2)] &= 3 [(9 + 1) - (8 - 2)] && \text{(powers)} \\
 &= 3 [10 - 6] && \text{(the interior parentheses)} \\
 &= 3 \times 4 && \text{(subtraction inside parentheses)} \\
 &= 12 && \text{(multiplication)}
 \end{aligned}$$

## Remark

In the problems containing fractions, we should perform the operations in the numerator and denominator before division.

**Example 4** Calculate the value of each of the following :

$$1 \quad \frac{36 - 6}{3 + 12}$$

$$2 \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2}$$

$$3 \quad 7 + 8 + \frac{4 + 12 - 2}{3^2 - 2} - (2^3 + 2)$$

**Solution**

$$1 \quad \frac{36 - 6}{3 + 12} = \frac{30}{15} = 2$$

$$2 \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2} = \frac{11 - 1}{25 - 20} = \frac{10}{5} = 2$$

$$\begin{aligned}
 3 \quad 7 + 8 + \frac{4 + 12 - 2}{3^2 - 2} - (2^3 + 2) &= 7 + 8 + \frac{14}{9 - 2} - (8 + 2) \\
 &= 7 + 8 + 2 - 10 = 7 + 4 - 10 = 1
 \end{aligned}$$

**TRY**

Calculate the value of each of the following :

$$1 \quad 20 + (12 - 2) \times 3^2 - 2$$

$$2 \quad \frac{6 \times 3 + 10 + 5}{2 - (10 - 2^2)}$$

2 - 5

16 | 91

Answers of try by yourself



6

## The square root of a perfect square rational number

### Definition :

The square root of the perfect square rational number "a" is the number whose square equals "a"

For example:

- The number 6 is a square root of the number 36 because :  $6^2 = 36$
- Also , the number -6 is a square root of the number 36 because :  $(-6)^2 = 36$

i.e. Finding the square root is the inverse operation of finding the square of a number.

It means that , to find the square root of a number , we search a number which , when multiplied by itself , equals this number.

### Generally

- The positive square root of the number a is symbolized by  $\sqrt{a}$
- The negative square root of the number a is symbolized by  $-\sqrt{a}$
- The two square roots of the number a is symbolized by  $\pm\sqrt{a}$  which means  $\sqrt{a}$  ,  $-\sqrt{a}$  , and each of them is the additive inverse of the other.

### Examples :

The positive square root of 25 is  $\sqrt{25} = 5$

The negative square root of 16 is  $-\sqrt{16} = -4$

The two square roots of 49 are  $\pm\sqrt{49} = \pm 7$



## Remarks

$$① \sqrt{0} = 0$$

② In the set of rational numbers it is meaningless to find  $\sqrt{a}$  if  $a$  is a negative rational number because there is no rational number if it is multiplied by itself, the result will be negative.

$$③ \sqrt{a^2} = |a|$$

For example:  $\bullet \sqrt{(-3)^2} = |-3| = 3$        $\bullet \sqrt{\left(-\frac{4}{5}\right)^2} = \left|-\frac{4}{5}\right| = \frac{4}{5}$

$$④ \sqrt{a^2 b^2} = \sqrt{(ab)^2} = |ab|$$

For example:  $\sqrt{a^4 b^6} = \sqrt{(a^2 b^3)^2} = |a^2 b^3|$

⑤ If  $X^2 = a$  where  $a \geq 0$ , then  $X = \pm \sqrt{a}$

**Example 1** Find each of the following in the simplest form :

$$1 \sqrt{36}$$

$$2 \sqrt{\frac{16}{25}}$$

$$3 \pm \sqrt{2\frac{1}{4}}$$

$$4 \sqrt{\left(-\frac{2}{7}\right)^2}$$

$$5 \sqrt{0.25}$$

$$6 \pm \sqrt{\frac{36}{10}}$$

$$7 \sqrt{16+9}$$

$$8 \sqrt{100-36}$$

$$9 \sqrt{\frac{36a^8}{49d^4}}$$

**Solution**

$$1 \sqrt{36} = 6 \text{ because } 6^2 = 36$$

$$2 - \sqrt{\frac{16}{25}} = -\frac{4}{5} \text{ because } \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$3 \pm \sqrt{2\frac{1}{4}} = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2}$$

$$4 \sqrt{\left(-\frac{2}{7}\right)^2} = \left|-\frac{2}{7}\right| = \frac{2}{7}$$

$$5 - \sqrt{0.25} = -\sqrt{\frac{25}{100}} = -\frac{5}{10} = -\frac{1}{2}$$

$$6 \pm \sqrt{\frac{36}{10}} = \pm \sqrt{\frac{36}{100}} = \pm \frac{6}{10} = \pm \frac{3}{5}$$

$$7 \sqrt{16+9} = \sqrt{25} = 5$$

$$8 \sqrt{100-36} = \sqrt{64} = 8$$

$$9 \sqrt{\frac{36a^8}{49d^4}} = \frac{6a^4}{7d^2}$$

**Notice that :**

When there is an addition or a subtraction operation under the square root, it must be performed first before finding the square root

## Unit 1

## TRY 1

Complete the following :

1)  $\sqrt{64} =$

2)  $-\sqrt{900} =$

3)  $-\sqrt{\frac{36}{25}} =$

4)  $\pm \sqrt{6\frac{1}{4}} =$

5)  $\sqrt{0.64} =$

6)  $\sqrt{100-64} =$

## Remark

In some cases , it is more easy to use factorization in finding the square root of a number , to do that we factorize the given number into its prime factors , then we take one factor from each two equal factors , then the product of these taken factors is the square root of this number.

Example 2 Find :  $\sqrt{441}$ 

$$\begin{aligned} \therefore 441 &= 3 \times 3 \times 7 \times 7 \\ \therefore \sqrt{441} &= \underbrace{3} \times \underbrace{7} \\ &= 21 \end{aligned}$$

$$\begin{array}{r|l} 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ 7 & 7 \\ & 1 \end{array}$$

## Example 3 Simplify each of the following to the simplest form :

1  $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times (\frac{2}{7})^2$

2  $(-\frac{3}{2})^2 \times \sqrt{\frac{64}{9}} \times (\frac{5}{2})^0$

3  $(2\frac{7}{9})^2 + \sqrt{\frac{25}{9}}$

## Solution

1  $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times (\frac{2}{7})^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$

2  $(-\frac{3}{2})^2 \times \sqrt{\frac{64}{9}} \times (\frac{5}{2})^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$

$$\begin{aligned} 3 \quad (2\frac{7}{9})^2 + \sqrt{\frac{25}{9}} &= (\frac{25}{9})^2 + \frac{5}{3} = ((\frac{5}{3})^2)^2 + \frac{5}{3} \\ &= (\frac{5}{3})^4 + \frac{5}{3} = (\frac{5}{3})^{4-1} = (\frac{5}{3})^3 = \frac{125}{27} \end{aligned}$$



## TRY 2

Simplify to the simplest form :

$$① \left(\frac{2}{3}\right)^2 \times \sqrt{\frac{81}{16}} \times \left(\frac{7}{9}\right)^0$$

$$② \frac{5}{7} \times \sqrt{\frac{49}{36}} + \left(-\frac{5}{3}\right)^2$$

## Example 4

The base length of a triangle is 16 cm. and its corresponding height is 8 cm. Find the side length of a square having the same area of that triangle.

∴ The area of the triangle =  $\frac{1}{2}$  of the base length  $\times$  its corresponding height.

∴ The area of the triangle =  $\frac{1}{2} \times 16 \times 8 = 64 \text{ cm}^2$

∴ The area of the square =  $64 \text{ cm}^2$

∴ The side length of the square =  $\sqrt{64} = 8 \text{ cm}$

## TRY 3

The area of a square is  $144 \text{ cm}^2$  Find its perimeter

3 The perimeter of square = 4.8 cm

2 1 :

(2) 0.3

4  $\frac{2}{3}$

(5) 0.8

1 1 8

(2) - 30

(6) 6

(3) 6

Answers of try by yourself

$$3 \text{ apples} + x = 5 \text{ apples}$$

$$x = ?$$

7

## Solving equations in ①

**The equation:**

is a mathematical statement which contains one variable as  $x$  (or more as  $x$  and  $y$ ) and contains equality relation  $=$ .

as :  $2x = 6$  ,  $x + 3 = 5$  ,  $2x - y = 3$  and  $x^2 = 25$

**The degree of the equation:**

is determined by the highest degree of the terms forming the equation.

For example:

- $5x + 2 = 7$  is an equation of the **first degree** in one unknown  $x$
- $x^2 + x - 3 = 0$  is an equation of the **second degree** in one unknown  $x$
- $2x + 3y = 5$  is an equation of the **first degree** in two unknowns  $x$  and  $y$

**The substitution set:**

is the set that contains the probable values of the unknown.

**The solution set (The S.S.):**

is the set whose elements satisfy the equality of the equation and it is a subset of the substitution set.



## Lesson Seven

For example:

If  $X + 3 = 5$  and the substitution set is  $\{2, 3\}$

• Putting  $X = 2$ , we get that the left side =  $2 + 3 = 5$  = the right side

i.e.  $X = 2$  is a solution of the equation.

• Putting  $X = 3$ , we get that the left side =  $3 + 3 = 6 \neq$  the right side.

i.e.  $X = 3$  is not a solution of the equation.

$\therefore$  The S.S. =  $\{2\}$  and it is a subset of the substitution set  $\{2, 3\}$

- The previous method for solving the equation is called substitution method and we notice that it is a long method and it may be impossible if the number of elements of the substitution set is infinite as we see in  $\mathbb{R}, \mathbb{Z}, \mathbb{Q}$

Therefore, we will use another easier method that will need studying the properties of the equality relation to enable us to put the unknown  $X$  in one side of the equation alone.

### The properties of the equality relation

- We can add any rational number to both sides of the equation.

For example: If  $X - 1 = 5$

, then  $X - 1 + 1 = 5 + 1$

i.e.  $[X = 6]$

- We can subtract any rational number from both sides of the equation.

For example: If  $X + 3 = 2$

, then  $X + 3 - 3 = 2 - 3$

i.e.  $[X = -1]$

- We can multiply both sides of the equation by the same rational number.

For example: If  $\frac{1}{5} X = 2$

, then  $\frac{1}{5} X \times 5 = 2 \times 5$

i.e.  $[X = 10]$

- We can divide both sides of the equation by the same rational number not equal to zero.

For example: If  $7 X = 14$

, then  $\frac{7 X}{7} = \frac{14}{7}$

i.e.  $[X = 2]$

Then by applying any of the previous properties on any equation, then we will get an equivalent equation to the origin equation that has the same solution.

Generally: If  $a, b$  and  $c$  are three rational numbers, then these numbers have the following properties:

- |  |                                |
|--|--------------------------------|
| 1 If $a = b$                             | then $a + c = b + c$           |
| 2 If $a + c = b + c$                     | then $a = b$                   |
| 3 If $a = b$                             | then $a \times c = b \times c$ |
| 4 If $a \times c = b \times c, c \neq 0$ | then $a = b$                   |

## Unit 1

The following examples show how to use the equality properties to solve an equation of the first degree in one unknown.

**Example 1** Find the solution set of the equation  $X + 5 = 4$  if the substitution set is :

1  $\mathbb{Z}$

2  $\mathbb{N}$

**Solution**

1 If the substitution set is  $\mathbb{Z}$

$$X + 5 = 4$$

Adding  $-5$  to both sides

( $-5$  is the additive inverse of  $5$ )

$$\therefore X + 5 + (-5) = 4 + (-5)$$

$$\text{i.e. } X + 0 = -1$$

$$\text{i.e. } X = -1$$

You can check the truth of your solution by substituting by  $X = -1$  in the origin equation  $\therefore$  you will get the left side  $= -1 + 5 = 4 =$  the right side

$$\therefore \text{The S.S.} = \{-1\}$$

2 If the substitution set is  $\mathbb{N}$

$$X + 5 = 4$$

$$\therefore X + 5 - 5 = 4 - 5$$

The subtraction  $(4 - 5)$  is impossible in  $\mathbb{N}$

$$\therefore \text{The S.S. in } \mathbb{N} \text{ is } \emptyset$$

**Another method :**

You can imagine that  $5$  is moved from the left side to the right side and became  $-5$

$$X + 5 = 4 \Rightarrow X = 4 - 5$$

Subtracting  $5$  from both sides

$$\therefore X = 4 - 5$$

**Example 2** Find the solution set of each of the following equations in  $\mathbb{Q}$  :

1.  $2X - 5 = 13$

2.  $2\frac{1}{2} - \frac{3}{2}X = 5$

**Solution**

1  $\therefore 2X - 5 = 13$

Adding  $5$  to both sides

( $5$  is the additive inverse of  $(-5)$ )

$$\therefore 2X - 5 + 5 = 13 + 5$$

$$\text{i.e. } 2X = 18$$

Dividing both sides by  $2$

$$\therefore \frac{2X}{2} = \frac{18}{2}$$

$$\therefore X = 9 \quad \therefore \text{The S.S.} = \{9\}$$

"Check the truth of the solution"

**Another method :**

You can imagine that  $2$  is moved from the left side to the right side and it became divisor

$$2X = 18 \Rightarrow X = \frac{18}{2}$$



## Lesson Seven

$$2. \quad 2\frac{1}{2} - \frac{3}{2}x = 5$$

Subtracting  $2\frac{1}{2}$  from both sides

$$\therefore 2\frac{1}{2} - \frac{3}{2}x - 2\frac{1}{2} = 5 - 2\frac{1}{2}$$

$$\therefore -\frac{3}{2}x = 2\frac{1}{2}$$

$$\frac{3}{2}x = -\frac{5}{2}$$

Multiplying both sides by  $(-\frac{2}{3})$  ( $-\frac{2}{3}$  is the multiplicative inverse of  $\frac{3}{2}$ )

$$\therefore \frac{3}{2}x \times (-\frac{2}{3}) = -\frac{5}{2} \times (-\frac{2}{3})$$

$$\therefore x = -\frac{5}{3}$$

$$\therefore \text{The S.S.} = \{-\frac{5}{3}\}$$

"Check the truth of the solution"

**Example 3** Find the S.S. of each of the following equations :

$$1. \quad 2(x+3) = 4, \text{ where } x \in \mathbb{Z}$$

$$2. \quad 5(x+2) - 1 = 19, \text{ where } x \in \mathbb{Q}$$

**Solution**

$$1. \quad \because 2(x+3) = 4$$

Dividing both sides by 2

$$\therefore \frac{2(x+3)}{2} = \frac{4}{2}$$

$$\therefore x+3 = 2$$

Adding  $(-3)$  to both sides

$$\therefore x+3-3 = 2-3$$

$$\therefore x = -1$$

$$\therefore \text{The S.S.} = \{-1\}$$

$$2. \quad \because 5(x+2) - 1 = 19$$

Using the distribution property

$$5(x+2) - 1 = 19 \Rightarrow 5x + 9 = 19$$

and  $5x + 9 = 19$  are equivalent equations.

$$\therefore 5x + 10 - 1 = 19$$

$$\therefore 5x + 9 = 19$$

Adding  $(-9)$  to both sides

$$\therefore 5x + 9 - 9 = 19 - 9$$

$$\therefore 5x = 10$$

Dividing both sides by 5

$$\therefore \frac{5x}{5} = \frac{10}{5}$$

$$\therefore x = 2$$

$$\therefore \text{The S.S.} = \{2\}$$

## Unit 1

**Example 4** Find in  $\mathbb{Q}$  the solution set of each of the following equations :

1  $3x + 4 = 2(x + 1)$

2  $2(x + 3) - (x - 2) = 4(x - 1) + 3$

**Solution**

Notice that the variable ( $x$ ) exists in the two sides , then we try to collect it in one side (say the left side)

1  $\therefore 3x + 4 = 2(x + 1)$

Using the distribution property

$\therefore 3x + 4 = 2x + 2$

Subtracting  $2x$  from both sides

$\therefore 3x - 2x + 4 = 2x - 2x + 2$

$\therefore x + 4 = 2$

Subtracting 4 from both sides

$\therefore x + 4 - 4 = 2 - 4$

$\therefore x = -2$

$\therefore$  The S.S. =  $\{-2\}$

2  $2(x + 3) - (x - 2) = 4(x - 1) + 3$

Using the distribution property  $\therefore 2x + 6 - x + 2 = 4x - 4 + 3$

$\therefore x + 8 = 4x - 1$

Subtracting  $x$  from both sides :  $\therefore x - x + 8 = 4x - x - 1$

$\therefore 8 = 3x - 1$

Adding 1 to both sides :

$\therefore 8 + 1 = 3x - 1 + 1$

$\therefore 9 = 3x$

Dividing both sides by 3

$\therefore \frac{9}{3} = \frac{3x}{3}$

$\therefore 3 = x$

$\therefore$  The S.S. =  $\{3\}$

**Another method :**

$(-4)$

$3x + 4 = 2x + 2$

$(-2x)$

$\therefore 3x - 2x = 2 - 4$

i.e.  $x = -2$

**Another method :**

$(+1)$

$x + 8 = 4x - 1$

$(-x)$

$\therefore 8 + 1 = 4x - x$

i.e.  $9 = 3x$

**TRY 1**

Find the solution set of each of the following equations :

1  $x - 5 = 2$  , where  $x \in \mathbb{I}$

2  $2x + 11 = 3$  , where  $x \in \mathbb{I}$

3  $2x - 3 = 5x + 6$  , where  $x \in \mathbb{Q}$



## Lesson Seven

## Using equations in solving word problems

To solve the word problems in algebra, we translate the verbal statements into algebraic symbols and expressions, and the following table shows some examples for that.

Verbal statement	Algebraic expression
• Two numbers, their sum is 9	$x, 9 - x$
• Two numbers, the difference between them is 4	$x, x - 4$ (or $x, x + 4$ )
• Two numbers, their product is 10	$x, \frac{10}{x}$
• Two numbers, one of them is twice the other	$x, 2x$ (or $x, \frac{1}{2}x$ )
• Two numbers, one of them is third of the other	$x, \frac{1}{3}x$ (or $x, 3x$ )
• Eight subtracted from three times of a number.	$3x - 8$
• Two numbers, one of them increases than twice of the other by 5	$x, 2x + 5$
• Three consecutive integers.	$x, x + 1, x + 2$
• Three consecutive even numbers.	$x, x + 2, x + 4$
• Three consecutive odd numbers	$x, x + 2, x + 4$

**Example 5** Two natural numbers, one of them is thrice of the other. If the sum of them is 16, find the two numbers.

**Solution**

- We give one of the two number the symbol  $x$
- Using the information given, we form a first degree equation in one unknown.
  - $\therefore$  The other number is thrice of the number  $x$
  - $\therefore$  The other number =  $3x$
  - $\therefore$  The sum of the two numbers = 16
  - $\therefore$  The equation is  $x + 3x = 16$
- We solve the equation we get to find the value of the unknown.
  - $\therefore x + 3x = 16 \quad \therefore 4x = 16 \quad \text{Dividing by 4: } \therefore x = 4$
  - i.e. one of the two numbers = 4, the other number =  $3 \times 4 = 12$
- We make sure that the solution is right by using the problem itself, not by using the equation.
  - $\therefore 12$  is the thrice of 4,  $12 + 4 = 16 \quad \therefore$  The solution is true.

## Unit 1

**Example 6** Three natural consecutive odd numbers whose sum is 27, find these numbers.

**Solution**

Let the smallest odd number =  $X$

∴ Each odd number exceeds the odd number just before it by 2

∴ The next odd number =  $X + 2$  and the third odd number =  $X + 4$

∴ The sum of the numbers = 27

$$\therefore X + (X + 2) + (X + 4) = 27$$

$$\therefore 3X + 6 = 27$$

$$\therefore 3X = 27 - 6$$

$$\therefore 3X = 21$$

$$\therefore X = \frac{21}{3}$$

$$\therefore X = 7$$

i.e. The numbers are 7, 9 and 11

**To check the solution :**

The numbers 7, 9 and 11 are natural consecutive odd numbers

$$\therefore 7 + 9 + 11 = 27$$

∴ The solution is true.

**Remember that**

- The perimeter of a rectangle = 2 (length + width)
- The perimeter of a square = side length  $\times$  4
- The perimeter of the triangle = the sum of its side lengths
- The area of the triangle =  $\frac{1}{2}$  the base length  $\times$  the height
- The sum of measures of the interior angles of the triangle =  $180^\circ$

**Example 7** A rectangle with length equals twice its width and its perimeter = 18 cm. Find the dimensions of the rectangle.

**Solution**

Let the width of the rectangle =  $X$  cm.

∴ Its length =  $2X$  cm.

∴ The perimeter of the rectangle = 2 (length + width)

$$\therefore 18 = 2(2X + X)$$

$$\therefore 18 = 2 \times 3X$$

$$\therefore 18 = 6X$$

$$\therefore X = 3$$

i.e. The width of the rectangle = 3 cm and its length = 6 cm.

**To check the solution :**

∴ The length of the rectangle = 6 cm equals twice its width 3 cm.

∴ the perimeter of the rectangle =  $2(6 + 3) = 2 \times 9 = 18$  cm.

∴ The solution is true.

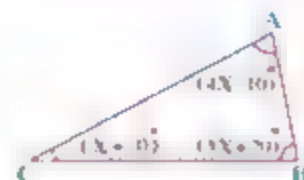


**Example 8** In the opposite figure :

ABC is a triangle in which  $m(\angle A) = (4x - 10)^\circ$

$\cdot m(\angle B) = (3x + 20)^\circ \cdot m(\angle C) = (x + 10)^\circ$

Find the measures of the angles of the triangle



**Solution**

$\therefore$  The sum of measures of the interior angles of the triangle  $= 180^\circ$

$$\therefore (4x - 10) + (3x + 20) + (x + 10) = 180$$

$$\therefore 8x + 20 = 180$$

Subtracting 20 from both sides

$$\therefore 8x = 180 - 20$$

$$\therefore 8x = 160$$

$$\therefore x = \frac{160}{8}$$

$$\therefore x = 20$$

$$\therefore m(\angle A) = (4 \times 20) - 10 = 80 - 10 = 70^\circ$$

$$\cdot m(\angle B) = (3 \times 20) + 20 = 60 + 20 = 80^\circ$$

$$\cdot m(\angle C) = 20 + 10 = 30^\circ$$

To check the solution :

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 70^\circ + 80^\circ + 30^\circ = 180^\circ$$

$\therefore$  The solution is true.



The difference between two integers is 4 and their sum is 14 , find the two numbers.

**2** The two numbers are : 9 , 5

3 1 1

2 1 4

1 1 7

Answers of try by yourself



8

Solving inequalities in  $\mathbb{Q}$ 

- We had studied before some concepts as the substitution set and the solution set in equations which they are also the same concepts for inequalities
- The solution set of the inequality is the set whose elements satisfy the inequality and it is a subset of the substitution set.

## Illustrated example

Find the solution set of the inequality  $X + 3 < 5$  if the substitution set is  $\{-1, 0, 1, 2, 3\}$ , then represent the S.S. on the number line.

## Solution

Substitute for the value of  $X$  by each element of the substitution set.

• At  $X = -1$

$\therefore -1 + 3 < 5$

$\therefore$  The left side  $= -1 + 3 = 2$

$\therefore -1$  is a solution to the inequality.

• At  $X = 0$

$\therefore 0 + 3 < 5$

$\therefore$  The left side  $= 0 + 3 = 3$

$\therefore 0$  is a solution to the inequality.

• At  $X = 1$

$\therefore 1 + 3 < 5$

$\therefore$  The left side  $= 1 + 3 = 4$

$\therefore 1$  is a solution to the inequality.

• At  $X = 2$

$\therefore 2 + 3 = 5$

$\therefore$  The left side  $= 2 + 3 = 5$

$\therefore 2$  is not a solution to the inequality.

• At  $X = 3$

$\therefore 3 + 3 > 5$

$\therefore$  The left side  $= 3 + 3 = 6$

$\therefore 3$  is not a solution to the inequality.

From the previous :

$\therefore$  The solution set  $= \{-1, 0, 1\}$



### Remark

The substitution method which is followed in the previous example sometimes be long and not easy and it may be impossible if the substitution set is infinite. Therefore we use another easier method for the solution and that requires studying the properties of inequalities.

### Properties of inequalities

We know that  $6 > -9$  is a true inequality.

But do the following operations lead to true inequalities ?

① Add 2 to the two sides of the inequality

$$\therefore 6 + 2 > -9 + 2 \longrightarrow 8 > -7 \text{ (true inequality)}$$

Generally: We can add a constant number to both sides of the inequality without change in the inequality relation.

② Subtract 7 from the two sides of the inequality

$$\therefore 6 - 7 > -9 - 7 \longrightarrow -1 > -16 \text{ (true inequality)}$$

Generally: We can subtract a constant number from the two sides of the inequality without change in the inequality relation.

③ Multiply the two sides of the inequality by 5 (positive number)

$$\therefore 6 \times 5 > -9 \times 5 \longrightarrow 30 > -45 \text{ (true inequality)}$$

Generally: Multiplying the two sides of the inequality by a positive number does not change the inequality relation.

④ Divide the two sides of the inequality by 3 (positive number)

$$\therefore \frac{6}{3} > \frac{-9}{3} \longrightarrow 2 > -3 \text{ (true inequality)}$$

Generally: Dividing the two sides of the inequality by a positive number does not change the inequality relation.

⑤ Multiply the two sides of the inequality by  $-1$  (negative number)

$$\therefore 6 \times (-1) > -9 \times (-1) \longrightarrow -6 > 9 \text{ (false inequality) because } -6 < 9$$

Generally: If we multiply the two sides of the inequality by a negative number, then we change the sign of the inequality to the opposite sign.

⑥ Divide the two sides of the inequality by  $-3$  (negative number)

$$\therefore \frac{6}{-3} > \frac{-9}{-3} \longrightarrow -2 > 3 \text{ (false inequality) because } -2 < 3$$

Generally: If we divide the two sides of the inequality by a negative number, then we change the inequality sign to the opposite sign.



## Unit 1

We can summarize the properties of inequality that noticed before as follows :  
Assuming that  $a > b$ ,  $c$  are three rational numbers , then :

- |   |                                    |
|---|------------------------------------|
| 1 If $a < b$                            | , then $a + c < b + c$             |
| 2 If $a < b$                            | , then $a - c < b - c$             |
| 3 If $a < b$ , $c$ is a positive number | , then $ac < bc$                   |
| 4 If $a < b$ , $c$ is a positive number | , then $\frac{a}{c} < \frac{b}{c}$ |
| 5 If $a < b$ , $c$ is a negative number | , then $ac > bc$                   |
| 6 If $a < b$ , $c$ is a negative number | , then $\frac{a}{c} > \frac{b}{c}$ |

## Remark

If  $a$  and  $b$  are two non-zero rational numbers have the same sign and  $a > b$  , then  $\frac{1}{a} < \frac{1}{b}$

**Example 1** Find the solution set of the inequality :  $x + 2 < 5$  , in each of the two following cases :

1 If  $x \in \mathbb{Z}$

2 If  $x \in \mathbb{N}$

, then represent the solution set on the number line in each case.

## Solution

$$x + 2 < 5$$

Subtracting 2 from the two sides

$$\therefore x + 2 - 2 < 5 - 2$$

$$\text{i.e. } x < 3$$

1 When  $x \in \mathbb{Z}$

The solution set is all the integers which are less than 3

$$\text{i.e. The S.S.} = \{2, 1, 0, -1, \dots\}$$



2 When  $x \in \mathbb{N}$

The solution set is all the natural numbers which are less than 3

$$\text{i.e. The S.S.} = \{2, 1, 0\}$$



## Lesson Eight

We notice from the previous example that :

The solution set of the inequality depends on the substitution set , we find that :

The solution set in  $\mathbb{R}$  differs from the solution set in  $\mathbb{Z}$

**Example 2** Find the solution set of the inequality :  $2x - 5 > 5$  , in each of the two following cases :

1 If  $x \in \mathbb{Q}$

2 If  $x \in \mathbb{Z}$

**Solution**

$$2x - 5 > 5$$

Adding 5 to both sides

$$\therefore 2x - 5 + 5 > 5 + 5$$

$$\therefore 2x > 10$$

Multiplying both sides by  $\frac{1}{2}$

$$\therefore \frac{1}{2} \times 2x > \frac{1}{2} \times 10$$

$$\text{i.e. } x > 5$$

1 When  $x \in \mathbb{Q}$

The S.S. is all the rational numbers which are greater than 5 , then we write it by characterized property method because it is difficult to list all its members.

$$\text{i.e. The S.S.} = \{x : x \in \mathbb{Q}, x > 5\}$$

2 When  $x \in \mathbb{Z}$

The solution set is all the integers which are greater than 5

$$\text{i.e. The S.S.} = \{6, 7, 8, \dots\}$$



**Example 3** Find in  $\mathbb{Q}$  the solution set of each of the following inequalities :

1  $4 - 2x \leq 2$

2  $7(x - 1) > 9x - 6$

**Solution**

1  $\because 4 - 2x \leq 2$

Adding  $-4$  to both sides

$$\therefore 4 + 4 - 2x \leq -4 + 2$$

$$\therefore -2x \leq -2$$

Dividing both sides by  $(-2)$

$$\therefore \frac{-2x}{-2} \geq \frac{-2}{-2} \text{ (Notice the change of inequality sign)}$$

$$\therefore x \geq 1$$

$$\text{i.e. The S.S.} = \{x : x \in \mathbb{Q}, x \geq 1\}$$

2  $\because 7(x - 1) > 9x - 6$

$$\therefore 7x - 7 > 9x - 6$$

Subtracting  $(9x)$  from both sides

$$\therefore 7x - 9x - 7 > 9x - 9x - 6$$

$$\therefore -2x - 7 > -6$$

Adding  $7$  to both sides

$$\therefore -2x - 7 + 7 > -6 + 7$$

$$\therefore -2x > 1$$

Dividing both sides by  $(-2)$

$$\therefore \frac{-2x}{-2} < \frac{1}{-2} \text{ (Notice the change of inequality sign)}$$

$$\therefore x < -\frac{1}{2}$$

$$\text{i.e. The S.S.} = \{x : x \in \mathbb{Q}, x < -\frac{1}{2}\}$$



## Lesson Eight

**Example 4** Find in  $\mathbb{Z}$  the solution set of the inequality  $-11 \leq 3x - 5 < 4$ , then represent it on the number line.

**Solution**

$$\therefore -11 \leq 3x - 5 < 4$$

Adding 5 to the three sides

$$\therefore -11 + 5 \leq 3x - 5 + 5 < 4 + 5 \qquad \therefore -6 \leq 3x < 9$$

Dividing all sides by 3

$$\therefore \frac{-6}{3} \leq \frac{3x}{3} < \frac{9}{3} \qquad \therefore -2 \leq x < 3$$

i.e. The S.S. =  $\{-2, -1, 0, 1, 2\}$



**TRY**

Find the solution set of each of the following inequalities :

- (1)  $2x - 3 \geq 5$ , where  $x \in \mathbb{Q}$
- (2)  $5x - 10 < 2x - 1$ , where  $x \in \mathbb{N}$

$$\{2, 0, 1, 2\}$$

$$(1) \{x: x \in \mathbb{Q}, x \geq 4\}$$

Answers / of try by yourself

UNIT

2

## Statistics and Probability



## ▶ Lessons of the unit :

1. Samples :
  - Systematic sample.
  - Random sample.
2. Probability :
  - Experimental probability.
  - Theoretical probability.

## ▶ Unit Objectives :

By the end of this unit, student should be able to :

- recognize the sample and how to select it.
- classify the samples according to the selecting method of its elements
- select a random sample from a society distributed randomly
- use a calculator to select a random sample
- perform a random experiment and write the sample space
- recognize the concept of the event
- calculate the probability of an event
- recognize the impossible event
- recognize the certain event.



**Pierre Simon Marquis  
de Laplace**  
(1749 - 1827)

He was a French mathematician and astronomer.

His first work was published in 1771 starting with differential equations however he had already started to think about the mathematical and philosophical concepts of probability and statistics.





1

## Samples

### Prelude

When we check the production of a factory to know if its products are manufactured according to the certain specifications , usually we don't check the whole production of this factory , but it is sufficient to check a part of this production under certain conditions , in order to represent the whole production , then we generalize the results for all the production.

This part is called "a sample".



### Definition

A sample is a small part from a large society that looks like this society and represents it well and is selected randomly.

- And notice that the selected sample should wholly represent the society (the object of study) and it shouldn't be based on a certain group and neglect the other , so that the results of the study can be near reality and we can make decisions according to these results , so we can generalise these results on the society as a whole

## Lesson One

## Types of samples

Samples are classified according to the way used in selecting its items , and in this lesson , we introduce two types of samples :

- 1 Systematic sample
- 2 Random sample.

## Systematic sample

Systematic sample is the sample whose elements are selected from the elements of a society distributed randomly by following a certain system or method in selection.

For example:

To select a systematic sample representing 10% of the marks of students in a preparatory school in the mid-year exam in maths , to study the standards of students , we will do as follows :

- 1 Students must be distributed randomly in a numbered list i.e. selection shouldn't be from certain classes as excellent students' classes or selecting certain classes and neglecting others.
- 2 We select in a regular way the tenth student in each 10 student from the students list i.e. we select the mark of the tenth , twentieth , thirtieth , ... students in the list.



## Remark

If the society (the object of study) is already divided into classes or groups as the school is divided into classes for boys and others for girls , then we select a part of each group to be represented in the sample so that the sample represents the society as a whole.

## Random sample

Random sample is the sample whose elements are selected from the elements of a society distributed randomly by following a random and irregular method of selecting

In this sample , each individual must get the same chance of selecting.

So , we can select its elements by two methods :

- Manual method.
- Using the scientific calculator.

## Unit 2

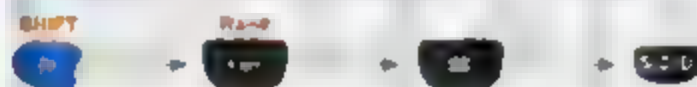
The first method (manual method) :

It is done as follows :

- 1 Every member of the society is given a number , then this number is written on a piece of paper such that all pieces of paper are of the same colour and size.
- 2 Each piece of paper is folded perfectly such that the written number does not appear and is put in a bag or a box and mixed together very well.
- 3 The selection is carried out by drawing a piece after piece without looking inside the paper till the operation of drawing is finished when the required number of selected members is done.

The second method (using the scientific calculator) :

This method depends on using the random number function on the scientific calculator shown in the opposite figure by pressing the following keys successively from the left :



Then a random number in the range 0.000 to 0.999 will appear every time. Take the apparent numbers without the decimal point. The numbers which are greater than the whole number of the society under study should be ignored and also if a number is repeated it must be ignored and taken once only.



In any survey , a 10% sample is considered adequate to provide reliable information about the whole society.

**Example**

A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.





**Solution**





∴ The number of workers in the factory = 300 workers.

∴ The number of the random sample =  $\frac{10}{100} \times 300 = 30$  workers.

Then we want to select 30 workers to do this survey. The selection operation can be carried out as follows :

- 1 Each worker in the factory is given a number from 1 to 300
- 2 Use the calculator to select 30 numbers randomly by the method mentioned before such that these numbers are included between 0 and 301 and the number that is more than 300 should be ignored.

For example:

By pressing the keys  →  →  →  successively.

- If we get the decimal 0.56 → then the number of the selected person is 56
- If we get the decimal 0.049 → then the number of the selected person is 49
- If we get the decimal [0.132] → then the number of the selected person is 132
- If we get the decimal 0.453 → it must be ignored because 453 is more than 300 and so on till we get 30 numbers.

Assuming that the calculator gave us the shown numbers in the opposite table → then the workers who carry these numbers are the selected sample to carry out this survey.

56	49	132	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103



## 2

## Probability

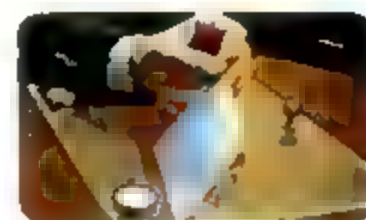
## Prelude

In our daily life , many times we ask ourselves about some affairs that may happen in the future and that we cannot give a certain result for them

## For example:

- If the Egyptian football team get to the finals of African nations championship , what is its chance for winning the cup ?
- If an Egyptian citizen puts himself up for parliamentary elections in one of elections zones , what is his chance for winning in the elections ?

All these questions' answers are expectations to what may happen (occur) in the future referring to previous experience , studies or observations.



When we answer these questions , we use some words as «may be , chance or probable».

In mathematics , we call this probability. In this lesson , we will study :

- 1 Experimental probability
- 2 Theoretical probability.

## Lesson Two

## Experimental probability

- If one of the Olympic swimmers wants to achieve a new record in the next Olympic Games , what is the probability that this swimmer achieves this record ?

The answer to this question cannot be got by expecting , hoping or by doing a survey of the opinions of the trainers or by asking the swimmer himself , but by trying.



i.e. The swimmer covers the needed distance in the race several times , then we record the number of times in which he could achieve the requested number and divide it by the total number of times , so the quotient is the probability of achieving the new record in the next Olympics.

The experimental probability depends on performing an experiment , then we record the results and use them to calculate the value of probability of an event occurrence using the rule :

$$\text{Experimental probability} = \frac{\text{Number of trials in which the outcome occurs}}{\text{Total number of trials}}$$

It is noticed that the more we carry out the experiment , the more we obtain an accurate value for the probability.

**Example 1** If we tossed a piece of coin with double face 200 times and the results of appearance of a head or a tail in each toss were recorded in a table as shown :



	Heads (H)	Tails (T)	Total
Statistics Tallies	IIII /	IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII IIII	
Frequency	106	94	200

Calculate :

- 1 The probability of appearance of a head.
- 2 The probability of appearance of a tail.



## Unit 2

## Solution

- 1 The probability of appearance of a head =  $\frac{\text{Number of getting heads}}{\text{Total number of tosses}}$   
 $= \frac{106}{200} = 0.53$
- 2 The probability of appearance of a tail =  $\frac{\text{Number of getting tails}}{\text{Total number of tosses}}$   
 $= \frac{94}{200} = 0.47$

## TRY

Roll a fair die 25 times and record the results of appearance of a number on the upper face in a table, then calculate :

- 1 The probability of appearance of the number 4
- 2 The probability of appearance of the number 3

## 2 Theoretical probability

In the previous , we carried out the experiment of tossing a piece of coin and we found that :

- The probability of appearance of a head = 0.53
- The probability of appearance of a tail = 0.47



But when we study this experiment theoretically , we find that :

If we tossed the coin piece once , then we obtain either a head or a tail.

i.e. the number of possible outcomes = 2

and there is one chance to obtain a head and also one chance to obtain a tail.

(i.e. all outcomes of the experiment have the same chance to happen).

i.e. the probability of appearance of a head =  $\frac{1}{2} = 0.50$   
 and the probability of appearance of a tail =  $\frac{1}{2} = 0.50$

## Notice that :

We can express the probability by percentage.  
 i.e. we write the probability of appearance of a head = 50%

## Lesson Two

## Remark

Notice the difference between the experimental probability of appearance of a head (0.53) and the theoretical probability of appearance of a head (0.50)

i.e. When the number of times of carrying out the experiment increases, the value of experimental probability approaches the value of theoretical probability.

## Definition of random experiment

Random experiment is an experiment in which we can specify all its possible outcomes before carrying it out but we cannot determine certainly which of them will occur

## Sample space

Sample space is the set of all possible outcomes of a random experiment and it is denoted by  $S$

For example:

- When we toss a piece of coin once, then the sample space is

$$S = \{H, T\}$$

- When we roll a fair die once observing the apparent number on the upper face, then the sample space is  $S = \{1, 2, 3, 4, 5, 6\}$



## Event

Event is a subset of the sample space.

For example:

If  $A$  is the event of appearance of an odd number when rolling a fair die once and observing the apparent number on the upper face, then  $A = \{1, 3, 5\}$ ,  $A \subset S$



## Generally

The probability of any event occurrence  $A \subset S$  is denoted by  $P(A)$  and it is given by using the relation :

$$P(A) = \frac{\text{The number of elements of the event} = n(A)}{\text{The number of elements of sample space} = n(S)} = \frac{n(A)}{n(S)}$$

## Unit 2

**Example 2** If a fair die is rolled once and we observe the apparent number on the upper face, find the probability of each of the following events :

- 1 A is the event of appearance of a number more than 4  
(Approximating the result to the nearest hundredth)
- 2 B is the event of appearance of an even number.
- 3 C is the event of appearance of a number equal to 5  
(Approximating the result to the nearest tenth)
- 4 D is the event of appearance of a number equal to 7
- 5 E is the event of appearance of a number less than 7

**Solution**

$$S = \{1, 2, 3, 4, 5, 6\}, n(S) = 6$$

$$1 \quad A = \{5, 6\}, n(A) = 2$$

$$\therefore P(A) = \frac{2}{6} = \frac{1}{3} \approx 0.33 \quad (\text{to the nearest hundredth})$$

$$2 \quad B = \{2, 4, 6\}, n(B) = 3 \quad \therefore P(B) = \frac{3}{6} = 0.5$$

$$3 \quad C = \{5\}, n(C) = 1$$

$$\therefore P(C) = \frac{1}{6} \approx 0.2 \quad (\text{to the nearest tenth})$$

$$4 \quad D = \{ \} \text{ or } \emptyset, n(D) = \text{zero}$$

$$\therefore P(D) = \frac{0}{6} = \text{zero} \quad (\text{the impossible event})$$

$$5 \quad E = \{1, 2, 3, 4, 5, 6\}, n(E) = 6$$

$$\therefore P(E) = \frac{6}{6} = 1 \quad (\text{the certain event})$$

**Remarks**

- 1 **The impossible event** : is the event that has no chance for occurring.  
i.e. the probability of the impossible event = Zero
- 2 **The certain event** : is the event that has all the possible outcomes.  
i.e. the probability of the certain event = 1
- 3 **The value of probability of any event** is not less than zero and not more than one  
i.e.  $0 \leq \text{the probability of an event occurrence} \leq 1$



## Lesson Two

**Example 3** From the set of digits {3, 4, 5}, form a two-digit number, then find the probability of each of the following events :

- 1 A « the event that the units digit is odd »
- 2 B « the event that the tens digit is even »
- 3 C « the event that the two digits are odd »
- 4 D « the event that the sum of the two digits = 8 »
- 5 E « the event that the product of the two digits = 20 »

**Solution**

$$S = \{33, 43, 53, 34, 44, 54, 35, 45, 55\}, n(S) = 9$$

- |  |   |
|--|---|
| 1 A = {33, 43, 53, 35, 45, 55}, n(A) = 6 | $\therefore P(A) = \frac{6}{9} = \frac{2}{3}$ |
| 2 B = {43, 44, 45}, n(B) = 3             | $\therefore P(B) = \frac{3}{9} = \frac{1}{3}$ |
| 3 C = {33, 53, 35, 55}, n(C) = 4         | $\therefore P(C) = \frac{4}{9}$               |
| 4 D = {53, 44, 35}, n(D) = 3             | $\therefore P(D) = \frac{3}{9} = \frac{1}{3}$ |
| 5 E = {54, 45}, n(E) = 2                 | $\therefore P(E) = \frac{2}{9}$               |

**Example 4** A bag has an amount of marbles of the same size and touch. If 2 marbles are red, 3 are blue and 5 are white and a marble is drawn randomly, calculate :

- 1 The probability of that (the drawn marble is red)
- 2 The probability of that (the drawn marble is blue)
- 3 The probability of that (the drawn marble is white)
- 4 The probability of that (the drawn marble is not blue)



## Unit 2

## Solution

The probability of occurrence of a certain outcome

$$= \frac{\text{The number of possible chances to get this outcome}}{\text{The total number of chances}}$$

$$\therefore \text{The total number of marbles} = 2 + 3 + 5 = 10$$

1 The probability of that (the drawn marble is red)

$$= \frac{\text{The number of red marbles}}{\text{The total number of marbles}} = \frac{2}{10} = \frac{1}{5}$$

2 The probability of that (the drawn marble is blue)

$$= \frac{\text{The number of blue marbles}}{\text{The total number of marbles}} = \frac{3}{10}$$

3 The probability of that (the drawn marble is white)

$$= \frac{\text{The number of white marbles}}{\text{The total number of marbles}} = \frac{5}{10} = \frac{1}{2}$$

4 The probability of that (the drawn marble is not blue)

$$= \frac{\text{The number of marbles which aren't blue}}{\text{The total number of marbles}} = \frac{10 - 3}{10} = \frac{7}{10}$$

## Remark 1

In the previous example , notice that :

$$P(\text{red marble}) = \frac{2}{10}, P(\text{blue marble}) = \frac{3}{10},$$

$$P(\text{white marble}) = \frac{5}{10} \therefore \frac{2}{10} + \frac{3}{10} + \frac{5}{10} = 1$$

$\therefore$  The sum of probabilities of all outcomes of a random experiment = 1

So , if the probability of occurrence of an event is  $a$  , then the probability that it doesn't occur =  $1 - a$

So , we can find the probability that the drawn marble is not blue as follows .

The probability that the drawn marble is not blue

$$= 1 - \text{the probability that it is blue} = 1 - \frac{3}{10} = \frac{7}{10}$$

## Lesson Two

**Example 5** A Class has some students who wear glasses and other students who don't wear glasses. If a student is chosen randomly from this class and the probability that this student wears glasses is 0.1



- 1 Find the probability that this student does not wear glasses.
- 2 If the number of students in this class is 30 students , find the expected number of students who wear glasses.

**Solution**

- 1 The probability that this student does not wear glasses  
 $= 1 - \text{the probability that the student wears glasses} = 1 - 0.1 = 0.9$
- 2  $\therefore$  The expected number of outcomes of an event = the probability of occurrence of this event  $\times$  the total number of all possible outcomes.  
 $\therefore$  The expected number of students who wear glasses  $= 0.1 \times 30$   
 $= 3 \text{ students.}$

**Example 6** A spinner game was divided into some equal sectors. 2 of them are green , 4 are blue and the rest are red. If the probability that the pointer stops pointing at a green sector is  $\frac{1}{6}$  , then find the number of red sectors.

**Solution**

- $\therefore$  The probability that the pointer stops pointing at a green sector  
 $= \frac{\text{The number of green sectors}}{\text{The number of all sectors}}$   
 $\therefore \frac{1}{6} = \frac{2}{\text{The number of all sectors}}$   
 $\therefore$  The number of all sectors  $= 2 \times 6 = 12 \text{ sectors.}$   
 $\therefore$  The number of red sectors  $= 12 - (2 + 4) = 6 \text{ sectors.}$



## Unit 2

## TRY 2

- 1) A box contains cards numbered from 1 to 15. If a card is drawn randomly, what is the probability that the written number on the card is divisible by 5?
- 2) An experiment has 3 outcomes. If the probability of occurrence of the first outcome is 0.3 and the probability of the second is 0.45, calculate the probability of the third outcome.
- 3) A farm has 2000 cows. If the probability of cow madness infection in this farm is 0.17, what is the expected number of infected cows?

- 1 Answer by yourself after doing experiment.
- 2 The probability that the written number on the card is divisible by 5 =  $\frac{15}{15} = \frac{1}{3}$
- 2 The probability of the third outcome =  $1 - (0.3 + 0.45) = 0.25$
- 3 The expected number of infected cows = 340 cows

or try by yourself

Answers

Second

# Geometry and Measurement



UNIT

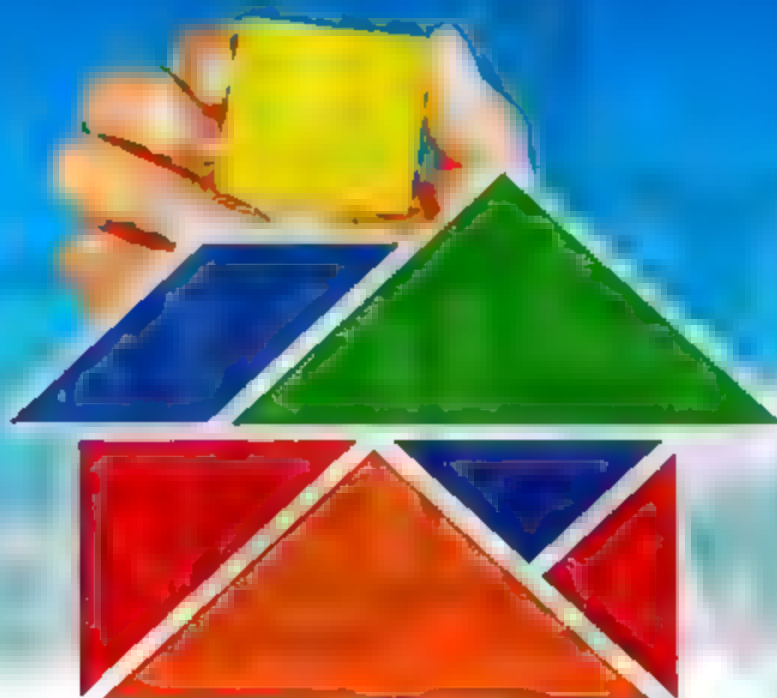
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Geometry and Measurement .... 64

UNIT

3

## Geometry and Measurement





### ► Lessons of the unit :

1. Deductive proof.
2. The polygon.
3. The parallelogram and its properties.
4. The special cases of the parallelogram.
5. The triangle : - Theorem (1) , exterior angle of the triangle.
6. Theorem (2) , theorem (3).
7. Pythagoras' theorem.
8. Geometric transformations.
9. Reflection in a straight line.
10. Reflection in a point.
11. Translation.
12. Rotation.

### ► Unit Objectives :

By the end of this unit, student should be able to :

- use the deductive proof to prove the verity of the theorems.
- recognize the polygon and the difference between the convex polygon and the concave polygon
- find the sum of the measures of the interior angles and the exterior angles of any polygon
- recognize the regular polygon and find the measure of its interior angle.
- recognize the parallelogram and its properties.
- deduce when the quadrilateral be a parallelogram
- recognize the special cases of the parallelogram (Rectangle - Rhombus - Square).
- deduce that the sum of measures of the interior angles of a triangle is  $180^\circ$
- recognize the exterior angle of a triangle and its measure.
- deduce the relation between the length of the line segment joining the midpoints of two sides of a triangle and the length of the third side
- recognize the Pythagoras' theorem.
- recognize the properties of the reflection in a straight line, the reflection in a point, the translation and the rotation
- find an image of a geometric figure by using the reflection, the translation and the rotation



**Euclid**

Euclid  
(325 B.C. - 265 B.C.)

He was a Greek mathematician. He lived in Alexandria. Euclid introduced the system of axioms. Since this time, geometry of Euclid was considered a model of logical proof.

#### Euclid's Axioms (notations) :

- Things which are equal to one thing are equal to each other.
- If equals are added to equals, then the sums are equal.
- Things which coincide with one another are equal to each other.
- The whole is greater than the part



## 1

## Deductive proof

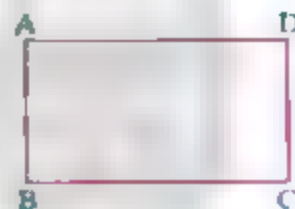
- Deductive proof is a theoretical method to prove theorems and reaching to results. In a deductive proof, we do not need to use geometric instruments in measuring, but we use definitions, properties, facts and previous theorems to get results, by writing mathematical statements, which means that when we write any statement, we have to mention the reason for which this statement is true.

For example:

If you know that

ABCD is a rectangle,

you can write the following



Mathematical statement	Reason
• ABCD is a rectangle	Given
• $AB = CD$	opposite sides in the rectangle are equal in length
• $m(\angle B) = 90^\circ$	Angles of the rectangle are right
• $\overline{AD} \parallel \overline{BC}$	opposite sides in the rectangle are parallel

### How to write the proof in geometry?

- Read carefully the problem, in order to determine what is "given", which are all information given in the problem, and "required", which is the question we need to answer in the problem.

- 2 Use the given information in the problem to draw a clear geometric figure (if the figure is not already given) and put on the figure the given information in the problem as : side lengths , angles measures and so on.
- 3 Write the given in form of points.
- 4 Write the required to be proved.
- 5 Think about a plan for the proof which is the main steps needed in order to arrive to the required.
- 6 Write the proof by writing a mathematical statements mention for each statement a reason that makes this statement is true.
- 7 Be sure that you answered the question of the problem.

In the following, some examples showing how to write the deductive proof :

- 1 If two straight lines intersect , then the measures of each two vertically opposite angles are equal.

**Given**  $\overline{AB}$  and  $\overline{CD}$  are two straight lines intersecting at E

**Required to prove (R.T.P)**  $m(\angle AED) = m(\angle BEC)$



**Proof**

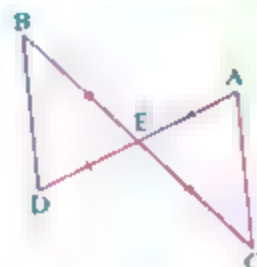
$\therefore \angle AED$  and  $\angle AEC$  are two adjacent angles  
 where  $\overline{EC} \cup \overline{ED} = \overline{CD}$   
 $\therefore m(\angle AED) + m(\angle AEC) = 180^\circ$   
 $\therefore \angle AEC$  and  $\angle BEC$  are two adjacent angles  
 where  $\overline{EA} \cup \overline{EB} = \overline{AB}$   
 $\therefore m(\angle AEC) + m(\angle BEC) = 180^\circ$   
 $\therefore m(\angle AED) + m(\angle AEC) = m(\angle AEC) + m(\angle BEC)$   
 $\therefore m(\angle AED) = m(\angle BEC)$  (Q.E.D.)

Similarly , you can prove that  $m(\angle AEC) = m(\angle BED)$

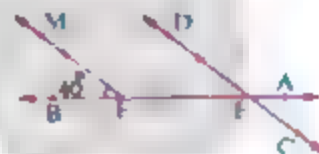
• Q.E.D. is an abbreviation for quod erat demonstrandum  
 It is a Latin abbreviation which means to be demonstrated



## Unit 3

**Example 1** In the opposite figure : $\overline{AD} \cap \overline{BC} = \{E\}$  where  $AE = DE$  and  $BE = CE$ Prove that :  $\triangle AEC \cong \triangle DEB$ **Solution****Given** $\overline{AD} \cap \overline{BC} = \{E\}$  where  $AE = DE$  ,  $BE = CE$ **R.T.P.** $\triangle AEC \cong \triangle DEB$ **Proof** $\therefore \overline{AD} \cap \overline{BC} = \{E\}$  $\therefore m(\angle AEC) = m(\angle DEB)$  (V.O.A) $\therefore$  In  $\triangle AEC$  and  $\triangle DEB$  : $\begin{cases} AE = DE \text{ (given)} \end{cases}$  $\begin{cases} CE = BE \text{ (given)} \end{cases}$  $\begin{cases} m(\angle AEC) = m(\angle DEB) \text{ (by proof)} \end{cases}$  $\therefore \triangle AEC \cong \triangle DEB$ 

(Q.E.D)

**In the opposite figure :** $\overline{AB} \cap \overline{CD} = \{E\}$  ,  $\overline{CD} \parallel \overline{FM}$  ,  $F \in \overline{AB}$  and  $m(\angle MFB) = 40^\circ$ Complete the following proof to find :  $m(\angle AEC)$ **Given****Required to find (R.T.F.)****Proof** $\therefore \overline{CD} \parallel \dots$  (given) and  $\overline{AB}$  is a transversal. $\therefore m(\angle DEB) = m(\angle \dots) = 40^\circ$  (corresponding angles) $\therefore \overline{AB} \cap \overline{CD} = \{E\}$  $\therefore m(\angle AEC) = m(\angle \dots)$  (V.O.A) $\therefore m(\angle AEC) = \dots^\circ$ 

(The req)

## Lesson One

**2** The sum of the measures of the accumulative angles at a point is equal to  $360^\circ$

**Given**  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$ ,  $\overrightarrow{OC}$  and  $\overrightarrow{OD}$  are rays that start at O

**R T P** The sum of the measures of the accumulative angles at O is  $360^\circ$

**Construction** Draw  $\overrightarrow{DO}$  and  $E \in \overrightarrow{DO}$

**Proof**

$$\begin{aligned} \therefore m(\angle EOB) + m(\angle BOA) + m(\angle AOD) &= 180^\circ \\ m(\angle EOC) + m(\angle COD) &= 180^\circ \\ \therefore m(\angle EOB) + m(\angle BOA) + m(\angle AOD) + m(\angle EOC) \\ &\quad + m(\angle COD) = 180^\circ + 180^\circ = 360^\circ \\ \therefore m(\angle AOB) + m(\angle BOC) + m(\angle COD) + m(\angle DOA) &= 360^\circ \end{aligned}$$

(Q.E.D.)

**Example 2** In the opposite figure :

$$m(\angle BAC) = 80^\circ, m(\angle DCE) = 120^\circ$$

$$\text{and } m(\angle ACE) = 140^\circ$$

Prove that :  $\overline{AB} \parallel \overline{CD}$

**Solution**

**Given**  $m(\angle BAC) = 80^\circ, m(\angle DCE) = 120^\circ,$   
 $m(\angle ACE) = 140^\circ$

**R.T.P.**  $\overline{AB} \parallel \overline{CD}$

**Proof**  $\therefore m(\angle DCA) + m(\angle DCE) + m(\angle ACE) = 360^\circ$   
 (accumulative angles at C)

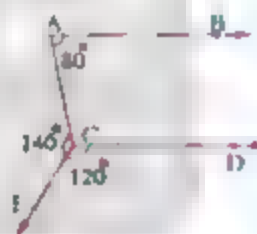
$$\therefore m(\angle DCA) = 360^\circ - (120^\circ + 140^\circ) = 100^\circ$$

$$\therefore m(\angle BAC) + m(\angle DCA) = 80^\circ + 100^\circ = 180^\circ$$

And they are interior angles in the same side of the transversal  $\overline{AC}$

$$\therefore \overline{AB} \parallel \overline{CD}$$

(Q.E.D.)



## Unit 3

## TRY 2

In the opposite figure :

$$m(\angle AOB) = 130^\circ, m(\angle BOC) = 70^\circ,$$

$$m(\angle COD) = 60^\circ \text{ and } \overline{OE} \text{ bisects } \angle AOD$$

Complete the following proof to prove that :

$\overline{OE}$  and  $\overline{OB}$  are on one straight line.



Given

R.T.P.

Proof  $\therefore m(\angle AOB) + m(\angle BOC) + m(\angle COD) + m(\angle AOD) = \quad^\circ$

(Accumulative angles at O)

$$\therefore m(\angle AOD) = \quad^\circ - \quad^\circ = \quad^\circ$$

$$\therefore \overline{OE} \text{ bisects } \angle \quad \text{ (given)}$$

$$\therefore m(\angle AOE) = \frac{1}{2} m(\angle \quad)$$

$$\therefore m(\angle AOE) = \frac{1}{2} \times \quad^\circ = \quad^\circ$$

$$\therefore m(\angle AOE) + m(\angle AOB) = \quad^\circ + \quad^\circ = \quad^\circ$$

$$\therefore \overline{OE} \text{ and } \overline{OB} \text{ are on one straight line.}$$

(Q.E.D.)

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2

## The polygon

## The polygon

It is a simple closed line that consists of three line segments , or more. The polygon is named according to the number of its sides.

Notice that :

The simple line is the line that does not cut itself

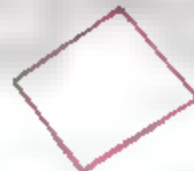
Examples for some polygons :

3 sides



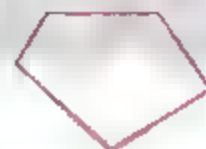
Triangle

4 sides



Quadrilateral

5 sides



Pentagon

6 sides



Hexagon

7 sides



Heptagon

8 sides

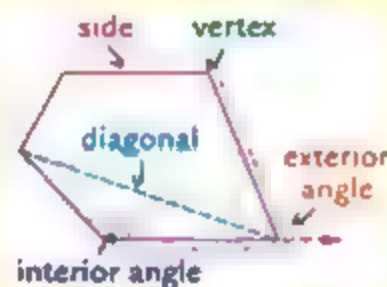


Octagon

## Unit 3

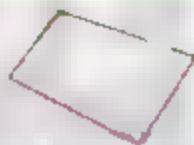
## Remarks

- 1 Each line segment of the line segments forming the polygon is called a **side**.
- 2 Each point resulted from intersecting of two adjacent sides of the polygon is called a **vertex**.
- 3 The sum of the side lengths of the polygon is called the **perimeter of the polygon**.
- 4 Each line segment joining two non-adjacent vertices of the polygon is called a **diagonal**.
- 5 The included angle between two adjacent sides of the polygon is called an **interior angle**.
- 6 The included angle between a side of the polygon and the extension of its adjacent side is called an **exterior angle**.
- 7 The number of sides of any polygon = the number of its vertices = the number of its interior angles.



## Convex polygon and concave polygon

- The polygon is **convex** if the measure of any of its interior angles is less than  $180^\circ$ .



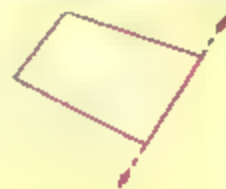
- The polygon is **concave** if the measure of one of its interior angles at least is greater than  $180^\circ$  "reflex angle".



## Remarks

- In the **convex polygon** :

If a straight line is drawn to pass through any two consecutive vertices, then the remaining vertices lie on one side of this straight line.



- In the **concave polygon** :

There are straight lines passing through two consecutive vertices and the remaining vertices lie on two different sides of the straight lines.







**The sum of measures of the interior angles of the polygon**

We knew before that : The sum of measures of the interior angles of the triangle equals  $180^\circ$

We can use that to deduce a general rule to find the sum of measures of the interior angles of any polygon whose number of sides is  $n$

If we draw from any vertex of the polygon all diagonals that pass through this vertex , then the surface of this polygon will be divided into a number of triangles as shown in the following table :

The polygon	The number of its sides	The number of the resulted triangles	The sum of measures of the interior angles of the polygon
 quadrilateral	4	2	$2 \times 180^\circ = 360^\circ$
 pentagon	5	3	$3 \times 180^\circ = 540^\circ$
 hexagon	6	4	$4 \times 180^\circ = 720^\circ$
 heptagon	7	5	$5 \times 180^\circ = 900^\circ$

From the previous , you notice that :

The number of resulted triangles = the number of the sides of the polygon - 2

Generally : If we draw all the possible diagonals that out of a vertex of a polygon having  $(n)$  sides, then the surface of this polygon will be divided into  $(n-2)$  triangles.

$\therefore$  The sum of measures of the interior angles of the triangle =  $180^\circ$

$\therefore$  The sum of measures of the interior angles of a polygon of  $n$  sides equals  $(n-2) \times 180^\circ$



## Unit 3

For example:

- The sum of measures of the interior angles of the octagon  $= (8 - 2) \times 180^\circ = 1080^\circ$
- The sum of measures of the interior angles of the enneagon (nonagon)  $= (9 - 2) \times 180^\circ = 1260^\circ$

**Example 1** Complete the following table :

Number of sides of the polygon	10	3	12	15
Sum of measures of its interior angles	.....			

Number of sides of the polygon	10	3	12	15
Sum of measures of its interior angles	$8 \times 180^\circ = 1440^\circ$	$1 \times 180^\circ = 180^\circ$	$10 \times 180^\circ = 1800^\circ$	$13 \times 180^\circ = 2340^\circ$

**Solution**

**Example 2** The sum of measures of the interior angles of a polygon is  $2160^\circ$ . Find the number of its sides.

**Solution**

$\therefore$  The sum of measures of the interior angles of a polygon of  $n$  sides equals  $(n - 2) \times 180^\circ$

$$\therefore 2160^\circ = (n - 2) \times 180^\circ \quad \therefore n - 2 = \frac{2160}{180} = 12 \quad \therefore n = 14$$

$\therefore$  The number of sides of this polygon is 14 sides



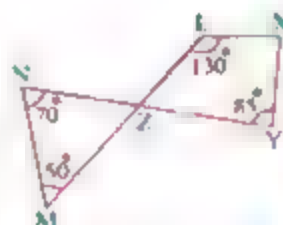
Complete the following table :

Number of sides of the polygon	11	16
Sum of measures of its interior angles	$900^\circ$	$540^\circ$

**Final answers**  
of try by yourself  
questions are at the  
end of each lesson  
to check your answer.

**Example 3** In the opposite figure :

$\overline{LM} \cap \overline{YN} = \{Z\}$ ,  $m(\angle M) = 50^\circ$ ,  
 $m(\angle N) = 70^\circ$ ,  $m(\angle Y) = 85^\circ$  and  
 $m(\angle L) = 130^\circ$   
Find :  $m(\angle X)$



## Lesson Two

## Solution

Given

$$m(\angle M) = 50^\circ, m(\angle N) = 70^\circ, m(\angle Y) = 85^\circ, m(\angle L) = 130^\circ$$

R.T.F.

$$m(\angle X)$$

Proof

In  $\triangle ZMN$ :

$$\therefore m(\angle M) = 50^\circ, m(\angle N) = 70^\circ$$

$$\therefore m(\angle NZM) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$$

$$\therefore m(\angle LZY) = m(\angle NZM)$$

(V.O.A)

$$\therefore m(\angle LZY) = 60^\circ$$

 $\therefore$  The figure XYZL is a quadrilateral.

$$\therefore \text{The sum of measures of its interior angles} = (4 - 2) \times 180^\circ$$

$$= 2 \times 180^\circ = 360^\circ$$

$$\therefore m(\angle X) = 360^\circ - (130^\circ + 85^\circ + 60^\circ) = 85^\circ$$

(The req)

**Example 4** If the ratio among the measures of the interior angles of a quadrilateral is  $2 : 3 : 3 : 4$ , find the smallest measure of these angles of that quadrilateral.

## Solution

 $\therefore$  The ratio among the measures of the interior angles of a quadrilateral is  $2 : 3 : 3 : 4$ 
 $\therefore$  The measures of the interior angles of this figure are  $2x, 3x, 3x$  and  $4x$ 
 $\therefore$  The sum of measures of the interior angles of the quadrilateral  $= (4 - 2) \times 180^\circ = 2 \times 180^\circ = 360^\circ$ 

$$\therefore 2x + 3x + 3x + 4x = 360^\circ \quad \therefore 12x = 360^\circ \quad \therefore x = \frac{360^\circ}{12} = 30^\circ$$

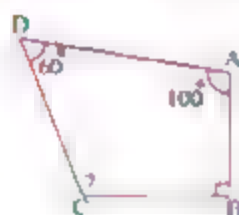
 $\therefore$  The smallest measure  $= 2x$ 

$$\therefore \text{The smallest measure of the angles} = 2 \times 30^\circ = 60^\circ$$

## TRY 2

By using the given data for each figure find the measure of the unknown angle:

1)



$$m(\angle C) = \quad^\circ$$

2)



$$m(\angle E) = \quad^\circ$$

## Unit 3

**The sum of measures of the exterior angles of the convex polygon which has  $n$  sides**

- We mentioned that the exterior angle of the polygon is the angle included between one side and the extension of its adjacent side, and although we can draw two exterior angles equal in measure at each vertex of the polygon, but the rule of the sum of measures of the exterior angles use only one exterior angle at each vertex.



- At any vertex of the polygon, we find that the sum of measures of the interior angle and the exterior angle equals  $180^\circ$

In the previous figure :  $m(\angle 1) + m(\angle 2) = 180^\circ$  as an example.

In the previous pentagon, we find that the sum of measures of the five exterior and five interior angles of the pentagon equals  $5 \times 180^\circ$

Since the sum of measures of the interior angles equals  $3 \times 180^\circ$

$\therefore$  The sum of measures of the five exterior angles of the pentagon  $= 2 \times 180^\circ = 360^\circ$

We can deduce that for any convex polygon of  $n$  sides as follows :

The sum of measures of the exterior angles + the sum of measures of the interior angles  $= n \times 180^\circ$

$\therefore$  The sum of measures of the exterior angles  $+ (n - 2) \times 180^\circ = n \times 180^\circ$

$\therefore$  The sum of measures of the exterior angles  $= n \times 180^\circ - (n - 2) \times 180^\circ$   
 $= 180^\circ n - 180^\circ n + 360^\circ = 360^\circ$

So we get :

The sum of measures of the exterior angles of a convex polygon of  $n$  sides  $= 360^\circ$   
 (taking into account one exterior angle at each vertex)

**The regular polygon** The polygon is regular if :

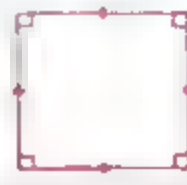
- All its sides are equal in length.
- All its angles are equal in measure.

As examples for the regular polygons :

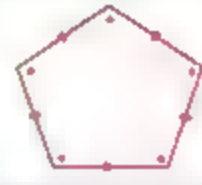
Equilateral triangle



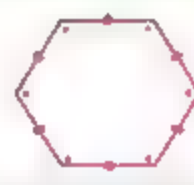
Square



Regular pentagon



Regular hexagon





## Lesson Two

**The measure of the interior angle of a regular polygon**

We knew that the sum of measures of the interior angles of a polygon of  $n$  sides  $= (n - 2) \times 180^\circ$

Then :

If the polygon is regular , then its interior angles (whose number is  $n$ ) are equal in measure.

$$\therefore \text{The measure of each interior angle of the regular polygon of } n \text{ sides} = \frac{(n-2) \times 180^\circ}{n}$$

For example:

- The measure of each interior angle of the equilateral triangle  $= \frac{(3-2) \times 180^\circ}{3} = 60^\circ$
- The measure of each interior angle of the square  $= \frac{(4-2) \times 180^\circ}{4} = 90^\circ$

**Example 5** Complete the following table :

Number of sides of the regular polygon	5	8	12	6
Measure of one of its interior angles				
Number of sides of the regular polygon	5	8	12	6
Measure of one of its interior angles	$\frac{3 \times 180^\circ}{5} = 108^\circ$	$\frac{6 \times 180^\circ}{8} = 135^\circ$	$\frac{10 \times 180^\circ}{12} = 150^\circ$	$\frac{4 \times 180^\circ}{6} = 120^\circ$

**Solution**

**Example 6** The measure of one of the interior angles of a regular polygon is  $144^\circ$ . Find the number of its sides.

**Solution**

$\therefore$  The measure of each interior angle of the regular polygon of  $n$  sides

$$= \frac{(n-2) \times 180^\circ}{n}$$

$$\therefore \frac{(n-2) \times 180^\circ}{n} = 144^\circ$$

$$\therefore 180^\circ n - 360^\circ = 144^\circ n$$

$$\therefore 36^\circ n = 360^\circ$$

$$\therefore \text{The number of sides} = 10 \text{ sides}$$

$$\therefore (n-2) \times 180^\circ = 144^\circ n$$

$$\therefore 180^\circ n - 144^\circ n = 360^\circ$$

$$\therefore n = 10$$

## Unit 3

## Another solution

- $\therefore$  The measure of the exterior angle of the polygon  
 $= 180^\circ - \text{the measure of the interior angle} = 180^\circ - 144^\circ = 36^\circ$   
 $\therefore$  The sum of the measures of the exterior angles  $= 360^\circ$   
 $\therefore$  The number of the exterior angles  $= \frac{360^\circ}{36^\circ} = 10$  angles  
 $\therefore$  The number of sides  $= 10$  sides.

## Notice that :

The number of the polygon sides = The number of its vertices  
 = The number of its interior angles = The number of its exterior angles

## Remark

The number of sides of the regular polygon in which the measure of one of its interior angles is  $X^\circ = \frac{360^\circ}{180^\circ - X}$

For example:

The number of sides of the regular polygon which the measure of one of its interior angles is  $144^\circ = \frac{360^\circ}{180^\circ - 144^\circ} = 10$  sides.



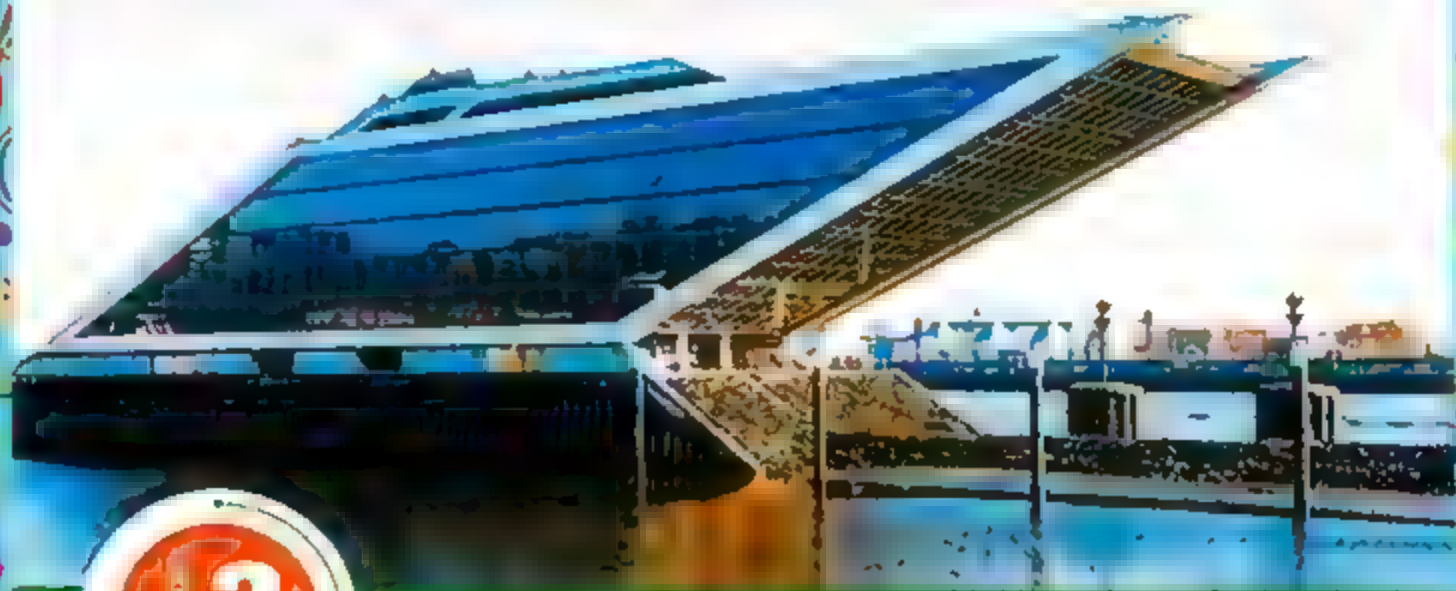
Complete the following table :

The number of sides of the regular polygon	3	10		
The measure of one of its interior angles			$135^\circ$	$160^\circ$

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

- 1 • The number of sides of the polygon : 7 • 5
- 2 • The sum of measures of the interior angles :  $1620^\circ$  •  $2520^\circ$
- 3 • The number of sides of the regular polygon : 8 • 18
- 4 • The measure of one of its interior angles :  $60^\circ$  •  $144^\circ$

Answers of "try by yourself"



3

## The parallelogram and its properties

- In primary stage you have studied the parallelogram and its properties and in this lesson you will remember first what you studied before , then you will study when a quadrilateral will be a parallelogram.

### Definition →

Parallelogram is a quadrilateral , in which each two opposite sides are parallel.

For example:

In the opposite figure :

If ABCD is a quadrilateral in which

$$\overline{AB} \parallel \overline{DC} \quad , \quad \overline{AD} \parallel \overline{BC}$$

• then ABCD is a parallelogram.





## Unit 3

## Properties of parallelogram

- 1 Each two opposite sides are equal in length.



- $AB = DC$
- $AD = BC$

- 2 Each two opposite angles are equal in measure.



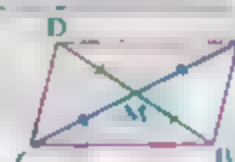
- $m(\angle A) = m(\angle C)$
- $m(\angle B) = m(\angle D)$

- 3 The sum of measures of each two consecutive angles is  $180^\circ$



- $m(\angle A) + m(\angle B) = 180^\circ$
- $m(\angle B) + m(\angle C) = 180^\circ$
- $m(\angle C) + m(\angle D) = 180^\circ$
- $m(\angle D) + m(\angle A) = 180^\circ$

- 4 The two diagonals bisect each other.



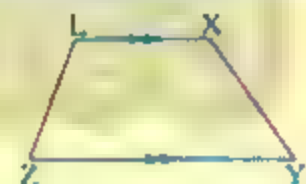
- $AM = CM$
- $BM = DM$

The perimeter of the parallelogram = The sum of two consecutive sides  $\times 2$

## Remark

A quadrilateral in which only two sides are parallel is called a trapezium, as shown in the opposite figure in which :

$$\overline{XL} \parallel \overline{YZ}$$



Example 1 In the opposite figure :

ABCD is a parallelogram in which :

$AB = 8 \text{ cm}$ ,  $BC = 6 \text{ cm}$ , and  $m(\angle B) = 120^\circ$

Find :

- 1 The length of each of  $\overline{CD}$  and  $\overline{DA}$
- 2 The measure of each of  $\angle D$ ,  $\angle A$  and  $\angle C$
- 3 The perimeter of ABCD



## Lesson Three

## Solution

Given

ABCD is a parallelogram ,  $AB = 8 \text{ cm}$  ,  $BC = 6 \text{ cm}$  and  $m(\angle B) = 120^\circ$ 

R.T.F.

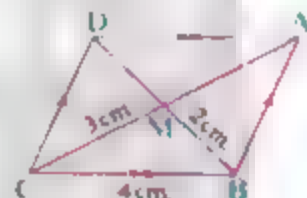
- 1 CD and DA
- 2  $m(\angle D)$  ,  $m(\angle A)$  and  $m(\angle C)$
- 3 The perimeter of ABCD

Proof

 $\therefore$  ABCD is a parallelogram $\therefore CD = AB = 8 \text{ cm}$  (Properties of a parallelogram)and  $DA = CB = 6 \text{ cm}$  (Properties of a parallelogram) (First req.) $\therefore m(\angle D) = m(\angle B) = 120^\circ$  (Properties of a parallelogram) $\therefore m(\angle A) + m(\angle B) = 180^\circ$  (Properties of a parallelogram) $\therefore m(\angle B) = 120^\circ$  $\therefore m(\angle A) = 180^\circ - 120^\circ = 60^\circ$  and  $m(\angle C) = m(\angle A) = 60^\circ$  (Second req.)The perimeter of ABCD =  $(AB + BC) \times 2$  $= (8 + 6) \times 2 = 14 \times 2 = 28 \text{ cm}$ . (Third req.)

Example 2 In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M. If  $BC = 4 \text{ cm}$  ,  $BM = 2 \text{ cm}$  , and  $MC = 3 \text{ cm}$  , then find the perimeter of  $\triangle AMD$



## Solution

Given

ABCD is a parallelogram whose diagonals intersect at M ,  $BC = 4 \text{ cm}$  ,  $BM = 2 \text{ cm}$  , and  $MC = 3 \text{ cm}$  .

R.T.F.

The perimeter of  $\triangle AMD$ 

Proof

 $\therefore$  ABCD is a parallelogram $\therefore AD = BC = 4 \text{ cm}$  . (Two opposite sides in a parallelogram) $\therefore$  The two diagonals bisect each other. $\therefore MD = MB = 2 \text{ cm}$  , and  $AM = MC = 3 \text{ cm}$  . $\therefore$  The perimeter of  $\triangle AMD = AD + MD + AM = 4 + 2 + 3 = 9 \text{ cm}$ 

(The req.)

## Unit 3



In the opposite figure :

ABCD is a parallelogram whose diagonals intersect at M

If  $BC = 5 \text{ cm}$ ,  $DC = 3 \text{ cm}$ ,  $DM = 2 \text{ cm}$ ,

and  $m(\angle ABC) = 127^\circ$ ,

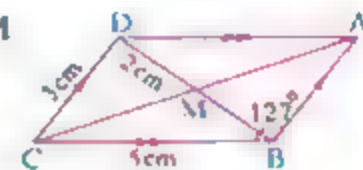
Complete the following :

1]  $AB = \dots\dots\dots \text{ cm}$ , and  $AD = \dots\dots\dots \text{ cm}$ ,

2]  $BD = \dots\dots\dots \text{ cm}$

3]  $m(\angle ADC) = \dots\dots^\circ$ ,  $m(\angle BAD) = \dots\dots^\circ$  and  $m(\angle BCD) = \dots\dots^\circ$

4] The perimeter of  $\square ABCD = \dots\dots\dots \text{ cm}$ .



When does a quadrilateral represent a parallelogram?

A quadrilateral represents a parallelogram if one of the following conditions satisfies

Each two opposite sides are parallel



Each two opposite sides are equal in length.



Two opposite sides are parallel and equal in length.



Each two opposite angles are equal in measure



The two diagonals bisect each other.



Example 3 In the opposite figure :

ABCD is a parallelogram,  $H \in \overline{AB}$

where :  $AB = BH$

Prove that : BHCD is a parallelogram.



**Solution**

Given

ABCD is a parallelogram and  $AB = BH$

R.T.F.

BHCD is a parallelogram.

Proof

$\because$  ABCD is a parallelogram

$\therefore AB = CD$

$\therefore AB = BH$  (Given)

$\therefore DC = BH$  (1)

$\therefore \overline{AB} \parallel \overline{DC}$ ,  $H \in \overline{AB}$

$\therefore \overline{BH} \parallel \overline{DC}$  (2)

From (1) and (2) :

$\therefore DC = BH$  and  $\overline{DC} \parallel \overline{BH}$

BHCD is a parallelogram.

(Q.E.D.)

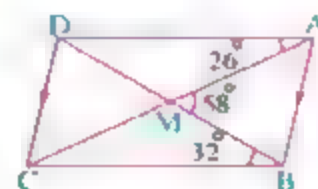


## Lesson Three



In the opposite figure :

ABCD is a quadrilateral , its diagonals intersect at M ,  
 $\overline{AB} \parallel \overline{CD}$  ,  $m(\angle AMB) = 58^\circ$  ,  $m(\angle MBC) = 32^\circ$   
 and  $m(\angle MAD) = 26^\circ$



Complete the following proof to prove that : ABCD is a parallelogram.

Given

R.T.P.

Proof

$$\because M \in \overline{AC}$$

$$\therefore m(\angle BMC) = 180^\circ - \dots^\circ = \dots^\circ$$

$$\therefore \text{In } \triangle BMC : m(\angle MCB) = 180^\circ - (\dots^\circ + \dots^\circ) = \dots^\circ$$

$$\therefore m(\angle MCB) = m(\angle \dots) \text{ and they are } \dots \text{ angles}$$

$$\therefore \overline{\dots} \parallel \overline{\dots} \quad \because \dots \parallel \dots \quad (\text{Given})$$

$$\therefore \text{ABCD is a parallelogram.}$$

(Q.E.D.)

14

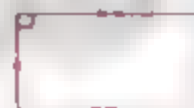
## The special cases of the parallelogram

We studied in the previous lesson that the parallelogram is a quadrilateral in which each two opposite sides are parallel, we notice that this condition is verified also in each of rectangle, rhombus and square.

So, we said that each of rectangle, rhombus and square is a special case of the parallelogram with the same properties of it which stated in the previous lesson, as well as some another properties for each figure. In this lesson, we will represent each figure of them and its properties.

## Reasoning

The rectangle is a parallelogram with a right angle.



## Properties of the rectangle

The rectangle has the same properties of the parallelogram and some additional properties as the following :

- The four angles of the rectangle are all equal in measure and the measure of each is  $90^\circ$

$$m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^\circ$$



11

## Lesson Four

- 2 The two diagonals of the rectangle are equal in length.

$AC = BD$  and as the two diagonals bisect each other  
 then  $AM = BM = CM = DM$



The perimeter of the rectangle = (length + width)  $\times$  2

## Rhombus

The rhombus is a parallelogram in which two adjacent sides are equal in length.

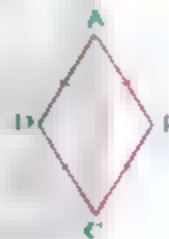


## Properties of the rhombus

The rhombus has the same properties of the parallelogram and some additional properties as the following :

- 1 The four sides of the rhombus are all equal in length.

$$AB = BC = CD = DA$$

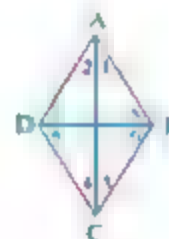


- 2 The two diagonals of the rhombus are perpendicular and bisect each of its interior angles.

$$\overline{AC} \perp \overline{BD}$$

$$m(\angle 1) = m(\angle 2) = m(\angle 3) = m(\angle 4)$$

$$m(\angle 5) = m(\angle 6) = m(\angle 7) = m(\angle 8)$$



The perimeter of the rhombus = the length of one side  $\times$  4



## Unit 3

## Square

The square is a parallelogram with a right angle and two adjacent sides are equal in length

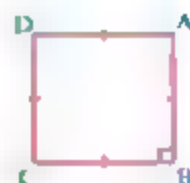


## Properties of the square

The square has the same properties of the parallelogram and some additional properties as the following :

- 1 The four sides of the square are all equal in length.

$$AB = BC = CD = DA$$



- 2 The four angles of the square are all equal in measure and each of them is of measure  $90^\circ$

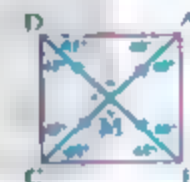
$$m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^\circ$$



- 3 The two diagonals of the square are equal in length , perpendicular and each diagonal bisects the two vertices angles which this diagonal joins into two angles each one of measure  $45^\circ$ .

$$\bullet AC = BD \text{ and hence } AM = BM = CM = DM$$

$$\bullet \overline{AC} \perp \overline{BD}$$



The perimeter of the square = the length of one side  $\times 4$

Notice that :

We can also define the square as follows :

- 1 A square is a rectangle with two adjacent sides equal in length.
- 2 A square is a rectangle with two perpendicular diagonals.
- 3 A square is a rhombus with a right angle.
- 4 A square is a rhombus with two diagonals equal in length

## Notice that :

To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram , as we see in the previous lesson , then

## The parallelogram is

## a rectangle

If :

One of its angles is a right angle

(or)

Its two diagonals are equal in length.

## a rhombus

If :

Two adjacent sides are equal in length

(or)

Its two diagonals are perpendicular

## a square

If :

One of its angles is right and two adjacent sides are equal in length.

(or)

One of its angles is right and its two diagonals are perpendicular

(or)

Its two diagonals are perpendicular and equal in length

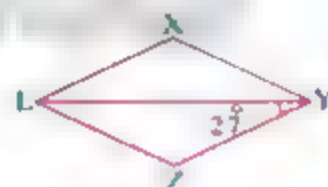
(or)

Two adjacent sides are equal in length and its two diagonals are equal in length

## Example 1 In the opposite figure :

XYZL is a rhombus in which  $m(\angle LYZ) = 27^\circ$

Calculate the measures of the angles of the rhombus XYZL



## Solution

Given

R.T.F.

Proof

XYZL is a rhombus in which  $m(\angle LYZ) = 27^\circ$

$m(\angle XYZ)$  ,  $m(\angle XLZ)$  ,  $m(\angle X)$  and  $m(\angle Z)$

$\therefore \overline{YL}$  is a diagonal in the rhombus XYZL

$\therefore \overline{YL}$  bisects  $\angle XYZ$

$$\therefore m(\angle XYZ) = 2 \times 27^\circ = 54^\circ$$

$\therefore$  Each two opposite angles in the rhombus are equal in measure.

$$\therefore m(\angle XLZ) = 54^\circ$$

Unit 3

∴ The rhombus is a special case of the parallelogram

∴ Each two consecutive angles are supplementary

$$\therefore m(\angle X) + m(\angle XYZ) = 180^\circ$$

$$\therefore m(\angle X) + 54^\circ = 180^\circ \quad \therefore m(\angle X) = 126^\circ$$

$$\therefore m(\angle Z) = 126^\circ$$

(The req.)

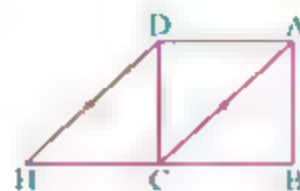
Try to solve this example by another method using the properties of the rhombus

**Example 2** In the opposite figure :

ABCD is a square,  $H \in \overline{BC}$  such that  $\overline{DH} \parallel \overline{AC}$

1 Prove that :  $CH = BC$

2 Find :  $m(\angle ADH)$



### Solution

Given

ABCD is a square and  $\overline{DH} \parallel \overline{AC}$

R.T.P.

$CH = BC$

R.T.F.

$m(\angle ADH)$

Proof

∴  $\overline{AD} \parallel \overline{BC}$  (two opposite sides in the square) and  $H \in \overline{BC}$

∴  $\overline{AD} \parallel \overline{CH}$

∴  $\overline{DH} \parallel \overline{AC}$  (given)

∴ ACHD is a parallelogram.

$$\therefore CH = AD$$

But  $AD = BC$  (two opposite sides in the square)

$$\therefore CH = BC$$

(First req.)

∴  $\overline{AC}$  is a diagonal in the square.

∴  $\overline{CA}$  bisects  $\angle BCD$

$$\therefore m(\angle BCD) = 90^\circ$$

$$\therefore m(\angle ACD) = 45^\circ$$

∴  $\overline{DH} \parallel \overline{AC}$  and  $\overline{CD}$  is their transversal.

$$\therefore m(\angle CDH) = m(\angle ACD) = 45^\circ \text{ (two alternate angles)}$$

$$\therefore m(\angle ADC) = 90^\circ \text{ (property of the square)}$$

$$\therefore m(\angle ADH) = m(\angle ADC) + m(\angle CDH)$$

$$= 90^\circ + 45^\circ = 135^\circ$$

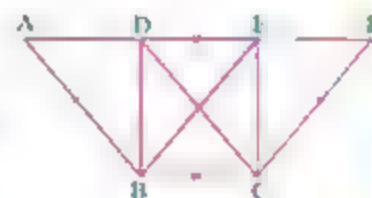
(Second req.)

**Example 3** In the opposite figure :

ABCD, EBCF are two parallelograms,

D and E belong to  $\overline{AF}$ ,  $AB = FC$ ,  $BC = DE$

Prove that : The figure DBCE is a rectangle.





## Lesson Four

## Solution

**Given :** ABCD and EBCF are two parallelograms ,  $AB = FC$  ,  $BC = DE$

**R.T.P.** The figure DBCE is a rectangle.

**Proof**

$\therefore$  ABCD is a parallelogram.

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore$  D and E belong to  $\overline{AF}$

$\therefore \overline{DE} \parallel \overline{BC}$

$\therefore DE = BC$

$\therefore$  DBCE is a parallelogram.

$\therefore$  ABCD is a parallelogram.

$\therefore AB = DC$

$\therefore$  EBCF is a parallelogram.

$\therefore FC = EB$  but  $AB = FC$

$\therefore DC = EB$

$\therefore$  DBCE is a parallelogram and its diagonals are equal in length.

$\therefore$  DBCE is a rectangle.

(Q.E.D.)

## TRY

Using the given in each figure , complete where M is the intersection point of the diagonals :

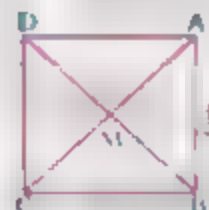
1



ABCD is a parallelogram :

- The perimeter of  $\Delta ABC =$  \_\_\_\_\_ cm
- $m(\angle AMB) = \dots^\circ$

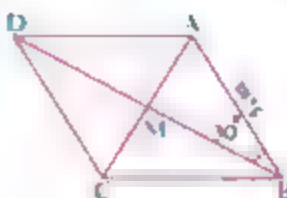
2



ABCD is a square :

- The perimeter of the square = \_\_\_\_\_ cm.
- $m(\angle BAC) = \dots^\circ$

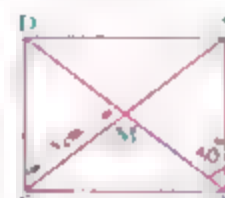
3



ABCD is a rhombus :

- $AD = \dots$  cm.
- $m(\angle BAM) = \dots^\circ$

4



ABCD is a rectangle :

- $BD = \dots$  cm.
- $m(\angle MCD) = \dots^\circ$

05 9 4

09 5 3

59 91 2

44 16 77

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5

## The triangle

## Theorem 1

The sum of the measures of the interior angles of a triangle is  $180^\circ$

Given  $\triangle ABC$  is a triangle

R.T.P.  $m(\angle A) + m(\angle B) + m(\angle ACB) = 180^\circ$

Construction Draw  $\overline{CX} \parallel \overline{AB}$

Proof  $\because \angle XCY$  is a straight angle

$$\therefore m(\angle XCA) + m(\angle ACB) + m(\angle YCB) = 180^\circ$$

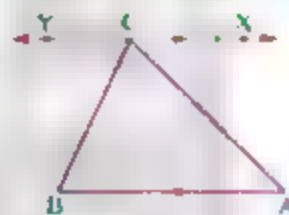
$$\because \overline{XY} \parallel \overline{AB}$$

$$\therefore m(\angle XCA) = m(\angle A) \quad (\text{alternate angles})$$

$$\therefore m(\angle YCB) = m(\angle B) \quad (\text{alternate angles})$$

$$\therefore m(\angle A) + m(\angle ACB) + m(\angle B) = 180^\circ$$

(Q.E.D.)



Example 1 In the opposite figure :

$\triangle ABC$  is a triangle in which  $m(\angle A) = 75^\circ$  ,

$m(\angle B) = 60^\circ$  ,

$X \in \overline{AB}$  and  $Y \in \overline{AC}$

Such that  $\overline{XY} \parallel \overline{BC}$

Find :  $m(\angle AYX)$



## Solution

Given

 $\overline{XY} \parallel \overline{BC}$ ,  $m(\angle A) = 75^\circ$  and  $m(\angle B) = 60^\circ$ 

R.T.F.

 $m(\angle AYX)$ 

Proof

 $\therefore m(\angle A) = 75^\circ$  and  $m(\angle B) = 60^\circ$  (given)

the sum of measures of the interior angles

of  $\triangle ABC = 180^\circ$  $\therefore m(\angle C) = 180^\circ - (75^\circ + 60^\circ)$ 

$$= 180^\circ - 135^\circ = 45^\circ$$

 $\therefore \overline{XY} \parallel \overline{BC}$  and  $\overline{AC}$  is a transversal. $\therefore m(\angle AYX) = m(\angle C) = 45^\circ$  (corresponding angles)

(The req.)

Try to solve  
the example by  
another method.

## Example 2 In the opposite figure :

 $\overline{BM}$  bisects  $\angle ABC$ ,  $\overline{CM}$  bisects  $\angle ACB$ and  $m(\angle BMC) = 125^\circ$ Find :  $m(\angle A)$ 

## Solution

Given

 $\overline{BM}$  bisects  $\angle ABC$ ,  $\overline{CM}$  bisects  $\angle ACB$  and  $m(\angle BMC) = 125^\circ$ 

R.T.F.

 $m(\angle A)$ 

Proof

 $\therefore$  The sum of measures of the interior angles of  $\triangle MBC = 180^\circ$ and  $m(\angle BMC) = 125^\circ$ 

$$\therefore m(\angle 1) + m(\angle 2) = 180^\circ - 125^\circ = 55^\circ$$

But  $m(\angle ABC) = 2 m(\angle 1)$  and  $m(\angle ACB) = 2 m(\angle 2)$ 

$$\therefore m(\angle ABC) + m(\angle ACB) = 2 \times 55^\circ = 110^\circ$$

 $\therefore$  The sum of measures of the interior angles of  $\triangle ABC = 180^\circ$ 

$$\therefore m(\angle A) = 180^\circ - 110^\circ = 70^\circ$$

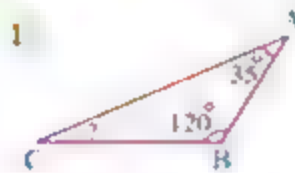
(The req.)



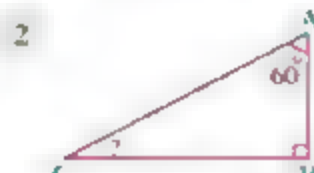
13



In each of the following figures, find the measure of the angle marked by (7) :



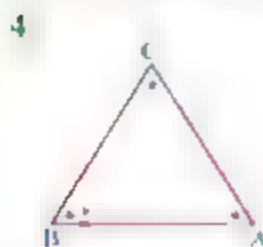
$$m(\angle C) = \text{ }^\circ$$



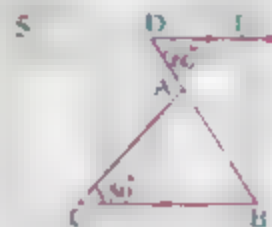
$$m(\angle C) = \text{ }^\circ$$



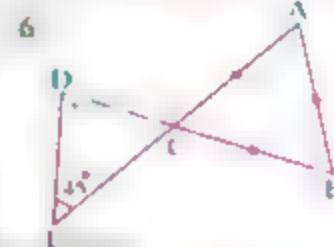
$$m(\angle A) = \text{ }^\circ$$



$$m(\angle B) = \text{ }^\circ$$



$$m(\angle BAC) = \text{ }^\circ$$



$$m(\angle D) = \text{ }^\circ$$

### The exterior angle of the triangle

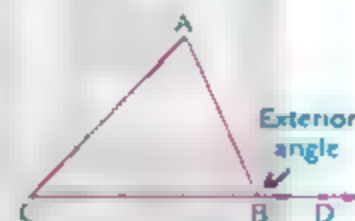
In the opposite figure :

If ABC is a triangle,  $D \in \overline{CB}$  and  $D \notin \overline{CB}$ , then  $\angle ABD$  is called an exterior angle of  $\triangle ABC$

Notice that :

In the opposite figure :

$\angle ACD$  is not an exterior angle of  $\triangle ABC$  because  $D \notin \overline{BC}$



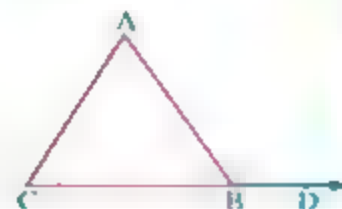
### The measure of an exterior angle of a triangle

The measure of an exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.

In the opposite figure :

If ABC is a triangle,  $D \in \overline{CB}$  and  $D \notin \overline{CB}$ , then  $m(\angle ABD) = m(\angle A) + m(\angle C)$

We can prove that as follows :



## Lesson Five

$$\therefore m(\angle A) + m(\angle C) + m(\angle ABC) = 180^\circ$$

$$m(\angle ABD) + m(\angle ABC) = 180^\circ$$

$$\therefore m(\angle ABD) + m(\angle ABC) = m(\angle A) + m(\angle C) + m(\angle ABC)$$

$$\therefore m(\angle ABD) = m(\angle A) + m(\angle C)$$

(Q.E.D)

Notice that :

The measure of the exterior angle of a triangle is greater than the measure of any interior angle of the triangle except its adjacent angle.

i.e. In the previous figure :  $m(\angle ABD) > m(\angle A)$  and  $m(\angle ABD) > m(\angle C)$

**Example 3** In the opposite figure :

ABC is a triangle ,  $D \in \overline{BC}$  and  $E \in \overline{AC}$

where  $\overline{BE}$  bisects  $\angle ABC$  ,

$$m(\angle A) = 80^\circ \text{ and}$$

$$m(\angle ACD) = 150^\circ$$

Find :

1  $m(\angle ABC)$

2  $m(\angle BEC)$



## Solution

**Given**  $\overline{BE}$  bisects  $\angle ABC$  ,  $m(\angle A) = 80^\circ$  and  $m(\angle ACD) = 150^\circ$

**R.T.F.** 1  $m(\angle ABC)$  2  $m(\angle BEC)$

**Proof**  $\therefore \angle ACD$  is an exterior angle of  $\triangle ABC$

$$\therefore m(\angle ACD) = m(\angle A) + m(\angle ABC)$$

$$\therefore 150^\circ = 80^\circ + m(\angle ABC)$$

$$\therefore m(\angle ABC) = 150^\circ - 80^\circ = 70^\circ \quad (\text{First req.})$$

$\therefore \overline{BE}$  bisects  $\angle ABC$  (given)

$$\therefore m(\angle ABE) = \frac{1}{2} m(\angle ABC) = \frac{70^\circ}{2} = 35^\circ$$

$\therefore \angle BEC$  is an exterior angle of  $\triangle ABE$

$$\therefore m(\angle BEC) = m(\angle A) + m(\angle ABE) = 80^\circ + 35^\circ = 115^\circ \quad (\text{Second req.})$$

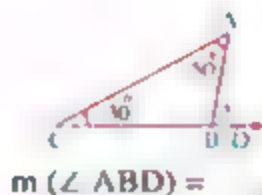
Try to solve the example by another method.

3

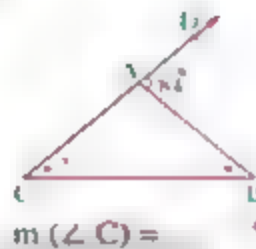
TRY 2

In each of the following figures, find the measure of the angle marked by (?):

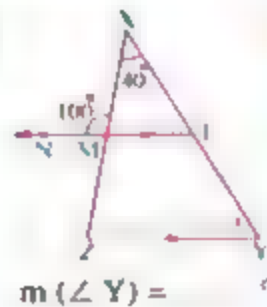
1



2

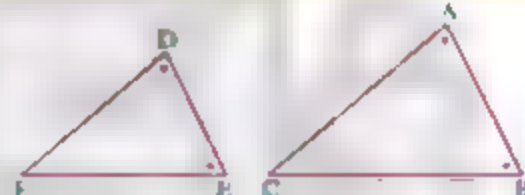


3



## Remark 1

If two angles of one triangle equal two angles of another triangle in measure, then the third angle of the first triangle is equal in measure to the third angle of the other triangle.

In  $\triangle ABC$  and  $DEF$ ,if  $m(\angle A) = m(\angle D)$  and  $m(\angle B) = m(\angle E)$ ,then  $m(\angle C) = m(\angle F)$ 

Example 4 In the opposite figure:

ABC and DEF are two triangles,  $E \in \overline{BC}$ , $F \in \overline{BC}$ ,  $\overline{DE} \parallel \overline{AB}$  and  $\overline{DF} \parallel \overline{AC}$ Prove that:  $m(\angle A) = m(\angle D)$ 

## Solution

Given

 $\overline{DE} \parallel \overline{AB}$  and  $\overline{DF} \parallel \overline{AC}$ 

R.T.P.

 $m(\angle A) = m(\angle D)$ 

Proof

 $\therefore \overline{DE} \parallel \overline{AB}$  and  $\overline{BC}$  is a transversal to them. $\therefore m(\angle 1) = m(\angle 2)$  (corresponding angles) $\therefore \overline{DF} \parallel \overline{AC}$  and  $\overline{BC}$  is a transversal to them. $\therefore m(\angle 3) = m(\angle 4)$  (corresponding angles)In  $\triangle ABC$  and  $DEF$ : $\therefore m(\angle 1) = m(\angle 2)$  and  $m(\angle 3) = m(\angle 4)$  $\therefore m(\angle A) = m(\angle D)$ 

(Q.E.D.)



## Remark 2

- If the sum of measures of two angles in a triangle equals  $90^\circ$ , then the third angle is right.
- If the sum of measures of two angles in a triangle is less than  $90^\circ$ , then the third angle is obtuse.
- If the sum of measures of two angles in a triangle is more than  $90^\circ$ , then the third angle is acute.

## Example 5 In the opposite figure :

ABC is a triangle in which  $m(\angle A) = 2m(\angle B) = 4x^\circ$   
and  $m(\angle C) = 7x^\circ$

Prove that :  $\angle C$  is an obtuse angle.



## Solution

Given  $m(\angle A) = 2m(\angle B) = 4x^\circ$  and  $m(\angle C) = 7x^\circ$

R.T.P.  $\angle C$  is an obtuse angle.

Proof  $\because 2m(\angle B) = 4x^\circ$

$$\therefore m(\angle B) = 2x^\circ$$

$$\therefore m(\angle A) + m(\angle B) = 4x^\circ + 2x^\circ = 6x^\circ$$

$$\because m(\angle C) = 7x^\circ$$

$$\therefore m(\angle A) + m(\angle B) < m(\angle C)$$

$$\therefore \angle C \text{ is an obtuse angle.}$$

(Q.E.D.)

## Remark 3

If the measure of an angle in a triangle equals the sum of measures of the other two angles, then the triangle is right-angled.

In the opposite figure :

If ABC is a triangle in which  $m(\angle A) + m(\angle C) = m(\angle B)$

$$\therefore \text{then } m(\angle B) = \frac{180^\circ}{2} = 90^\circ$$

i.e.  $\Delta ABC$  is right-angled at B





**Example 6** ABC is a triangle in which  $m(\angle A) : m(\angle B) : m(\angle C) = 2 : 3 : 5$   
Prove that the triangle is right-angled, then mention the right angle

**Solution**

**Given**

ABC is a triangle in which  $m(\angle A) : m(\angle B) : m(\angle C) = 2 : 3 : 5$

**R.T.P.**

$\Delta ABC$  is right-angled and mention the right angle

**Proof**

$\therefore m(\angle A) + m(\angle B)$  is equivalent to 5 parts and  $m(\angle C)$   
is equivalent to 5 parts

$$\therefore m(\angle A) + m(\angle B) = m(\angle C)$$

$\therefore$  The sum of measures of the interior angles of a triangle  $= 180^\circ$

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

$$\therefore m(\angle A) + m(\angle B) = m(\angle C) = 90^\circ$$

$\therefore \Delta ABC$  is right-angled at C

(Q.E.D.)

2 1 80°  
4 60°  
1 1 25°

2 42°  
5 70°  
2 30°

3 60°  
6 75°  
3 45°

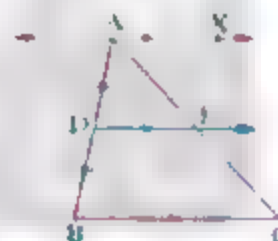


## Follow The triangle

## Theorem 2

The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.

Given	D is the midpoint of $\overline{AB}$ , $\overline{DE} \parallel \overline{BC}$
R.T.P.	E is the midpoint of AC
Construction	Draw $\overline{AX} \parallel \overline{BC}$
Proof	$\therefore \overline{AX} \parallel \overline{DE} \parallel \overline{BC}$ $\therefore \overline{AB}$ and $\overline{AC}$ are two transversals to them at D and E respectively. $\therefore AD = DB \quad \therefore AE = EC$ $\therefore E$ is the midpoint of $\overline{AC}$



Q.E.D.

**Example 1** In the opposite figure :

ABCD is a parallelogram ,

$H \in \overline{AD}$  such that  $AD = DH$ ,  $\overline{HC} \cap \overline{AB} = \{O\}$

Prove that :

1  $HC = CO$

2  $AB = BO$





Unit 3

## Solution

Given

ABCD is a parallelogram ,  $AD = DH$  and  $\overline{HC} \cap \overline{AB} = \{O\}$ 

R.T.P.

1  $HC = CO$ 2  $AB = BO$ 

Proof

In  $\triangle HAO$  $\therefore D$  is the midpoint of  $\overline{HA}$  (given) , $\overline{DC} \parallel \overline{AO}$  (definition of the parallelogram) $\therefore C$  is the midpoint of  $\overline{HO}$ i.e.  $HC = CO$  (theorem)

(Q.E.D. 1)

 $\therefore C$  is the midpoint of  $\overline{OH}$  (proved) $\therefore CB \parallel HA$  (definition of the parallelogram) $\therefore B$  is the midpoint of  $\overline{AO}$ i.e.  $AB = BO$  (theorem)

(Q.E.D. 2)

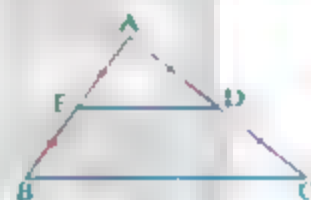
## Corollary

The line segment joining the midpoints of two sides of a triangle is parallel to the third side.

In the opposite figure :

If ABC is a triangle in which D  
is the midpoint of  $\overline{AC}$  ,

E is the midpoint of  $\overline{AB}$  , then :  $\overline{ED} \parallel \overline{BC}$



Exemplo 2 In the opposite figure :

X is the midpoint of  $\overline{AB}$  , $\overline{XY} \parallel \overline{BC}$  and Z is the midpoint of  $\overline{DC}$ Prove that :  $\overline{YZ} \parallel \overline{AD}$ 

## Solution

Given

X is the midpoint of  $\overline{AB}$  , Z is the midpoint of  $\overline{CD}$  and  $\overline{XY} \parallel \overline{BC}$ 

R.T.P.

 $\overline{YZ} \parallel \overline{AD}$ 

Proof

In  $\triangle ABC$  : $\therefore AX = XB$  ,  $\overline{XY} \parallel \overline{BC}$  $\therefore AY = YC$  (theorem)In  $\triangle ACD$  :  $\therefore AY = YC$  (proved) $\therefore DZ = ZC$  (given) $\therefore \overline{YZ} \parallel \overline{AD}$  (corollary)

(Q.E.D.)

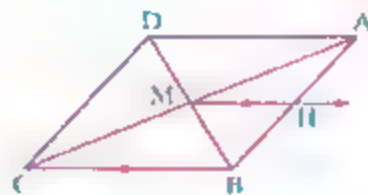


In the opposite figure :

ABCD is a parallelogram and M is the point of intersection of its two diagonals.

Draw  $\overline{MH} \parallel \overline{BC}$  to cut  $\overline{AB}$  at H

Complete the solution to prove that :  $AH = HB$



Given

R.T.P.

Proof

$\therefore$  ABCD is a parallelogram

$\therefore$  M is the midpoint of

In  $\triangle ABC$  :

$\therefore$  M is the midpoint of ..... and //

$\therefore$  ..... (theorem)

(Q.E.D.)

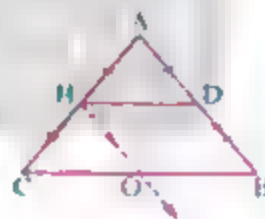


The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

Given : ABC is a triangle , D is the midpoint of  $\overline{AB}$  , H is the midpoint of  $\overline{AC}$

R.T.P. :  $DH = \frac{1}{2} BC$

Construction : Draw  $\overline{HO} \parallel \overline{AB}$  to cut  $\overline{BC}$  at O



Proof : D is the midpoint of  $\overline{AB}$  , H is the midpoint of  $\overline{AC}$

$\therefore \overline{DH} \parallel \overline{BC}$  (corollary)

$\therefore \overline{HO} \parallel \overline{AB}$  (construction) , H is the midpoint of AC

$\therefore$  O is the midpoint of  $\overline{BC}$

$\therefore BO = \frac{1}{2} BC$

$\therefore$  The figure DHOB is a parallelogram

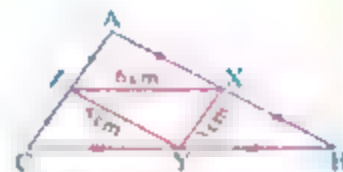
$\therefore DH = BO = \frac{1}{2} BC$

(Q.E.D.)

## Unit 3

**Example 3** In the opposite figure :

$ABC$  is a triangle in which  $X$ ,  $Y$  and  $Z$  are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{CA}$  respectively. If  $XY = 3$  cm,  $YZ = 5$  cm, and  $ZX = 6$  cm, then find the perimeter of  $\Delta ABC$

**Solution****Given**

$ABC$  is a triangle in which  $X$ ,  $Y$  and  $Z$  are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{CA}$  respectively,  $XY = 3$  cm,  $YZ = 5$  cm and  $ZX = 6$  cm

**R.T.F.**

The perimeter of  $\Delta ABC$

**Proof**

In  $\Delta ABC$ :

$\therefore X$  is the midpoint of  $\overline{AB}$  and  $Z$  is the midpoint of  $\overline{AC}$

$\therefore XZ = \frac{1}{2} BC$  (theorem)

$\therefore BC = 6 \times 2 = 12$  cm.

Similarly  $\therefore X$  is the midpoint of  $\overline{AB}$  and  $Y$  is the midpoint of  $\overline{BC}$

$\therefore XY = \frac{1}{2} AC$

$\therefore AC = 3 \times 2 = 6$  cm.

Similarly  $\therefore Y$  is the midpoint of  $\overline{BC}$  and  $Z$  is the midpoint of  $\overline{AC}$

$\therefore YZ = \frac{1}{2} AB$

$\therefore AB = 5 \times 2 = 10$  cm.

$\therefore$  The perimeter of  $\Delta ABC = AB + BC + CA = 10 + 12 + 6 = 28$  cm.

(The req.)

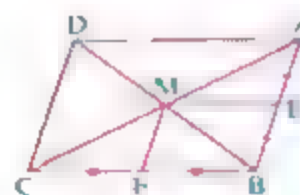
**Example 4** In the opposite figure :

$ABCD$  is a parallelogram in which :

$\overline{AC} \cap \overline{BD} = \{M\}$ ,  $E$  is the midpoint of  $\overline{AB}$

$F$  is the midpoint of  $\overline{BC}$

Prove that : The figure  $EBFM$  is a parallelogram

**Solution****Given**

$ABCD$  is a parallelogram,  $E$  is the midpoint of  $\overline{AB}$

$F$  is the midpoint of  $\overline{BC}$

**R.T.P.**

The figure  $EBFM$  is a parallelogram



## Lesson Six

Proof

- ∵ ABCD is a parallelogram whose diagonals intersect at M
- ∴ M is the midpoint of each of  $\overline{AC}$  and  $\overline{BD}$
- ∴ In  $\triangle ABC$  :
- ∴ E is the midpoint of  $\overline{AB}$  , M is the midpoint of  $\overline{AC}$
- ∴  $\overline{EM} \parallel \overline{BC}$
- ∴  $\overline{EM} \parallel \overline{BF}$
- ∴  $EM = \frac{1}{2} BC$  (theorem)
- ∴  $EM = BF$
- ∴ The figure EBFM is a parallelogram.

(Q.E.D)

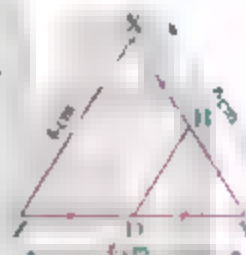


In the opposite figure :

XYZ is an equilateral triangle whose side is of length 6 cm. ,

D is the midpoint of  $\overline{YZ}$  and H is the midpoint of  $\overline{XY}$ 

Complete the following solution to prove that :

 $\triangle HYD$  is an equilateral triangle and find its perimeter.

Given

R.T.P.

Proof

∴ D is the midpoint of ..... ∴  $YD =$  ..... cm∴ H is the midpoint of ..... ∴  $YH =$  ..... cm.In  $\triangle XYZ$  .

∴ D is the midpoint of ..... and H is the midpoint of .....

∴  $DH = \frac{1}{2} \dots\dots\dots = \dots\dots\dots$  cm. (3)

From (1) , (2) and (3) :

∴  $\triangle HYD$  is ..... and its perimeter = ..... cm. (The req)

7

## Pythagoras' theorem

In the opposite figure :

- If ABC is a right-angled triangle at A in which :

$AB = 4$  length units ,  $AC = 3$  length units

$BC = 5$  length units , then :

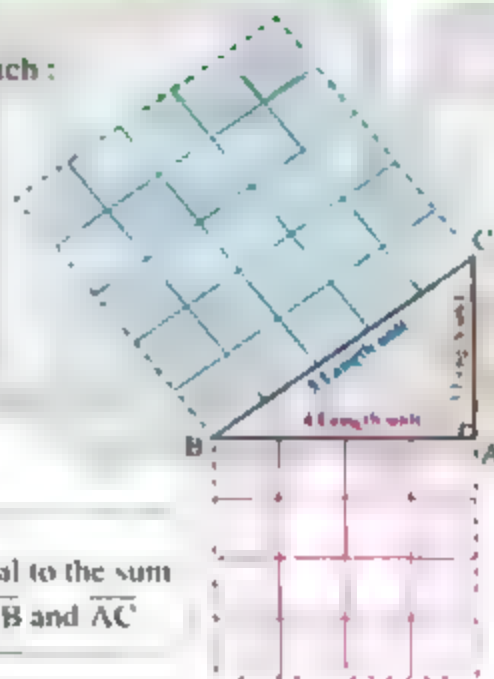
- The area of the square drawn on  $\overline{AB}$   
 $= (AB)^2 = 16$  square unit
- The area of the square drawn on  $\overline{AC}$   
 $= (AC)^2 = 9$  square unit.
- The area of the square drawn on  $\overline{BC}$   
 $= (BC)^2 = 25$  square unit.

i.e. ,

The area of the square drawn on  $\overline{BC}$  is equal to the sum of the areas of the two squares drawn on  $\overline{AB}$  and  $\overline{AC}$

In other words :

$$(BC)^2 = (AB)^2 + (AC)^2$$



The verbal formula of this relation is defined by Pythagoras' theorem.

### Pythagoras' theorem

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.



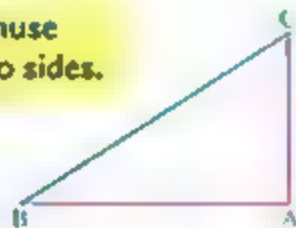
## Lesson Seven

- We can also write the previous theorem as follows :

In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

i.e. If ABC is a right-angled triangle at A, then :

$$(BC)^2 = (AB)^2 + (AC)^2$$



• From the previous relation, we can deduce the following two relations :

$$(AB)^2 = (BC)^2 - (AC)^2$$

$$(AC)^2 = (BC)^2 - (AB)^2$$

**Example 1** In each of the following figures :

Find the side length which is denoted by sign (?) :



Fig. (1)

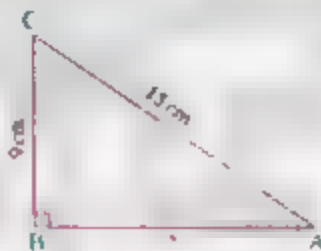


Fig. (2)

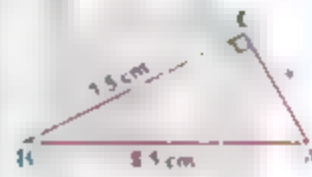


Fig. (3)

**Solution**

In Fig. (1) :

∵ Δ ABC is right-angled at A

$$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (15)^2 + (20)^2 = 225 + 400 = 625$$

$$\therefore BC = \sqrt{625} = 25 \text{ cm.}$$

In Fig. (2) :

∵ Δ ABC is right-angled at B

$$\therefore (AB)^2 = (AC)^2 - (BC)^2 = (15)^2 - (9)^2 = 225 - 81 = 144$$

$$\therefore AB = \sqrt{144} = 12 \text{ cm.}$$

In fig. (3) :

∵ Δ ABC is right-angled at C

$$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (8.5)^2 - (7.5)^2 = 72.25 - 56.25 = 16$$

$$\therefore AC = \sqrt{16} = 4 \text{ cm.}$$



33

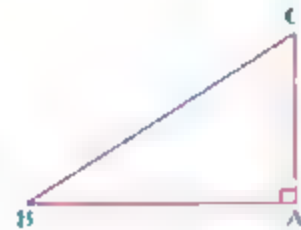


In the opposite figure :

 $\Delta ABC$  is right-angled at A

Complete the following table :

AB	8 cm	12 cm	12 cm	20 cm		
AC	6 cm.	9 cm.	.....		12 cm	4.5 cm
BC			13 cm	25 cm	20 cm	7.5 cm

**Example 2** In the opposite figure :

ABCD is a quadrilateral in which :

 $m(\angle B) = m(\angle ACD) = 90^\circ$  $\therefore AB = 4$  cm,  $\therefore BC = 3$  cm, and  $AD = 13$  cm.Find : The length of each of  $\overline{AC}$  and  $\overline{CD}$ **Solution****Given**  $m(\angle B) = m(\angle ACD) = 90^\circ$ ,  $AB = 4$  cm,  $BC = 3$  cm, and  $AD = 13$  cm.**R.T.F.** The length of each of  $\overline{AC}$  and  $\overline{CD}$ **Proof**  $\therefore \Delta ABC$  is a right-angled triangle at B $\therefore (AC)^2 = (AB)^2 + (BC)^2$  (Pythagoras' theorem) $\therefore (AC)^2 = (4)^2 + (3)^2 = 16 + 9 = 25$  $\therefore AC = \sqrt{25} = 5$  cm.

(First req.)

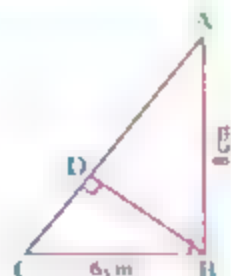
 $\therefore \Delta ACD$  is a right-angled triangle at C $\therefore (CD)^2 = (AD)^2 - (AC)^2$  (Pythagoras' theorem) $\therefore (CD)^2 = (13)^2 - (5)^2 = 169 - 25 = 144$  $\therefore CD = \sqrt{144} = 12$  cm.

(Second req.)

## Lesson Seven

**Example 3** In the opposite figure :

$\triangle ABC$  is a right-angled triangle at  $B$ ,  $D \in \overline{AC}$   
 such that  $\overline{BD} \perp \overline{AC}$ ,  $AB = 8$  cm and  $CB = 6$  cm  
 Find : The length of  $\overline{BD}$

**Solution**

Given

 $\triangle ABC$  is right-angled at  $B$ ,  $BD \perp AC$ ,  $AB = 8$  cm and  $CB = 6$  cm

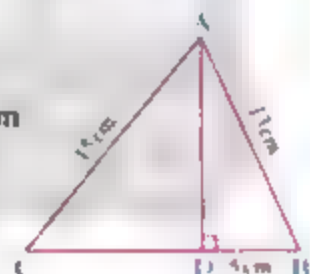
R.T.F.

The length of  $\overline{BD}$ 

Proof

 $\therefore \triangle ABC$  is right-angled at  $B$  $\therefore (AC)^2 = (AB)^2 + (BC)^2$  (Pythagoras' theorem) $\therefore (AC)^2 = 64 + 36 = 100$   $\therefore AC = \sqrt{100} = 10$  cm. $\therefore$  The area of  $\triangle ABC = \frac{1}{2} BC \times AB = \frac{1}{2} \times 6 \times 8 = 24$  cm<sup>2</sup> $\therefore$  The area of  $\triangle ABC = \frac{1}{2} AC \times BD$  $\therefore 24 = \frac{1}{2} \times 10 \times BD$   $\therefore 24 = 5 BD$  $\therefore BD = \frac{24}{5} = 4.8$  cm. (The req.)

In the opposite figure :

 $\triangle ABC$  is a triangle in which  $AB = 13$  cm. $AC = 15$  cm,  $D \in \overline{BC}$  such that  $\overline{AD} \perp \overline{BC}$  and  $BD = 5$  cmComplete the following proof to find : The length of  $\overline{DC}$  $\therefore \triangle ABD$  is right-angled at  $D$  $\therefore (AD)^2 = (AB)^2 - (BD)^2 = (13)^2 - (5)^2 =$  $\therefore AD = \sqrt{\dots\dots\dots} = \dots\dots\dots$  cm. $\therefore \triangle ADC$  is right-angled at  $D$  $\therefore (DC)^2 = (AC)^2 - (AD)^2 = (15)^2 - (\dots\dots\dots)^2 =$  $\therefore DC = \sqrt{\dots\dots\dots} = \dots\dots\dots$  cm. (The req.) $\bullet BC = 10$  cm,  $\bullet BC = 15$  cm $\bullet AB = 16$  cm,  $\bullet AB = 6$  cm. $\bullet AC = 5$  cm,  $\bullet AC = 15$  cm

of try by yourself

Answers

## Unit 3

Enrichment information (for reading only) :

You can get three numbers representing the lengths of sides of a right-angled triangle as follows :

- 1 If  $M$  is an even number bigger than 2, then the numbers  $M, \left(\frac{M}{2}\right)^2 - 1, \left(\frac{M}{2}\right)^2 + 1$

represent three lengths of sides of a right-angled triangle as shown in the following table :

$M$	$\left(\frac{M}{2}\right)^2 - 1$	$\left(\frac{M}{2}\right)^2 + 1$	Side lengths of the right-angled triangle
4	$\frac{16}{4} - 1 = 3$	$\frac{16}{4} + 1 = 5$	4, 3, 5
6	$\frac{36}{4} - 1 = 8$	$\frac{36}{4} + 1 = 10$	6, 8, 10
8	$\frac{64}{4} - 1 = 15$	$\frac{64}{4} + 1 = 17$	8, 15, 17
10	$\frac{100}{4} - 1 = 24$	$\frac{100}{4} + 1 = 26$	10, 24, 26

- 2 If  $M$  is an odd number bigger than 2, then the numbers  $M, \frac{M^2 - 1}{2}, \frac{M^2 + 1}{2}$

represent three lengths of sides of a right angled triangle as shown in the following table :

$M$	$\frac{M^2 - 1}{2}$	$\frac{M^2 + 1}{2}$	Side length of the right angled triangle
3	$\frac{9 - 1}{2} = 4$	$\frac{9 + 1}{2} = 5$	3, 4, 5
5	$\frac{25 - 1}{2} = 12$	$\frac{25 + 1}{2} = 13$	5, 12, 13
7	$\frac{49 - 1}{2} = 24$	$\frac{49 + 1}{2} = 25$	7, 24, 25
9	$\frac{81 - 1}{2} = 40$	$\frac{81 + 1}{2} = 41$	9, 40, 41



## 8

## Geometric transformations

There are many types of geometric transformations.

In this lesson, you will recognize the meaning of the geometric transformation, also recognize quickly three types of them :

1 Reflection

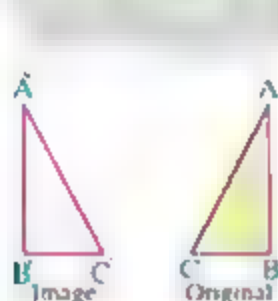
2 Translation

3 Rotation

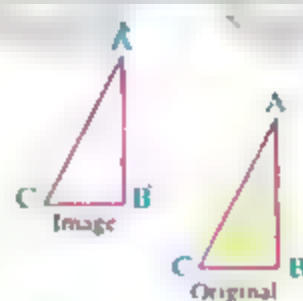
And you will study each of them in details next lessons.

### The concept of the geometric transformation.

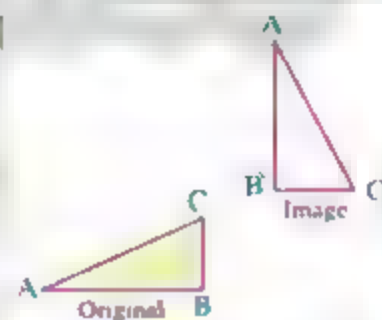
In each of the following figures , notice the Image of  $\triangle ABC$  :



$\triangle ABC$  has been reflected



$\triangle ABC$  has been translated



$\triangle ABC$  has been rotated

In each of the previous figures , notice that :

- The point A has been transferred to  $\hat{A}$
- The point B has been transferred to  $\hat{B}$

## Unit 3

- The point C has been transferred to  $\bar{C}$

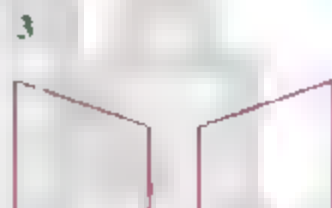
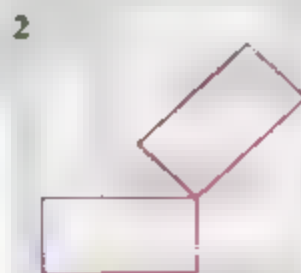
Thus, all the points of  $\Delta ABC$  have been transferred to another position, then we say that  $\Delta ABC$  has been transformed from a position to another position

From the previous, we deduce that

If all the points of a geometrical figure have moved according to a certain system, then we will obtain an image in a new position to the same figure, then we say that this figure has been under the effect of a geometric transformation.

i.e. The geometric transformation maps each point P in the plane onto an image point  $\bar{P}$  in the same plane

**Example 1** Write the geometric transformation which affected on the figure (reflection - translation - rotation) in each of the following :



**Solution**

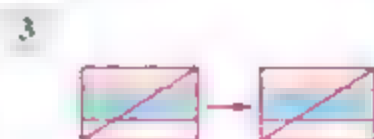
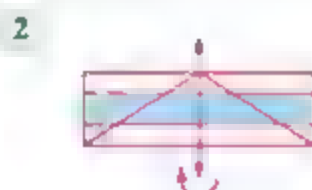
1 Translation.

2 Rotation.

3 Reflection

## TRY

Describe the type of the geometric transformation (reflection, translation or rotation) in each of the following figures :



## Lesson Eight

**Example 2** Draw the image of each of the following figures according to the illustrated geometric transformation , then describe its type :

1  $(X, y) \longrightarrow (X - 4, y + 1)$



2  $(X, y) \longrightarrow (-X, -y)$



3  $(X, y) \longrightarrow (X, -y)$



**Solution**

1  $(X, y) \longrightarrow (X - 4, y + 1)$

i.e.  $X$  transferred to  $X - 4$  and

$y$  transferred to  $y + 1$  , therefore we get

$A(2, 0) \longrightarrow \hat{A}(2 - 4, 0 + 1)$

i.e.  $\hat{A}(-2, 1)$

$B(4, 2) \longrightarrow \hat{B}(4 - 4, 2 + 1)$

i.e.  $\hat{B}(0, 3)$

$C(1, 3) \longrightarrow \hat{C}(1 - 4, 3 + 1)$

i.e.  $\hat{C}(-3, 4)$

From the graph , it is shown that  $\Delta ABC$  has been translated to become  $\Delta \hat{A}\hat{B}\hat{C}$



## Unit 3

$$2 \quad (x, y) \longrightarrow (-x, -y)$$

i.e.

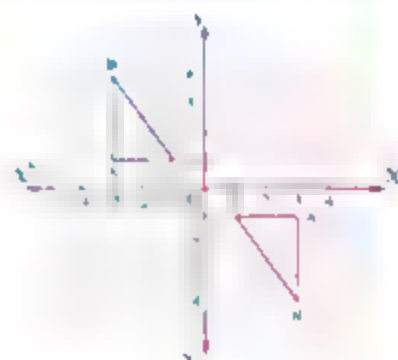
$$A(-3, 1) \longrightarrow \hat{A}(3, -1)$$

[Note :  $-(-3) = 3$ ]

$$B(-3, 4) \longrightarrow \hat{B}(3, -4)$$

$$C(-1, 1) \longrightarrow \hat{C}(1, -1)$$

From the graph, it is shown that  $\Delta ABC$  has been rotated to become  $\Delta \hat{A}\hat{B}\hat{C}$



$$3 \quad (x, y) \longrightarrow (x, -y)$$

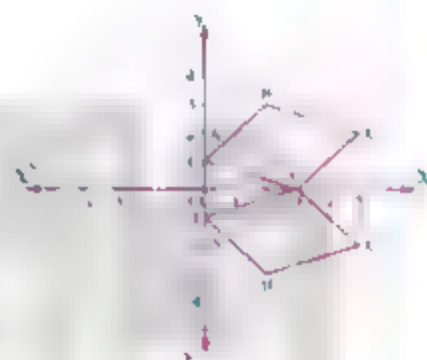
$$A(0, 1) \longrightarrow \hat{A}(0, -1)$$

$$B(2, 3) \longrightarrow \hat{B}(2, -3)$$

$$C(5, 2) \longrightarrow \hat{C}(5, -2)$$

$$D(3, 0) \longrightarrow \hat{D}(3, 0)$$

From the graph, it is shown that the shape ABCD has been reflected to become  $\hat{A}\hat{B}\hat{C}\hat{D}$



**Example 3** Draw the image of the quadrilateral ABCD where A(1, 1), B(4, 1), C(4, 3), D(1, 5) according to each of the following transformations, then describe its type :

$$1 \quad (x, y) \longrightarrow (-x, y)$$

$$2 \quad (x, y) \longrightarrow (-y, x)$$

$$3 \quad (x, y) \longrightarrow (x, y - 5)$$

**Solution**

$$1 \quad (x, y) \longrightarrow (-x, y)$$

i.e.

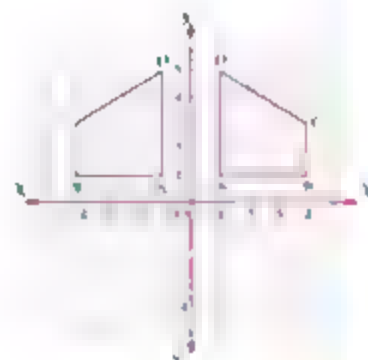
$$A(1, 1) \longrightarrow \hat{A}(-1, 1)$$

$$B(4, 1) \longrightarrow \hat{B}(-4, 1)$$

$$C(4, 3) \longrightarrow \hat{C}(-4, 3)$$

$$D(1, 5) \longrightarrow \hat{D}(-1, 5)$$

The transformation is reflection.



## Lesson Eight

2  $(X, y) \longrightarrow (-y, X)$

i.e.

$A(1, 1) \longrightarrow \hat{A}(-1, 1)$

$B(4, 1) \longrightarrow \hat{B}(-1, 4)$

$C(4, 3) \longrightarrow \hat{C}(-3, 4)$

$D(1, 5) \longrightarrow \hat{D}(-5, 1)$

The transformation is rotation.

3  $(X, y) \longrightarrow (X, y - 5)$

i.e.

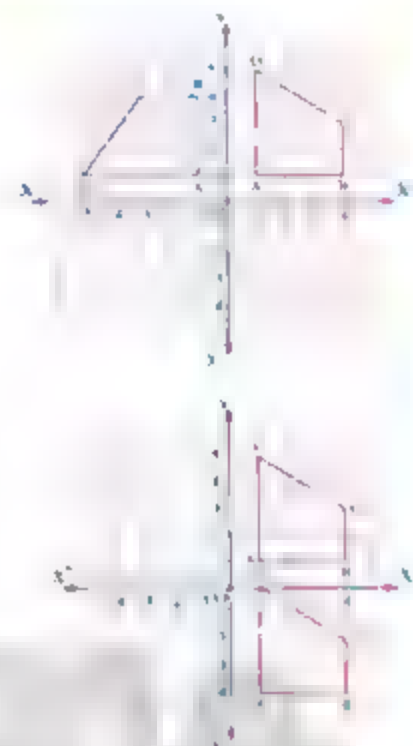
$A(1, 1) \longrightarrow \hat{A}(1, -4)$

$B(4, 1) \longrightarrow \hat{B}(4, -4)$

$C(4, 3) \longrightarrow \hat{C}(4, -2)$

$D(1, 5) \longrightarrow \hat{D}(1, 0)$

The transformation is translation



## TRY 2

Map each of the following shapes due to the geometric transformation above it, then describe its type :

1  $(X, y) \longrightarrow (-X, y)$



2  $(X, y) \longrightarrow (X, y)$



3  $(X, y) \longrightarrow (X, y + 2)$



3 translation

3 translation

2 reflection

2 reflection

2 rotation

1 rotation

Answers of try 2



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## Reflection in a straight line

### Prelude

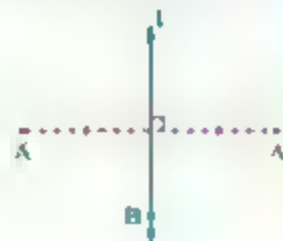
If you stand in front of a plane mirror, then you will see your picture (image) reflected in the mirror in the same size and details and you will notice also that the distance between the image and the mirror equals the real distance between you and the mirror. If you approach the mirror, then you will find that your image approaches also the mirror.



### Definition of reflection in a straight line

Reflection in the straight line  $L$  maps each point  $A$  to the point  $\hat{A}$  in the same plane such that :

- 1 If  $A \notin L$ , then the straight line  $L$  is the perpendicular bisector to the line segment  $\overline{A\hat{A}}$
- 2 If  $B \in L$ , then  $B$  is reflected onto itself  
i.e.  $\hat{B}$  coincides  $B$





## Lesson Nine

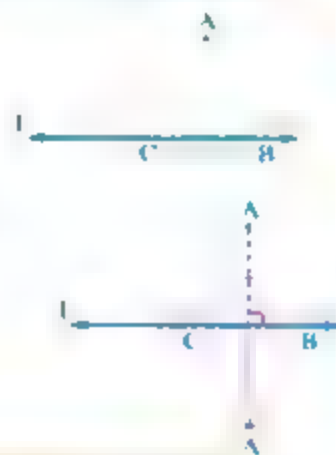
## Finding the image of a point by reflection in a given straight line

- To find  $\hat{A}$  which is the image of  $A$  by reflection in the straight line  $L$ , we do as follows :

- 1 Draw an arc of a circle with centre  $A$  to cut  $L$  at  $B$  and  $C$
- 2 With the same radius length taking  $B$  and  $C$  as centres, draw two arcs in the other side of the straight line  $L$  to intersect at  $\hat{A}$ , then  $\hat{A}$  is the image of  $A$  by reflection in  $L$ .

Check by measuring that :

$\overline{AA'} \perp L$  and  $L$  bisects  $\overline{AA'}$



## Finding the image of a polygon by reflection in a given straight line

- To find the image of a polygon as  $\triangle ABC$  by reflection in the straight line  $L$ , we do as follows :

- 1 Find the image of each vertex of  $\triangle ABC$  by reflection in the straight line  $L$  as we did before ( $\hat{A}$  is the image of  $A$ ,  $\hat{B}$  is the image of  $B$  and  $\hat{C}$  is the image of  $C$ )
- 2 Draw  $\hat{A}\hat{B}$ ,  $\hat{B}\hat{C}$  and  $\hat{C}\hat{A}$ , then  $\triangle \hat{A}\hat{B}\hat{C}$  is the image of  $\triangle ABC$  by reflection in the straight line  $L$ .

Check by measuring that :

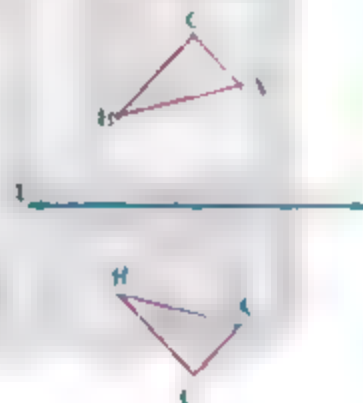
- $AB = \hat{A}\hat{B}$ ,  $BC = \hat{B}\hat{C}$  and  $CA = \hat{C}\hat{A}$
- $m(\angle A) = m(\angle \hat{A})$ ,  $m(\angle B) = m(\angle \hat{B})$  and  $m(\angle C) = m(\angle \hat{C})$

From the previous, we deduce that :

**Reflection** is a geometrical transformation which transforms the geometrical shape into another one congruent to it (equal to it in its side lengths and angle measures), but the direction of reading the shape is the opposite direction of reading the image.

Notice that :

The reading of  $\triangle ABC$  is clockwise  while the reading of  $\triangle \hat{A}\hat{B}\hat{C}$  is anticlockwise .



$\triangle ABC \cong \triangle \hat{A}\hat{B}\hat{C}$

## Unit 3

## Properties of reflection in a straight line

Illustrated  
example

Draw the image of the rectangle ABCD in which  $AB = 4 \text{ cm}$ ,  
 $BC = 2 \text{ cm}$ , by reflection in  $\overleftrightarrow{AB}$

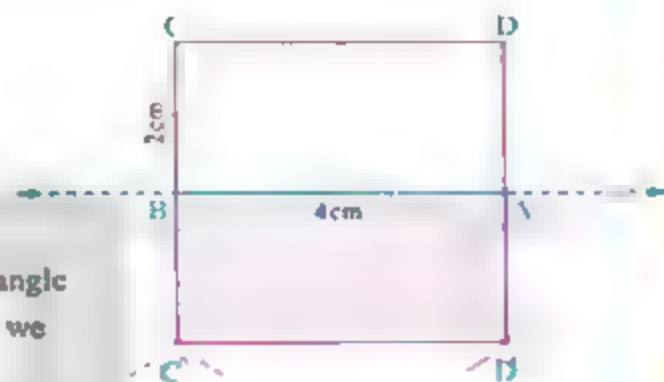
Solution

First

We draw the rectangle  
ABCD in which :  
 $AB = 4 \text{ cm}$ , and  $BC = 2 \text{ cm}$ .

Second

To find the image of the rectangle  
ABCD by reflection in  $\overleftrightarrow{AB}$ , we  
do as follows :



- 1 The images of A and B by reflection in  $\overleftrightarrow{AB}$  are the same because they belong to  $\overleftrightarrow{AB}$
- 2 We find the image of D by reflection in  $\overleftrightarrow{AB}$ , let it be  $D'$ , the image of C by reflection in  $\overleftrightarrow{AB}$ , let it be  $C'$ , then we get the rectangle  $AB'C'D'$  to be the image of the rectangle ABCD by reflection in  $\overleftrightarrow{AB}$

Notice that

- 1  $AD = A'D'$ ,  $DC = D'C'$ ,  $CB = C'B$  and AB is a common side

i.e. Reflection in a straight line reserves the lengths of the line segments

- 2  $m(\angle BAD) = m(\angle B A D')$   
 $\therefore m(\angle ABC) = m(\angle A B C')$   
 $\therefore m(\angle C) = m(\angle C')$  and  $m(\angle D) = m(\angle D')$

i.e. Reflection in a straight line reserves the measures of the angles.

- 3 From the rectangle ABCD  $AD \parallel BC$   
 $\therefore$  from the rectangle  $AB'C'D'$   $A'D' \parallel B'C'$   
 $\therefore$  The images of the two parallel line segments are also two parallel line segments.

i.e. Reflection in a straight line reserves the parallelism.

- 4 The reading of the rectangle ABCD is in the clockwise direction while the reading of the rectangle  $AB'C'D'$  is in anticlockwise direction.

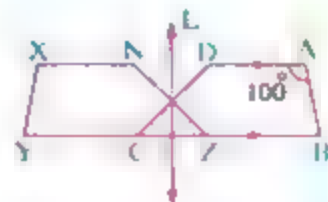
i.e. Reflection in a straight line doesn't reserve the orientation of the vertices of the figure

- 5 If a point lies on  $\overline{DC}$  and we find its image by reflection in  $\overleftrightarrow{AB}$ , we find its image lie on  $\overline{D'C'}$

i.e. Reflection in a straight line reserves the betweenness.

**Example 1** In the opposite figure :

ABCD is a quadrilateral in which  $m(\angle A) = 100^\circ$  and  $AD \parallel BC$ . If the figure XYZN is the image of the figure ABCD by reflection in the straight line L. Find :  $m(\angle Y)$



**Solution**

$\therefore$  The figure XYZN is the image of the figure ABCD by reflection in L

$\therefore$  X is the image of A by reflection in L

$\therefore m(\angle X) = m(\angle A) = 100^\circ$

(because reflection in the straight line reserves the measures of angles)

$\therefore \overline{XN}$  is the image of  $\overline{AD}$  and  $\overline{YZ}$  is the image of  $\overline{BC}$  by reflection in L

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore \overline{XN} \parallel \overline{YZ}$  (because reflection in a straight line reserves parallelism)

$\therefore m(\angle X) + m(\angle Y) = 180^\circ$  (two interior angles in the same side of the transversal)

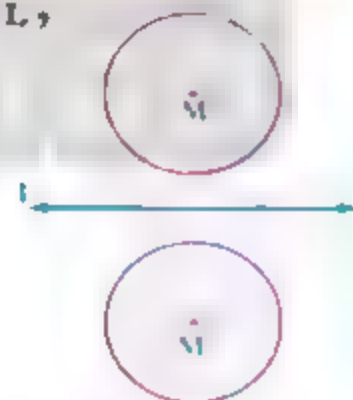
$\therefore m(\angle Y) = 180^\circ - 100^\circ = 80^\circ$

(The req.)

### Finding the Image of a circle by reflection in a given straight line

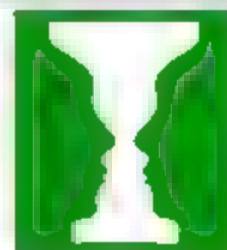
To find the image of a circle M by reflection in the straight line L, we do as follows :

- 1 Find the image of the centre M by reflection in L as we did before, say  $M'$
- 2 Use the compasses with radius length equal to the radius length of the circle M to draw a circle with centre  $M'$  that will be the image of the circle M by reflection in L



**Optical illusion :**

Look at the picture. What do you see ?



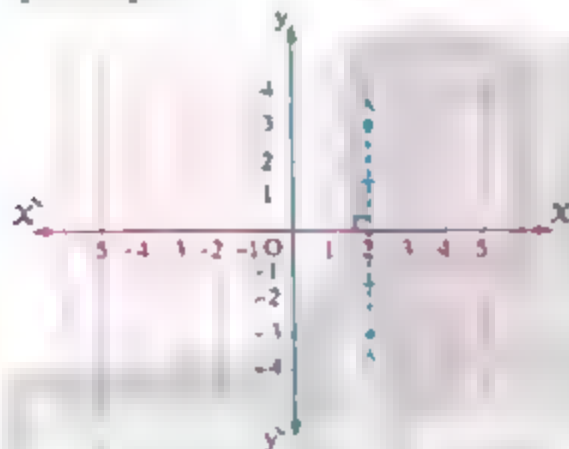


## Unit 3

## Reflection in the Cartesian coordinates plane.

## Reflection in the X-axis

- To find the image of the point A (2, 3) by reflection in  $\overleftrightarrow{XX}$  (the X-axis): draw  $\overleftrightarrow{AA}$  such that  $\overleftrightarrow{XX}$  is the line of symmetry of it.



Then we find that :

$$A(2, 3) \longrightarrow \hat{A}(2, -3)$$

i.e. The reflection in the X-axis changes the sign of the 2<sup>nd</sup> projection (y-coordinates)

$$A(X, y) \xrightarrow{\text{by reflection in the X-axis}} \hat{A}(X, -y)$$

For example:

$$(2, 4) \xrightarrow{\text{by reflection in the X-axis}} (2, -4)$$

$$(-2, -1) \xrightarrow{\text{by reflection in the X-axis}} (-2, 1)$$

$$(-5, 3) \xrightarrow{\text{by reflection in the X-axis}} (-5, -3)$$

$$(7, -2) \xrightarrow{\text{by reflection in the X-axis}} (7, 2)$$

## Reflection in the y-axis

- To find the image of the point A (2, 3) by reflection in  $\overleftrightarrow{yy}$  (the y-axis): draw  $\overleftrightarrow{AA}$  such that  $\overleftrightarrow{yy}$  is the line of symmetry of it.



Then we find that :

$$A(2, 3) \longrightarrow \hat{A}(-2, 3)$$

i.e. The reflection in the y-axis changes the sign of the 1<sup>st</sup> projection (X-coordinates)

$$A(X, y) \xrightarrow{\text{by reflection in the y-axis}} \hat{A}(-X, y)$$

For example:

$$(2, 4) \xrightarrow{\text{by reflection in the y-axis}} (-2, 4)$$

$$(-2, -1) \xrightarrow{\text{by reflection in the y-axis}} (2, -1)$$

$$(-5, 3) \xrightarrow{\text{by reflection in the y-axis}} (5, 3)$$

$$(7, -2) \xrightarrow{\text{by reflection in the y-axis}} (-7, -2)$$

## Remarks

- The image of the point  $(x, 0)$  by reflection in the  $x$ -axis is itself because it lies on the  $x$ -axis.  
For example:  $(5, 0) \xrightarrow[\text{the } x\text{-axis}]{\text{by reflection in}} (5, 0)$
- The image of the point  $(0, y)$  by reflection in the  $y$ -axis is itself because it lies on the  $y$ -axis.  
For example:  $(0, -3) \xrightarrow[\text{the } y\text{-axis}]{\text{by reflection in}} (0, -3)$
- The image of the point  $(0, 0)$  by reflection in the  $x$ -axis or the  $y$ -axis is itself because it lies on both two axes.



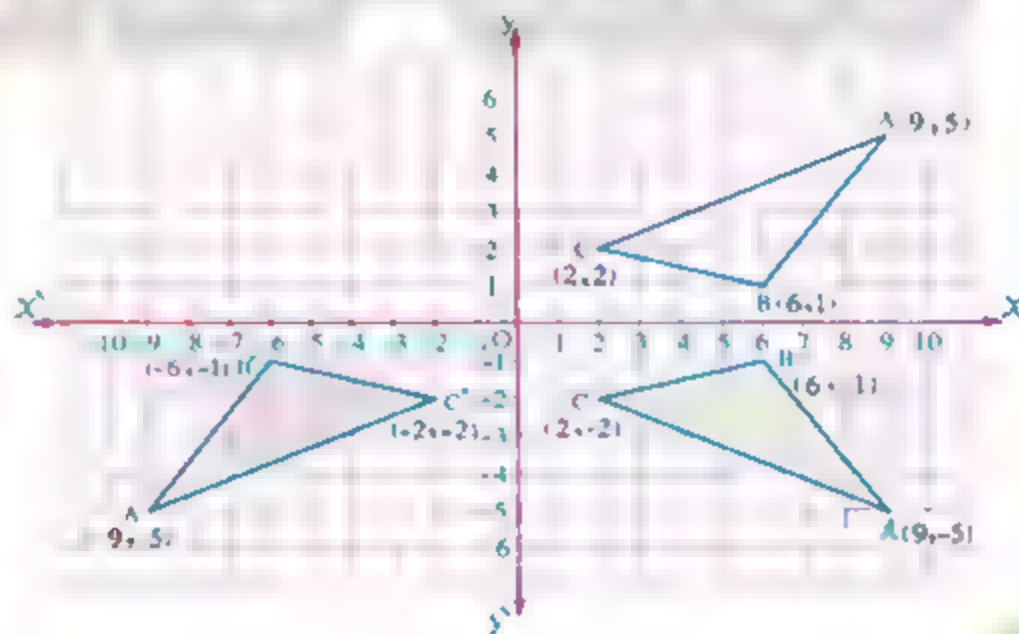
Complete the following table :

The point	$(5, 1)$	$(2, -3)$	$(-1, 4)$	$(-2, -6)$	$(0, -1)$	$(3, 0)$	$(0, 0)$
Its image by reflection in the $x$ -axis							
Its image by reflection in the $y$ -axis							

**Example 2** Draw on a square lattice the triangle  $ABC$  where  $A(9, 5)$ ,  $B(6, 1)$  and  $C(2, 2)$  :

- Draw  $\triangle A'B'C'$  which is the image of  $\triangle ABC$  by reflection in the  $x$ -axis
- Draw  $\triangle A''B''C''$  which is the image of  $\triangle A'B'C'$  by reflection in the  $y$ -axis

Solution



3

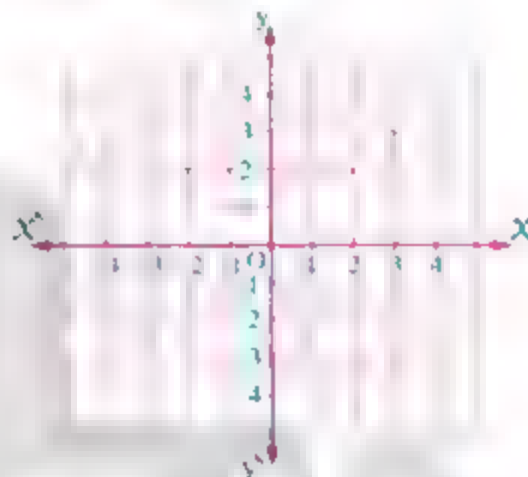
## TRY 2

Draw  $\triangle ABC$  where  $A(1, 1)$ ,  $B(4, 1)$  and  $C(3, 3)$ , then draw its image by reflection in :

1 The X-axis



2 The y-axis



## Symmetry

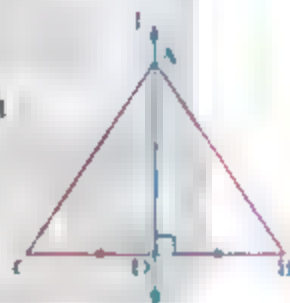
In the opposite figure :

$ABC$  is a triangle,  $\overline{AD} \perp \overline{BC}$ ,  $D$  is the midpoint of  $\overline{BC}$ , we find that

- The image of  $A$  by reflection in  $L$  is itself ( $A$ )
- The image of  $B$  by reflection in  $L$  is  $C$
- The image of  $C$  by reflection in  $L$  is  $B$

i.e. The image of  $\triangle ABC$  by reflection in  $L$  is  $\triangle ACB$

We can say that  $\triangle ABC$  is transformed to itself by reflection in the straight line  $L$ , therefore the straight line  $L$  is called the axis of symmetry of  $\triangle ABC$



From the previous, we can deduce the definition of the axis of symmetry as follows :





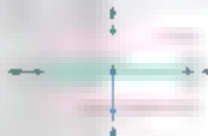
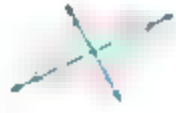

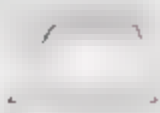




If the reflection in a line transforms the figure to itself, then this line is called an axis of symmetry of the figure.

## Remark

The axis of symmetry divides the figure into two congruent figures.



## The axes of symmetry of some geometric figures

The figure			
Number of axes of symmetry	1	3	Zero (does not exist)
The figure			
Number of axes of symmetry	Zero (does not exist)	2	2
The figure			
Number of axes of symmetry	4	Zero (does not exist)	1
The figure			
Number of axes of symmetry	An infinite number	5	6

3

**Example 3** On a square lattice, determine the point A, B and C in each case, then find the image of  $\Delta ABC$  by reflection in  $\overline{XX}$  (X-axis).

Mention if  $\overline{XX}$  is an axis of symmetry of  $\Delta ABC$  or not :

1  $A(1, 3), B(3, 1), C(0, -1)$

2  $A(1, 2), B(4, 0), C(1, -2)$

**Solution**

1  $A(1, 3) \longrightarrow \hat{A}(1, -3)$

$B(3, 1) \longrightarrow \hat{B}(3, -1)$

$C(0, -1) \longrightarrow \hat{C}(0, 1)$

$\therefore \Delta \hat{A}\hat{B}\hat{C}$  is the image of  $\Delta ABC$  by reflection in X-axis

$\bullet \overline{XX}$  is not an axis of symmetry of  $\Delta ABC$  because it does not transform the figure ABC to itself by reflection.

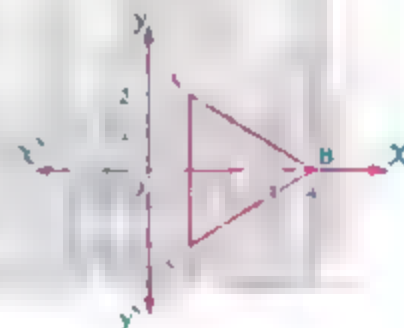
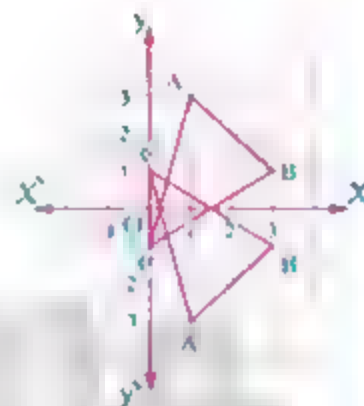
2  $A(1, 2) \longrightarrow C(1, -2)$

$B(4, 0) \longrightarrow B(4, 0)$  itself

$C(1, -2) \longrightarrow A(1, 2)$

$\therefore \Delta CBA$  is the image of  $\Delta ABC$  by reflection in  $\overline{XX}$  (X-axis)

$\bullet \overline{XX}$  is the axis of symmetry of  $\Delta ABC$  because it transforms  $\Delta ABC$  by reflection in it to itself.



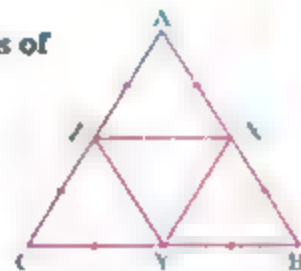
## Lesson Nine

## TRY 3

In the opposite figure :

$\triangle ABC$  is an equilateral triangle.  $X$ ,  $Y$  and  $Z$  are the midpoints of its sides. Complete the following :

- 1] The image of  $\triangle AXZ$  by reflection in  $\overline{XZ}$  is
- 2] The image of the figure  $AZYX$  by reflection in  $\overline{AY}$  is
- 3]  $\triangle ABC$  is the image of  $\triangle ACB$  by reflection in
- 4] The number of axes of symmetry of the figure  $ABYZ$  is
- 5] The number of axes of symmetry of  $\triangle ABC$  is



- 3] 1]  $\triangle YXZ$
- 2] Figure  $AXYZ$
- 3]  $\overline{AY}$

- 2] 1] The X-axis
- 2] The y-axis

- 1] • The image by reflection in the X-axis  
 $(5, -1) \rightarrow (2, 3) \rightarrow (-1, -4) \rightarrow (-2, 6) \rightarrow (0, 1) \rightarrow (3, 0) \rightarrow (0, 0)$   
 • The image by reflection in the y-axis  
 $(-5, 1) \rightarrow (-2, -3) \rightarrow (1, 4) \rightarrow (2, -6) \rightarrow (0, -1) \rightarrow (-3, 0) \rightarrow (0, 0)$

Answers of try by yourself





10

## Reflection in a point

### Definition of reflection in a point

Reflection in a point  $M$  maps each point  $A$  in the plane to the point  $\hat{A}$  in the same plane where  $M$  is the midpoint of the line segment  $\overline{AA'}$  ,  
the point  $M$  is called the centre of reflection and the image of  $M$  by reflection in  $M$  is itself.



### Finding the image of a point by reflection in a given point

- To find the image of a point as  $A$  by reflection in  $M$  ,  
we do as follows :

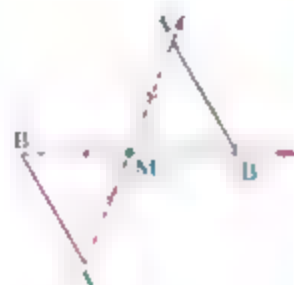
- 1 Draw  $\overline{AM}$
- 2 Using the compasses with a radius length equals  $MA$  ,  
then use  $M$  as a centre and draw an arc to intersect  $\overline{AM}$   
at a point as  $\hat{A}$  ,  
then  $\hat{A}$  is the image of the point  $A$  by reflection in the point  $M$
- 3 From the previous , we found that :  $MA = M\hat{A}$



**Finding the image of a line segment by reflection in a given point**

• To find the image of a line segment  $\overline{AB}$  by reflection in  $M$ , we do as follows :

- 1 Find the image of  $A$  by reflection in  $M$  to be  $\hat{A}$  as we mentioned before.
- 2 Similarly find the image of  $B$  by reflection in  $M$  to be  $\hat{B}$
- 3 Draw  $\overline{\hat{A}\hat{B}}$  to be the image of  $\overline{AB}$  by reflection in the point  $M$



**Notice that :**

$$\hat{A}\hat{B} = AB \text{ and } \hat{A}\hat{B} \parallel AB$$

i.e. The image of a line segment by reflection in a point is a line segment parallel to the original one and its length equals the length of the original line segment.

**Finding the image of a polygon by reflection in a given point**

• To find the image of a polygon as the triangle  $ABC$  by reflection in  $M$ , we do as follows :

- 1 Find the image of each vertex of the vertices of the triangle  $ABC$  by reflection in the point  $M$  as we mentioned before to be :



$\hat{A}$  is the image of  $A$ ,  $\hat{B}$  is the image of  $B$  and  $\hat{C}$  is the image of  $C$

- 2 Draw  $\overline{\hat{A}\hat{B}}$ ,  $\overline{\hat{B}\hat{C}}$  and  $\overline{\hat{C}\hat{A}}$  to get  $\Delta \hat{A}\hat{B}\hat{C}$  which is the image of  $\Delta ABC$  by reflection in the point  $M$

**Notice that :**

$$\Delta ABC = \Delta \hat{A}\hat{B}\hat{C}, \text{ therefore it is said that the reflection in a point is isometric}$$

**From the previous, we deduce that :**

Reflection in a point is a geometric transformation that maps the geometric figure to another geometric figure congruent to it and has the same orientation of its vertices

## Unit 3

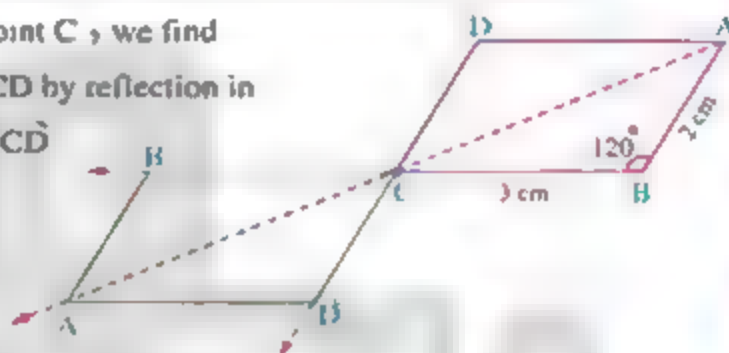
## Properties of reflection in a point

## Illustrated example

Draw the parallelogram ABCD in which  $AB = 2 \text{ cm}$ ,  $BC = 3 \text{ cm}$ , and  $m(\angle B) = 120^\circ$ , then draw its image by reflection in the point C and show what you observe.

## Solution

Finding the image of each vertex of the vertices of  $\square ABCD$  by reflection in the point C, we find the image of  $\square ABCD$  by reflection in the point C is  $\square \hat{A}\hat{B}\hat{C}\hat{D}$



## Notice that :

1  $\hat{A}\hat{B} = AB$ ,  $\hat{B}\hat{C} = BC$ ,  $\hat{C}\hat{D} = CD$  and  $\hat{D}\hat{A} = DA$

i.e.

Reflection in a point reserves the lengths of the line segments

2  $m(\angle \hat{A}) = m(\angle A)$ ,  $m(\angle \hat{B}) = m(\angle B)$ ,  
 $m(\angle \hat{B}\hat{C}\hat{D}) = m(\angle BCD)$  and  $m(\angle \hat{D}) = m(\angle D)$

i.e.

Reflection in a point reserves the measures of the angles.

3 From the parallelogram ABCD :  $\overline{AB} \parallel \overline{DC}$ ,  
From the parallelogram  $\hat{A}\hat{B}\hat{C}\hat{D}$  :  $\overline{\hat{A}\hat{B}} \parallel \overline{\hat{D}\hat{C}}$   
 $\therefore$  The images of the two parallel line segments are also two parallel line segments.

i.e.

Reflection in a point reserves the parallelism.

4 The reading of the parallelogram ABCD is in the clockwise direction and the reading of the parallelogram  $\hat{A}\hat{B}\hat{C}\hat{D}$  is in the clockwise direction also.

i.e.

Reflection in a point reserves the orientation of the vertices of the figure.

5 Putting a point belongs to  $\overline{AB}$ , we find its image by reflection in C belongs to  $\overline{\hat{A}\hat{B}}$

i.e.

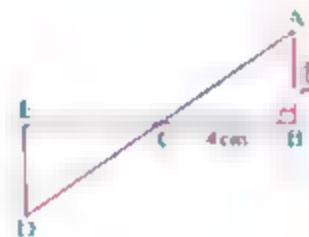
Reflection in a point reserves the betweenness.



**Example 1** In the opposite figure :

$ABC$  is a triangle in which  $m(\angle B) = 90^\circ$ ,  
 $AB = 3$  cm,  $BC = 4$  cm, and  $\triangle DEC$   
 is the image of  $\triangle ABC$  by reflection in  $C$

- 1 Find : The length of  $\overline{DC}$
- 2 Prove that :  $\overline{AB} \parallel \overline{DE}$



**Solution**

In  $\triangle ABC$  :  $\because m(\angle B) = 90^\circ$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25 \text{ (Pythagoras' theorem)}$$

$$\therefore AC = 5 \text{ cm.}$$

$\because \triangle DEC$  is the image of  $\triangle ABC$  by reflection in  $C$

$$\therefore DC = AC = 5 \text{ cm.}$$

(because reflection reserves the lengths of the line segments) (First req.)

$\because m(\angle E) = m(\angle B) = 90^\circ$  (properties of reflection in a point) and they are alternate angles

$$\therefore \overline{AB} \parallel \overline{DE}$$

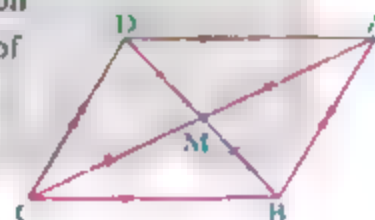
(Second req.)

### Using reflection in a point to prove that a quadrilateral is a parallelogram

We mentioned before that the image of a line segment by reflection in a point is a line segment parallel to it and has the same length of the original line segment.

If  $\overline{CD}$  is the image of  $\overline{AB}$  by reflection

in the point  $M$ , then  $\overline{AB} \parallel \overline{DC}$  and  $AB = DC$



We can prove that the quadrilateral  $ABCD$  is a parallelogram by several methods as follows :

$$1 \because AB = DC \text{ and } \overline{AB} \parallel \overline{DC}$$

$\therefore$  The quadrilateral  $ABCD$  is a parallelogram

$$2 \because \overline{CD} \text{ is the image of } \overline{AB} \text{ by reflection in the point } M$$

$$\therefore MA = MC \text{ and } MB = MD$$

$\therefore$  The quadrilateral  $ABCD$  is a parallelogram



#### Remember that

The quadrilateral in which two opposite sides are parallel and equal in length is a parallelogram.



#### Remember that

The quadrilateral whose diagonals bisect each other is a parallelogram

3

3  $\therefore MA = MC$  and  $MB = MD$

$\therefore \overline{AD}$  is the image of  $\overline{CB}$  by reflection in the point M

$\therefore \overline{CD}$  is the image of  $\overline{AB}$  by reflection in the point M

$\therefore \overline{AB} \parallel \overline{DC}$  and  $\overline{AD} \parallel \overline{BC}$

$\therefore$  The quadrilateral ABCD is a parallelogram

4  $\therefore \overline{AD}$  is the image of  $\overline{CB}$  and  $\overline{CD}$  is the image of  $\overline{AB}$  by reflection in the point M

$\therefore AD = CB$  and  $CD = AB$

$\therefore$  The quadrilateral ABCD is a parallelogram

### Remember that

The quadrilateral in which each two opposite sides are parallel is a parallelogram

### Remember that

The quadrilateral in which each two opposite sides are equal in length is a parallelogram.

**Example 2** In the opposite figure :

ABCD is a parallelogram , M is the point of intersection of its diagonals ,  $E \in AD$  and

$\overline{EM} \cap \overline{BC} = \{F\}$

Prove that :

- 1 F is the image of E by reflection in M
- 2 The quadrilateral AFCE is a parallelogram.



**Solution**

$\therefore$  ABCD is a parallelogram.

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore$  In  $\Delta AME$  and  $\Delta CMF$  :

$m(\angle DAC) = m(\angle BCA)$  (alternate angles)

$m(\angle AME) = m(\angle FMC)$  (V.O.A)

$AM = MC$  (properties of parallelogram)

$\therefore \Delta AME \cong \Delta CMF$  , then we deduce that  $EM = FM$

$\therefore F \in \overline{EM}$

$\therefore$  F is the image of E by reflection in the point M

(Q.E.D. 1)

$\therefore AM = CM$  and  $A \in \overline{CM}$

$\therefore$  A is the image of C by reflection in the point M

$\therefore \overline{AF}$  is the image of  $\overline{CE}$  by reflection in the point M

$\therefore AF = CE$  and  $\overline{AF} \parallel \overline{CE}$

$\therefore$  The quadrilateral AFCE is a parallelogram

(Q.E.D. 2)

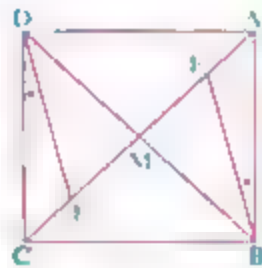


## Lesson Ten

**Example 3** In the opposite figure :

ABCD is a square whose diagonals intersect at M ,  
 $E \in \overline{AC}$  and  $F \in \overline{AC}$  where  $m(\angle ABE) = m(\angle CDF)$   
 Prove that :

- 1  $\triangle ABE$  is the image of  $\triangle CDF$  by reflection in M
- 2 The quadrilateral EBFD is a rhombus

**Solution**

In  $\triangle ABE$  and  $\triangle CDF$  :

- $AB = CD$  (properties of the square)
- $m(\angle BAE) = m(\angle DCF) = 45^\circ$  (properties of the square)
- $m(\angle ABE) = m(\angle CDF)$  (given)

$\therefore \triangle ABE \cong \triangle CDF$  , then we deduce that  $AE = CF$

$\therefore AM = CM$  (properties of the square)

$\therefore AM - AE = CM - CF$

$\therefore EM = FM$  ,  $E \in \overline{FM}$   $\therefore E$  is the image of  $F$  by reflection in M (1)

$AM = CM$  ,  $A \in \overline{CM}$   $\therefore A$  is the image of  $C$  by reflection in M

$BM = DM$  ,  $B \in \overline{DM}$   $\therefore B$  is the image of  $D$  by reflection in M (2)

$\therefore \triangle ABE$  is the image of  $\triangle CDF$  by reflection in M (Q.E.D. 1)

From (1) and (2) :

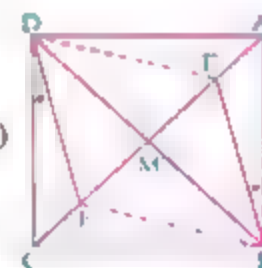
$\therefore \overline{EB}$  is the image of  $\overline{FD}$  by reflection in M

$\therefore \overline{EB} \parallel \overline{FD}$  and  $EB = FD$

$\therefore$  The quadrilateral EBFD is a parallelogram.

$\therefore \overline{DB} \perp \overline{EF}$  (properties of the square)

$\therefore$  The quadrilateral EBFD is a rhombus. (Q.E.D. 2)



Draw any triangle ABC , then draw its image  $\hat{A}\hat{B}\hat{C}$  by reflection in C , then prove that the quadrilateral AB $\hat{A}\hat{B}$  is a parallelogram.



## Unit 3

## Reflection in the origin point

- If the point  $C$  is a point in the Cartesian coordinates plane where  $C (2, 3)$
- To find the image of the point  $C$  by reflection in the origin point  $O$  using the same previous method , we will find it  $\hat{C} (-2, -3)$



**Notice that :** The signs of the two projections of the ordered pair  $(2, 3)$  have been changed , hence we can define the reflection in the origin point as follows :

## Definition

If  $A (X, y)$  is a point in the Cartesian coordinates plane , then the image of the point  $A$  by reflection in the origin point  $O$  is  $\hat{A} (-X, -y)$

i.e. Reflection in the origin point converts the sign of each of the coordinates of the point.

$$\therefore \text{The image of the point } (X, y) \xrightarrow{\text{by reflection in the origin point}} (-X, -y)$$

**For example:**

- The image of the point  $(2, 3) \xrightarrow{\text{by reflection in the origin point}} (-2, -3)$
- The image of the point  $(-4, 1) \xrightarrow{\text{by reflection in the origin point}} (4, -1)$
- The image of the point  $(5, -2) \xrightarrow{\text{by reflection in the origin point}} (-5, 2)$
- The image of the point  $(-3, -6) \xrightarrow{\text{by reflection in the origin point}} (3, 6)$

## Remark

The image of the point  $(0, 0)$  by reflection in the origin point is itself.

**Example 4** Draw  $\triangle ABC$  where  $A(4, 1)$ ,  $B(2, 4)$  and  $C(-1, 3)$ , then map draw image by reflection in the origin point.

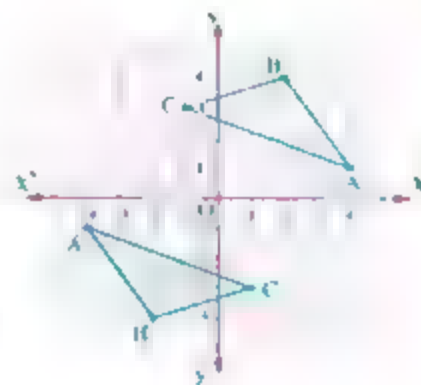
**Solution**

$$\therefore (X, y) \xrightarrow[\text{the origin point}]{\text{by reflection in}} (-X, -y)$$

$$\therefore A(4, 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{A}(-4, -1)$$

$$B(2, 4) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{B}(-2, -4)$$

$$C(-1, 3) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{C}(1, -3)$$



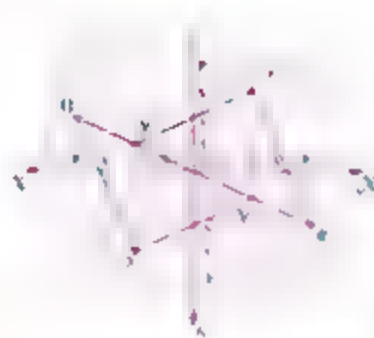
**TRY 2**

Draw on a square lattice  $\triangle ABC$ , where  $A(-2, 1)$ ,  $B(4, -2)$  and  $C(2, 3)$ , then draw its image by reflection in the origin point:

$$A(-2, 1) \xrightarrow[\text{the origin point}]{\text{by reflection in}} \hat{A}(\quad, \quad)$$

$$B(4, -2) \longrightarrow \hat{B}(\quad, \quad)$$

$$C(\quad, \quad) \longrightarrow \hat{C}(\quad, \quad)$$



2

Prove by yourself  
[Hint:  $\overline{AB} = \overline{A'B}$ ,  $\overline{AB} \parallel \overline{A'B}$ ]



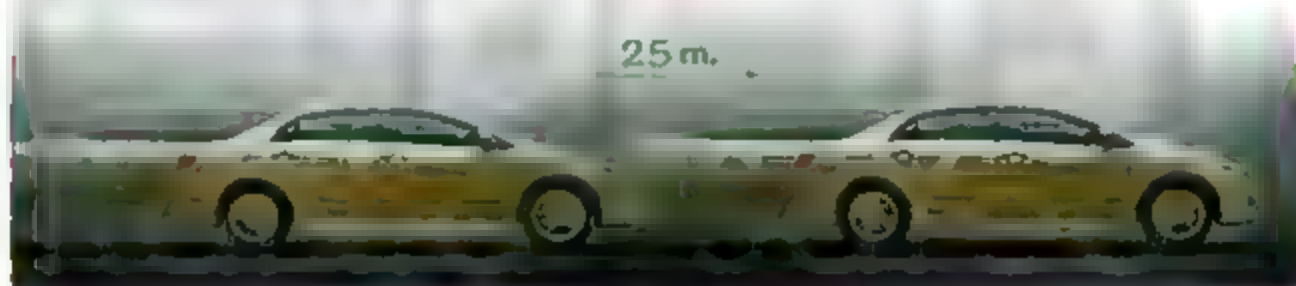
1



## Translation

### Prelude

If a car moved a distance 25 metres in a straight line forward , then we say that :  
The car translated for a distance 25 metres forward.



i.e. To determine the new position of the car after its movement , we should know two important elements which are :

- 1 The magnitude of the translation (25 metres).
- 2 The direction of the translation (forward in a straight line).

According to this, we can say that :

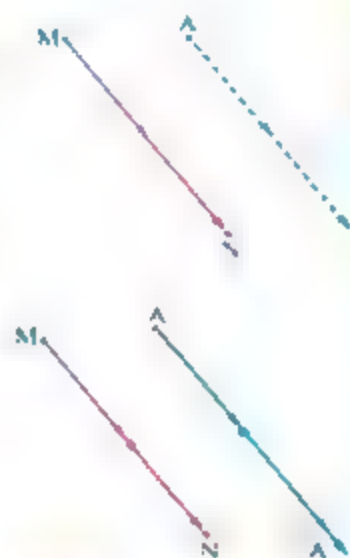
Translation is a geometrical transformation which maps each point A in the plane to another point  $\hat{A}$  in the same plane with a constant distance in a certain direction.



## Translation in the plane

## Finding the image of a point by a given translation

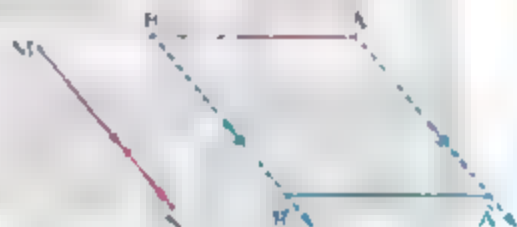
- To find  $\hat{A}$  which is the image of  $A$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$ , we do as follows :
  - 1 Draw from  $A$  a ray parallel to  $\overrightarrow{MN}$  and in the same direction
  - 2 By the compasses in  $A$  as a centre with radius  $= MN$ , draw an arc to intersect the ray drawn from  $A$  at the point  $\hat{A}$  ( $A\hat{A} = MN$  and  $\overrightarrow{AA} \parallel \overrightarrow{MN}$ )
- Then  $\hat{A}$  is the image of  $A$  by translation of magnitude  $MN$  in the direction of  $\overrightarrow{MN}$



## Finding the image of a line segment by a given translation

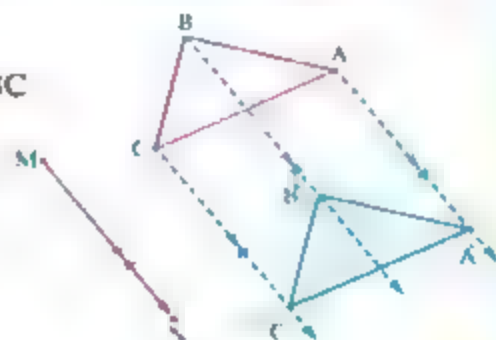
- To find the image of  $\overline{AB}$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$ , we do as follows :
  - 1 Find the image of the point  $A$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$  as we mentioned before, say  $\hat{A}$
  - 2 Similarly, we find the image of the point  $B$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$ , say  $\hat{B}$
  - 3 Draw  $\overline{\hat{A}\hat{B}}$  to be the image of  $\overline{AB}$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$

Check that :  $AB = \hat{A}\hat{B}$  and  $\overline{AB} \parallel \overline{\hat{A}\hat{B}}$



## Finding the image of a polygon by a given translation

- To find the image of a polygon as  $\triangle ABC$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$ , we do as follows :
  - 1 Find the image of each vertex of the vertices of  $\triangle ABC$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$  as we mentioned before (say  $\hat{A}$  for  $A$ ,  $\hat{B}$  for  $B$  and  $\hat{C}$  for  $C$ )
  - 2 Draw  $\overline{\hat{A}\hat{B}}$ ,  $\overline{\hat{B}\hat{C}}$  and  $\overline{\hat{C}\hat{A}}$  then  $\triangle \hat{A}\hat{B}\hat{C}$  is the image of  $\triangle ABC$  by translation  $MN$  in the direction of  $\overrightarrow{MN}$



3

Check that :

$$AB = \hat{A}\hat{B}, BC = \hat{B}\hat{C} \text{ and } CA = \hat{C}\hat{A}$$

$$m(\angle A) = m(\angle \hat{A}), m(\angle B) = m(\angle \hat{B}), m(\angle C) = m(\angle \hat{C})$$

From the previous , we deduce that translation is a geometrical transformation which maps the geometrical figure to another geometrical figure congruent to it.

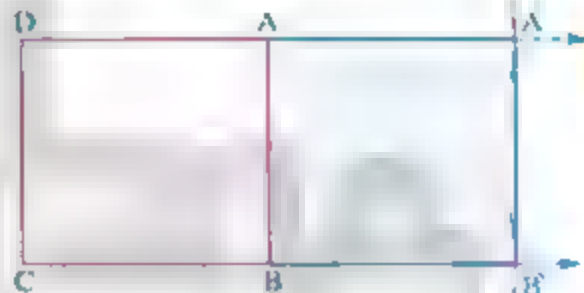
### Properties of translation

Illustrated  
example

Draw the square ABCD whose side length is 3 cm. , then draw its image by translation AB in the direction  $\overrightarrow{DA}$

**Solution**

The square  $\hat{A}\hat{B}\hat{B}\hat{A}$  is the image of the square ABCD by translation AB in the direction  $\overrightarrow{DA}$



Notice that :

1  $\hat{A}\hat{B} = AB, AB = DC$

i.e.

Translation reserves the lengths of the line segments.

2  $m(\angle \hat{A}) = m(\angle BAD), m(\angle \hat{B}) = m(\angle CBA)$

i.e.

Translation reserves the measures of the angles.

3 From the square ABCD :  $\overline{AB} \parallel \overline{DC}$  , from the square  $\hat{A}\hat{B}\hat{B}\hat{A}$  :  $\overline{\hat{A}\hat{B}} \parallel \overline{\hat{A}\hat{B}}$

i.e.

Translation reserves the parallelism.

$\therefore$  The images of the two parallel line segments are also two parallel line segment.

4 The reading of the square ABCD is in the clockwise direction and the reading of the square  $\hat{A}\hat{B}\hat{B}\hat{A}$  is in the clockwise direction also.

i.e.

Translation reserves the orientation of the vertices of the figure.

5 If you take a point lies on  $\overline{AB}$  and find its image by the previous translation , you will find its image lies on  $\overline{\hat{A}\hat{B}}$

i.e.

Translation reserves the betweenness.

**Example 1** In the opposite figure :

ABC is a triangle, X is the image of B by translation of a distance CA in the direction of  $\vec{CA}$ ,

Y is the image of C by translation of a distance BA in the direction of  $\vec{BA}$



1 Prove that :  $\triangle XAB \equiv \triangle AYC$

2 Determine the translation which makes  $\triangle AYC$  the image of  $\triangle XAB$

**Solution**

X is the image of B by translation of a distance CA in the direction  $\vec{CA}$

$\therefore BX = CA$  and  $\vec{BX} \parallel \vec{CA}$

$\therefore m(\angle 1) = m(\angle 2)$  (alternate angles) (1)

Y is the image of C by translation of a distance BA in the direction  $\vec{BA}$

$\therefore CY = BA$ ,  $\vec{CY} \parallel \vec{BA}$

$\therefore m(\angle 2) = m(\angle 3)$  (alternate angles) (2)

From (1) and (2) : We deduce that  $m(\angle 1) = m(\angle 3)$

$\therefore$  In  $\triangle XAB$ ,  $\triangle AYC$  :

$$\begin{cases} BX = CA \text{ (properties of translation)} \\ BA = CY \text{ (properties of translation)} \\ m(\angle 1) = m(\angle 3) \text{ (proved)} \end{cases}$$

$\therefore \triangle XAB \equiv \triangle AYC$

(First req.)

The translation which makes  $\triangle AYC$  is the image of  $\triangle XAB$  is the translation of a distance BC in the direction of  $\vec{BC}$

(Second req.)

**Using the properties of translation to prove that a quadrilateral is a parallelogram**

We noticed that :

The image of a line segment by a translation is another line segment parallel to it and has the same length of the original line segment.

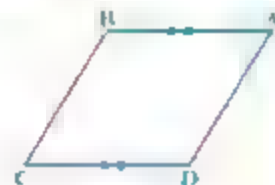
For example:

In the opposite figure :

If  $\vec{AB}$  is the image of  $\vec{DC}$  by a translation,

then :  $\vec{AB} \parallel \vec{DC}$  and  $AB = DC$

i.e. The figure ABCD is a parallelogram.

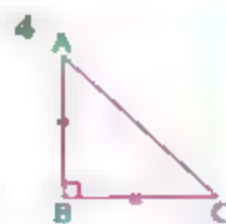
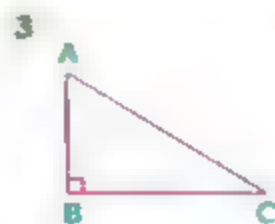
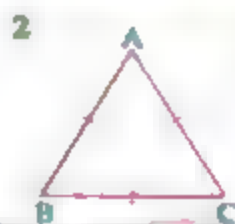
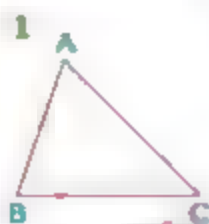
**Remember that**

The parallelogram is a quadrilateral in which two opposite sides are parallel and equal in length.



**Example 2** In each of the following figures :

Draw  $\triangle \hat{A}\hat{B}\hat{B}$  as the image of  $\triangle ABC$  by translation of a distance  $CB$  in the direction of  $\overrightarrow{CB}$ , then show the kind of the figure  $ABB\hat{A}$  in each case :



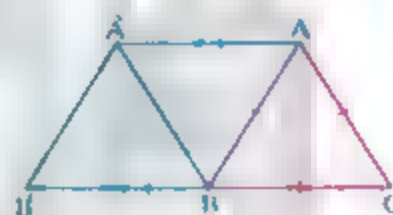
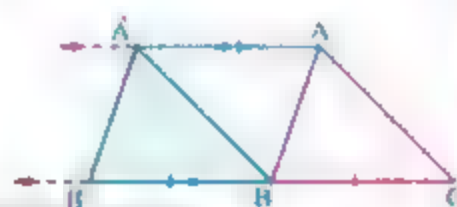
**Solution**

- 1  $\because \triangle \hat{A}\hat{B}\hat{B}$  is the image of  $\triangle ABC$   
 $\therefore \overline{A\hat{B}}$  is the image of  $\overline{AB}$   
 $\therefore \overline{A\hat{B}} \parallel \overline{AB}, \hat{A}\hat{B} = AB$   
 $\therefore$  The figure  $ABB\hat{A}$  is a parallelogram.

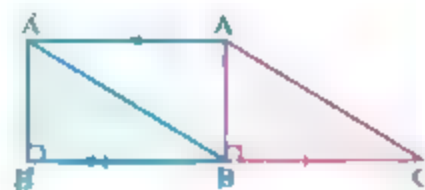
- 2  $\because \triangle \hat{A}\hat{B}\hat{B}$  is the image of  $\triangle ABC$   
 $\therefore \overline{A\hat{B}}$  is the image of  $\overline{AB}$   
 $\therefore \overline{A\hat{B}} \parallel \overline{AB}, \hat{A}\hat{B} = AB$   
 $\therefore$  The figure  $ABB\hat{A}$  is a parallelogram.

- $\because \triangle ABC = \triangle \hat{A}\hat{B}\hat{B}$   
 $\therefore \triangle \hat{A}\hat{B}\hat{B}$  is an equilateral triangle  
 $\therefore \hat{A}\hat{B} = \hat{B}\hat{B}$   
 $\therefore$  The figure  $ABB\hat{A}$  is a rhombus.

- 3  $\because \triangle \hat{A}\hat{B}\hat{B}$  is the image of  $\triangle ABC$   
 $\therefore \overline{A\hat{B}}$  is the image of  $\overline{AB}$   
 $\therefore \overline{A\hat{B}} \parallel \overline{AB}, \hat{A}\hat{B} = AB$   
 $\therefore$  The figure  $ABB\hat{A}$  is a parallelogram.  
 $\because m(\angle \hat{B}) = m(\angle B) = 90^\circ$   
 $\therefore$  The figure  $ABB\hat{A}$  is a rectangle.



**Remember that**  
 The rhombus is a parallelogram in which two adjacent sides are equal in length



**Remember that**  
 The rectangle is a parallelogram one of its angles is right.

4  $\therefore \triangle A'B'B$  is the image of  $\triangle ABC$

$\therefore \overline{A'B'}$  is the image of  $\overline{AB}$

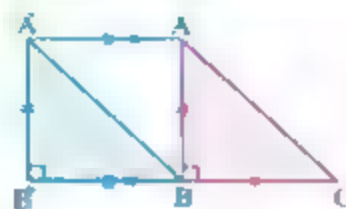
$\therefore \overline{A'B'} \parallel \overline{AB}$ ,  $A'B' = AB$

$\therefore$  The figure  $ABB'A'$  is  
a parallelogram.

$\therefore m(\angle B') = m(\angle B) = 90^\circ$

$\therefore A'B' = B'B$

$\therefore$  The figure  $ABB'A'$  is a square.



### Remember that

The square is a parallelogram in which one of its angles is right and two adjacent sides are equal in length.

**Example 3** Draw the rectangle ABCD. Take the point  $E \in \overline{AD}$ , then find the image of E by translation DA in the direction of  $\overline{AD}$ . Then prove that the figure EBC'E' is a parallelogram.

### Solution

We take  $E' \in \overline{AD}$  such that  $E'E = AD$ , then  $E'$  is the image of E by translation DA in the direction of  $\overline{AD}$

$\therefore$  ABCD is a rectangle.

$\therefore \overline{AD} \parallel \overline{BC}$  and  $AD = BC$

$\therefore$  C is the image of B by translation AD in the direction of  $\overline{AD}$

$\therefore E'$  is the image of E by the same translation

$\therefore \overline{E'C}$  is the image of  $\overline{EB}$  by translation AD in the direction of  $\overline{AD}$

$\therefore EB = E'C$  and  $\overline{EB} \parallel \overline{E'C}$

$\therefore$  The figure EBC'E' is a parallelogram.



Draw the parallelogram ABCD, plot  $E \in \overline{AD}$  where  $\overline{BE} \perp \overline{AD}$ , then draw the image of  $\triangle ABE$  by translation AD in the direction of  $\overline{AD}$ :

- 1 Prove that the figure EBC'E' is a rectangle. (where  $E'$  is the image of E by the previous translation)
- 2 Determine the distance and the direction of the translation which transforms  $\overline{BC}$  to  $\overline{EE'}$

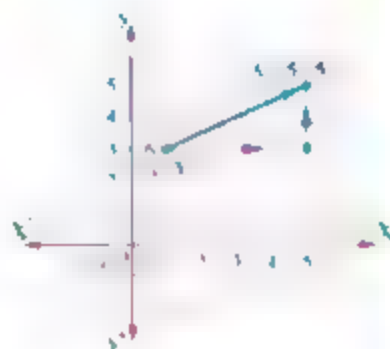
3

## Translation in the Cartesian plane

If  $A(1, 3)$  is a point in the orthogonal coordinates plane and to find its image  $\tilde{A}$  by translation with magnitude 4 length units in the direction of  $\overrightarrow{OX}$  followed by a translation with magnitude 2 length units in the direction of  $\overrightarrow{OY}$

From the graph, we get  $\tilde{A}$  to be the point  $(5, 5)$

i.e.  $\tilde{A}(1 + 4, 3 + 2)$



According to this :

Translation in the orthogonal Cartesian coordinates plane transforms each point by a displacement  $a$  in the direction of the  $X$ -axis followed by a displacement  $b$  in the direction of the  $y$ -axis

i.e. The image of the point  $A(X, y) \longrightarrow$  the point  $\tilde{A}(X + a, y + b)$

**Example 4** Find the images of the points  $A(2, 5)$ ,  $B(-4, 3)$  and  $C(2, 0)$  by translation  $(X, y) \longrightarrow (X + 2, y - 3)$

**Solution**

$(X, y) \longrightarrow (X + 2, y - 3)$ , then :

• The image of  $A(2, 5)$  is  $\tilde{A}(2 + 2, 5 - 3)$

i.e.  $\tilde{A}(4, 2)$

• The image of  $B(-4, 3)$  is

$\tilde{B}(-4 + 2, 3 - 3)$

i.e.  $\tilde{B}(-2, 0)$

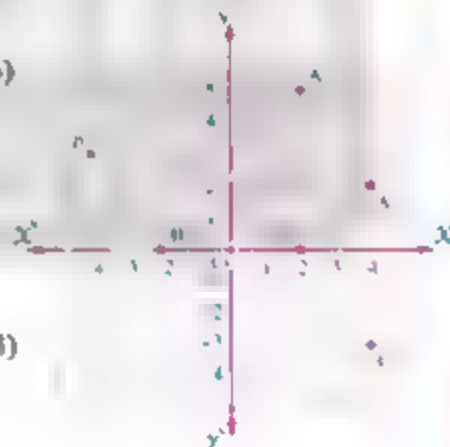
• The image of  $C(2, 0)$  is  $\tilde{C}(2 + 2, 0 - 3)$

i.e.  $\tilde{C}(4, -3)$

Notice that :

The translation  $(X, y) \longrightarrow (X + 2, y - 3)$

transforms each point to another point by a right horizontal displacement of 2 units and a vertical displacement of 3 units downwards.



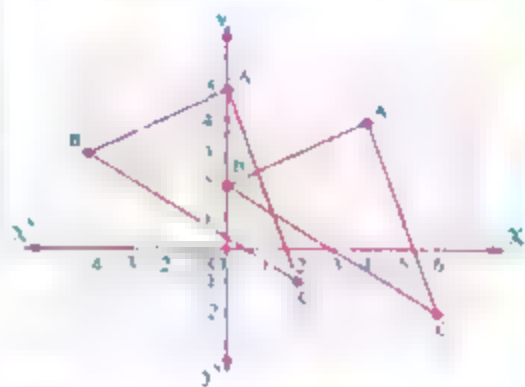
**Example 5** Draw on a square lattice  $\triangle ABC$  where  $A(4, 4)$ ,  $B(0, 2)$ ,  $C(6, -2)$ , then find its image by translation  $(X, y) \longrightarrow (X - 4, y + 1)$



## Solution

The point	Its image by the translation
$(x, y)$	$(x - 4, y + 1)$
$A(4, 4)$	$\tilde{A}(0, 5)$
$B(0, 2)$	$\tilde{B}(-4, 3)$
$C(6, -2)$	$\tilde{C}(2, -1)$

$\therefore \triangle \tilde{A}\tilde{B}\tilde{C}$  is the image of  $\triangle ABC$  by translation  $(x, y) \longrightarrow (x - 4, y + 1)$



## Remark

The translation  $(x, y) \longrightarrow (x + a, y + b)$  can be written as the translation  $(a, b)$  for example

The translation  $(x, y) \longrightarrow (x + 2, y - 1)$  can be written as the translation  $(2, -1)$



On a square lattice, draw  $\triangle ABC$  where

$A(-3, 2)$ ,  $B(-1, 1)$ ,  $C(-2, 0)$

, then find its image by translation :

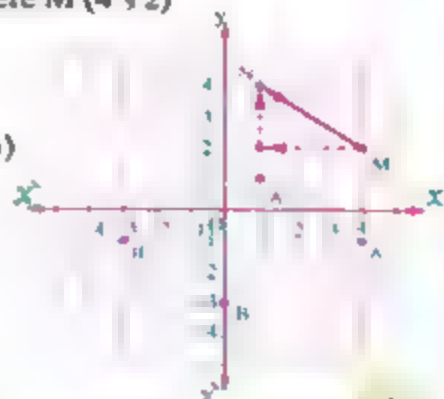
$(x, y) \longrightarrow (x + 2, y + 1)$



**Example 6** Find the image of each of the two points  $A(4, -1)$  and  $B(0, -3)$  by translation with magnitude  $\overline{MN}$  in the direction of  $\overline{MN}$  where  $M(4, 2)$  and  $N(1, 4)$

## Solution

- By noticing the opposite graph, we find that the translation with magnitude  $\overline{MN}$  in the direction of  $\overline{MN}$  where  $M(4, 2)$  and  $N(1, 4)$  is equivalent to :
  - Horizontal displacement from 4 to 1  
i.e. a displacement = 3 units to the left  $(-3)$
  - Vertical displacement from 2 to 4  
i.e. a displacement = 2 units upwards  $(2)$   
i.e.  $(x, y) \longrightarrow (x - 3, y + 2)$



thus we get :

$$A(4, -1) \longrightarrow \hat{A}(4-3, -1+2)$$

$$\text{i.e. } \hat{A}(1, 1)$$

$$B(0, -3) \longrightarrow \hat{B}(0-3, -3+2)$$

$$\text{i.e. } \hat{B}(-3, -1)$$

#### Notice that :

The translation with magnitude  $MN$  in the direction of  $\overrightarrow{MN}$  where  $M(4, 2)$  and  $N(1, 4)$  is equivalent to :

- A horizontal displacement (in the  $x$ -axis direction) from 4 to 1  $= 1 - 4 = -3$
  - A vertical displacement (in the  $y$ -axis direction) from 2 to 4  $= 4 - 2 = 2$
- i.e. The rule of translation is  $(x, y) \longrightarrow (x-3, y+2)$

**Example 7** Draw the image of  $\triangle ABC$  where  $A(5, 2)$ ,  $B(4, 5)$  and  $C(2, 2)$  by translation  $BC$  in the direction of  $\overrightarrow{BC}$  and write the rule of the translation.

$$\bullet B(4, 5), C(2, 2)$$

$\therefore$  The translation  $BC$  in the direction of  $\overrightarrow{BC}$  is equivalent to

- Horizontal displacement  $= 2 - 4 = -2$
- Vertical displacement  $= 2 - 5 = -3$

Thus the rule of translation

$$\text{is } (x, y) \longrightarrow (x-2, y-3)$$

Thus :

$$A(5, 2) \longrightarrow \hat{A}(5-2, 2-3)$$

$$\text{i.e. } \hat{A}(3, -1)$$

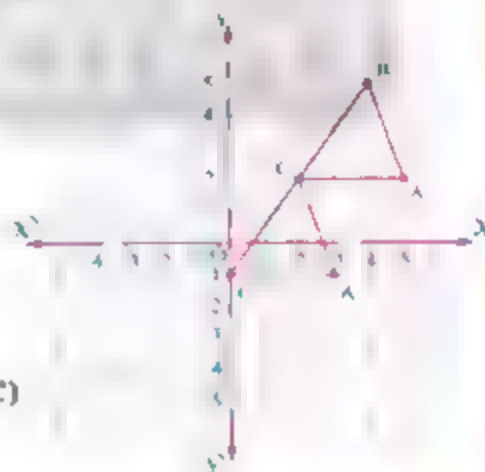
$$B(4, 5) \longrightarrow \hat{B}(4-2, 5-3)$$

$$\text{i.e. } \hat{B}(2, 2) \quad (\text{Notice : } \hat{B} \text{ coincides } C)$$

$$C(2, 2) \longrightarrow \hat{C}(2-2, 2-3)$$

$$\text{i.e. } \hat{C}(0, -1)$$

i.e.  $\triangle \hat{A}\hat{C}\hat{C}$  is the image of  $\triangle ABC$  by translation  $BC$  in the direction of  $\overrightarrow{BC}$





Draw the square ABCD where A (4 , -2) , B (4 , -5) , C (1 , -5) and D (1 , -2) ,  
then find its image by translation CA in the direction of  $\overrightarrow{CA}$

**Example 6** If the image of the point A (-3 , 2) by translation in the Cartesian coordinates plane is  $\hat{A}$  (2 , -2) :

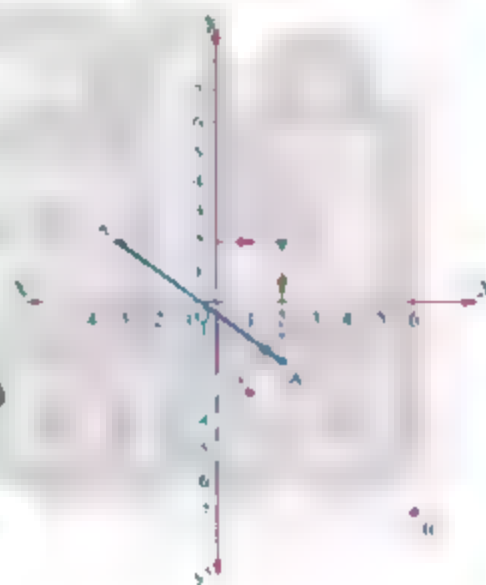
- 1 Find the rule of translation.
- 2 Find the image of B (1 , -3) by the same translation.

### Solution

- 1 By noticing the following graph , we find that the translation which makes  $\hat{A}$  (2 , -2) the image of A (-3 , 2) is equivalent to :

- Horizontal displacement of 5 units to the right side (5)
  - Vertical displacement of 4 units downwards (-4)
- $\therefore$  The rule of translation is  
 $(X , y) \longrightarrow (X + 5 , y - 4)$

- 2 B (1 , -3)  $\longrightarrow \hat{B}$  (1 + 5 , -3 - 4)  
 i.e.  $\hat{B}$  (6 , -7)





3

**Example 9** If  $\hat{A}(7, -2)$  is the image of A by the translation whose rule is  $(X, y) \longrightarrow (X-3, y+1)$ , find A

**Solution**Let A be  $(X, y)$ 

$$\therefore A(X, y) \longrightarrow \hat{A}(X-3, y+1)$$

$$\therefore \hat{A}(7, -2)$$

$$\therefore (X-3, y+1) = (7, -2)$$

$$\therefore X-3 = 7$$

$$\therefore X = 10$$

$$y+1 = -2$$

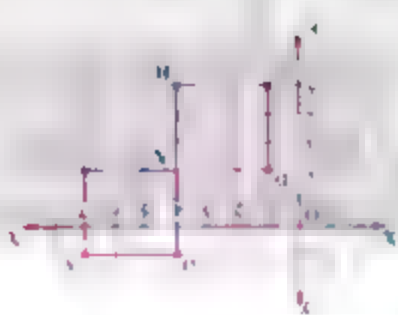
$$\therefore y = -3$$

$$\therefore A(10, -3)$$

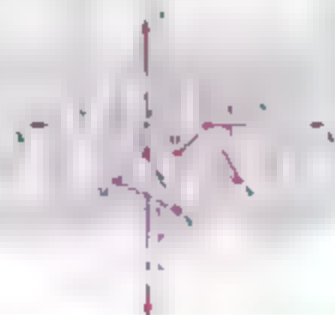
Notice that :

$$\text{If } (X, y) = (a, b),$$

$$\text{then } X = a, y = b$$



3

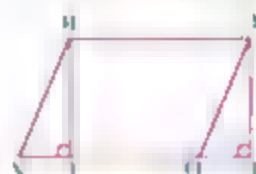


2

2 Translation with magnitude BE, in the direction of BE

$$(\text{Hint : } BE = CE, BE \parallel CE, m(\angle BEC) = 90^\circ)$$

1 Prove by yourself



1

Prove by yourself

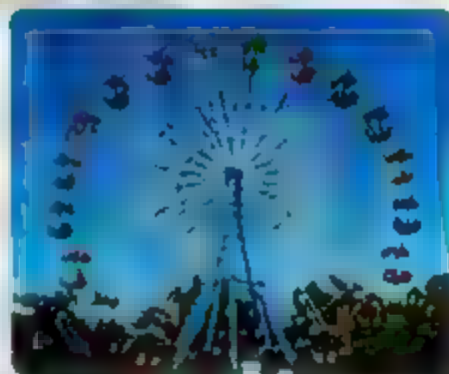


12

## Rotation

## Prelude

If you stand in front of Ferris wheel in the funfair , you will find that the carriage moves in a circular motion around a fixed point in the direction of clockwise or in the direction of anticlockwise , this motion is called "rotation"



## Definition of rotation

If  $M$  is a fixed point in the plane , then the rotation around  $M$  with an angle of measure  $\theta$  is a geometric transformation transforming each point  $A$  in the plane to another point  $\hat{A}$  in the same plane such that  $m(\angle AMA) = \theta$  ,  $MA = M\hat{A}$  this rotation is denoted by  $R(M, \theta)$  where :

- $M$  is the centre of rotation.
- $\theta$  is the measure of the angle of rotation.



According to this definition , the rotation is determined completely if we know the following elements

- 1 The centre of the rotation
- 2 The measure of the angle of the rotation ( $\theta$ )
- 3 The direction of rotation.

## Unit 3

## Remark

The measure of rotation angle is positive  
if the rotation is anticlockwise and it is negative  
if the rotation is clockwise.

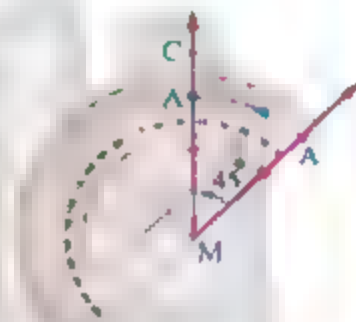


## Rotation in the plane

## Finding the image of a given point by a given rotation

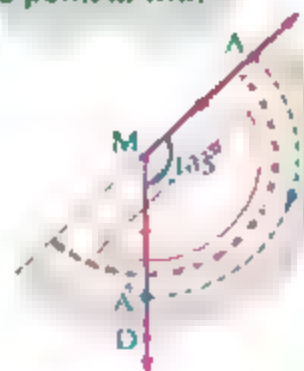
Firstly : Finding the image of the point A by rotation around the point M with an angle of measure  $45^\circ$  i.e.  $R(M, 45^\circ)$  :

- Draw the ray  $\overrightarrow{MA}$
- Put the protractor with its straight edge on  $\overrightarrow{MA}$  and in the anticlockwise direction , then draw  $\overrightarrow{MC}$  such that  $m(\angle AMC) = 45^\circ$
- Use the compasses at the point M as a centre with radius = MA , draw an arc to cut  $\overrightarrow{MC}$  at  $\hat{A}$  , then  $\hat{A}$  is the image of the point A by rotation around M with an angle of measure  $45^\circ$



Secondly : Finding the image of the point A by rotation around the point M with an angle of measure  $(-135^\circ)$  i.e.  $R(M, -135^\circ)$  :

- Repeat the same previous steps , then draw  $\overrightarrow{MD}$  in the clockwise direction such that  $m(\angle AMD) = 135^\circ$  , then determine on it the point  $\hat{A}$  such that  $M\hat{A} = MA$  , then  $\hat{A}$  is the image of A by rotation around M with an angle of measure  $(-135^\circ)$



## Remark

If  $\hat{A}$  is the image of A by rotation around M with an angle  $\theta$  , then A is the image of  $\hat{A}$  by rotation around M with an angle of measure  $(-\theta)$



## Lesson Twelve

## Finding the image of a polygon by a given rotation

The opposite figure shows how to find the image of  $\triangle ABC$  by the rotation  $R(A, -120^\circ)$  by finding the image of each vertex of its vertices, then  $\triangle A'B'C'$  is the image of  $\triangle ABC$  by rotation  $R(A, -120^\circ)$

Notice that :  $\triangle A'B'C' \equiv \triangle ABC$



## Remark

From the previous figure, the image of the point A by rotation  $R(A, -120^\circ)$  is itself because it is the centre of rotation.

## Properties of rotation

Through our study of rotation, we found that the rotation is a geometric transformation that maps the figure to another congruent figure to it

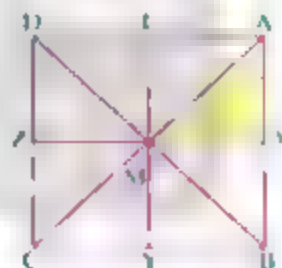
Therefore it is said that the rotation in the plane is isometric

, thus we can deduce some of properties and add other properties through our study of the following illustrated example.

## Illustrated example

In the opposite figure :

ABCD is a square whose diagonals intersect at M, X, Y, Z and L are the midpoints of its sides  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$  and  $\overline{DA}$  respectively. Find :



- 1 The image of  $\triangle AXM$  by rotation  $R(M, 90^\circ)$ , then mention what you observe.
- 2 The image of each of  $\overline{AB}$  and  $\overline{DC}$  by rotation  $R(M, 90^\circ)$ , then mention what you observe.
- 3 The image of each of B, Y and C by rotation  $R(M, 180^\circ)$ , then mention what you observe.

## Solution

- 1  $\therefore$  D is the image of A by rotation  $R(M, 90^\circ)$ , L is the image of X by rotation  $R(M, 90^\circ)$  and M is the image of itself (The centre of rotation).

$\therefore \triangle DLM$  is the image of  $\triangle AXM$  by rotation  $R(M, 90^\circ)$

We notice that :

•  $DL = AX$  ,  $LM = XM$  and  $DM = AM$

i.e. Rotation in the plane reserves the lengths of the line segments.

•  $m(\angle DLM) = m(\angle AXM)$  ,  
 $m(\angle LDM) = m(\angle XAM)$  and  
 $m(\angle DML) = m(\angle AMX)$

i.e. Rotation in the plane reserves the measures of the angles.

• Reading  $\Delta AXM$  is in the clockwise direction and reading its image  $\Delta DLM$  is in the clockwise direction also.

i.e. Rotation in the plane reserves the orientation of the vertices of the figure.

2.  $\therefore B$  is the image of  $A$  by rotation  $R(M, 90^\circ)$  ,  $C$  is the image of  $B$  by rotation  $R(M, -90^\circ)$

$\therefore \overline{BC}$  is the image of  $\overline{AB}$  by rotation  $R(M, -90^\circ)$

$\therefore A$  is the image of  $D$  by rotation  $R(M, -90^\circ)$  ,  $D$  is the image of  $C$  by rotation  $R(M, -90^\circ)$

$\therefore \overline{AD}$  is the image of  $\overline{DC}$  by rotation  $R(M, -90^\circ)$

We notice that :

•  $\overline{AB} \parallel \overline{DC}$  and  $\overline{BC} \parallel \overline{AD}$

i.e. Rotation in the plane reserves the parallelism.

3.  $\therefore D$  is the image of  $B$  ,  $L$  is the image of  $Y$  and  $A$  is the image of  $C$  by rotation  $(M, 180^\circ)$

We notice that :

•  $Y \in \overline{BC}$   
 and  $L$  (The image of  $Y$ )  $\in \overline{AD}$

i.e. Rotation in the plane reserves the betweenness

•  $B, Y, C$  are collinear ,  $D, L, A$  are also collinear.

i.e. Rotation in the plane reserves the collinearity.

## Lesson Twelve

**Example 1** In the opposite figure :

ABC is a triangle Find :

- 1 The point D as the image of B by rotation

$$R(A, m(\angle B))$$

- 2 The point E as the image of C by rotation  $R(A, m(\angle C))$

Then prove that : The points D , A and E are collinear



**Solution**

$\therefore$  D is the image of B by the rotation  $R(A, m(\angle B))$

$$\therefore m(\angle DAB) = m(\angle B) \quad (1)$$

$\therefore$  E is the image of C by the rotation

$$R(A, m(\angle C))$$

$$\therefore m(\angle EAC) = m(\angle C) \quad (2)$$

Adding (1) and (2), we deduce that :

$$m(\angle DAB) + m(\angle EAC) = m(\angle B) + m(\angle C)$$

Adding  $m(\angle BAC)$  to both sides :

$$\therefore m(\angle DAB) + m(\angle EAC) + m(\angle BAC) = m(\angle B) + m(\angle C) + m(\angle BAC)$$

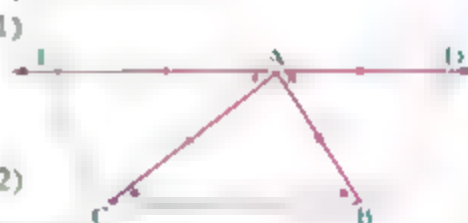
$\therefore$  The sum of measures of the interior angles of the triangle =  $180^\circ$

$$\therefore m(\angle B) + m(\angle C) + m(\angle BAC) = 180^\circ$$

$$\therefore m(\angle DAB) + m(\angle EAC) + m(\angle BAC) = 180^\circ$$

$\therefore$  D , A and E are collinear.

Q.E.D.



**Example 2** In the opposite figure :

If the figure XYBD is the image of the figure

DBCE by the rotation  $R(A, 50^\circ)$  ,

$BC = 6 \text{ cm}$  ,  $\overline{DE} \parallel \overline{BC}$

- 1 Find the length of :  $\overline{BY}$

- 2 Prove that :  $\overline{DX} \parallel \overline{BY}$



**Solution**

$\therefore$  The figure XYBD is the image of the figure DBCE by the rotation  $R(A, 50^\circ)$

$\therefore$  B is the image of C and Y is the image of B by this rotation.

$\therefore \overline{BY}$  is the image of  $\overline{CB}$  by this rotation.

$\therefore BY = CB = 6 \text{ cm}$ .

(First req.)



## Unit 3

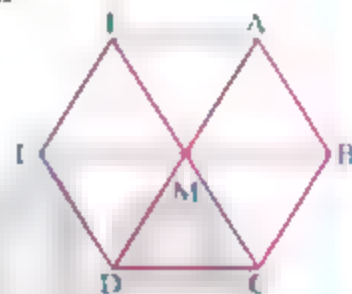
- ∴ The figure XYBD is the image of the figure DBCE by rotation  $R(A, 50^\circ)$
- ∴  $\overline{XD}$  and  $\overline{YB}$  are the images of  $\overline{DE}$  and  $\overline{BC}$  by this rotation respectively.
- ∴  $\overline{DE} \parallel \overline{BC}$
- ∴  $\overline{XD} \parallel \overline{BY}$  (Second req)



**In the opposite figure :**

**ABCDEF is a regular hexagon. Complete the following :**

- 1 The image of the point A by rotation around M with an angle of measure  $180^\circ$  is
- 2 The image of AB by rotation around M with an angle of measure  $(-60^\circ)$  is
- 3 The image of  $\triangle CMD$  by rotation around M with an angle of measure  $120^\circ$  is



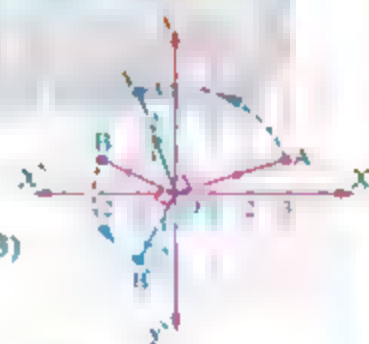
### Rotation in the Cartesian plane

#### First: Rotation about the origin $O(0, 0)$ with an angle of measure $90^\circ$

The opposite figure shows the two images of the two points A (3, 1) and B (-2, 1) by rotation  $R(O, 90^\circ)$

By noticing the figure , we find that :

- The image of the point A (3, 1) by rotation  $R(O, 90^\circ)$  → the point  $\hat{A}(-1, 3)$
- The image of the point B (-2, 1) by rotation  $R(O, 90^\circ)$  → the point  $\hat{B}(-1, -2)$



From the previous , we deduce the following rule :

The image of the point  $(X, y)$  by rotation  $R(O, 90^\circ)$  → the point  $(-y, X)$

## Remarks

- The image of the point  $(X, y)$  by rotation  $R(O, -90^\circ)$  is the point  $(y, -X)$

For example: The image of the point  $(2, -3)$  by rotation  $R(O, -90^\circ)$  is the point  $(-3, -2)$

- Rotation about the origin point with an angle of measure  $270^\circ$  is equivalent to rotation about the origin point with an angle of measure  $(-90^\circ)$

For example: The image of the point  $(2, -3)$  by rotation  $R(O, 270^\circ)$  is the point  $(-3, -2)$

Second Rotation about the origin point  $(O)$  with an angle of measure  $180^\circ$ 

The opposite figure shows the two images of the two points  $A(3, 1)$  and  $B(-2, 1)$  by rotation  $R(O, 180^\circ)$



By noticing the figure, we find that :

- The image of the point  $A(3, 1)$  by rotation  $R(O, 180^\circ)$  is the point  $A'(-3, -1)$
- The image of the point  $B(-2, 1)$  by rotation  $R(O, 180^\circ)$  is the point  $B'(2, -1)$

From the previous, we deduce the following rule :

The image of the point  $(X, y)$  by rotation  $R(O, 180^\circ)$  is the point  $(-X, -y)$

## Remarks

- The image of the point  $A(X, y)$  by rotation  $R(O, 180^\circ)$  is the same image of the point  $A$  by rotation  $R(O, -180^\circ)$
- The image of the point  $A(X, y)$  about the origin point with an angle of measure  $\pm 360^\circ$  is the same point  $A(X, y)$
- Rotation with an angle of measure  $90^\circ$  is called a  $\frac{1}{4}$  turn.
- Rotation with an angle of measure  $180^\circ$  is called a  $\frac{1}{2}$  turn.
- Rotation with an angle of measure  $360^\circ$  is called the identity rotation because it returns the figure to its original position.

## Unit 3

**Example 3** Complete the following table :

	The point	Its image by rotation $R(O, \pm 180^\circ)$	Its image by rotation $R(O, 90^\circ)$
1	(3, 2)		
2	(-3, 4)		
3	(-2, 1)		
4		(5, -2)	
5			(6, 0)

**Solution**

1 (3, 2) → (-3, -2)

2 (-3, 4) → (3, -4)

3 (-2, 1) → (2, -1)

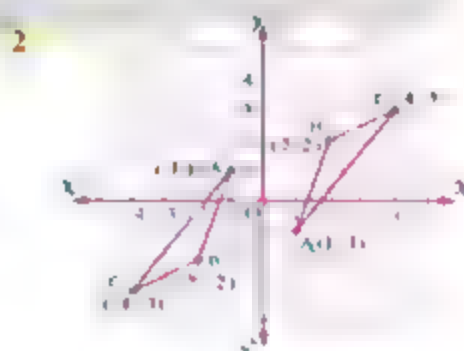
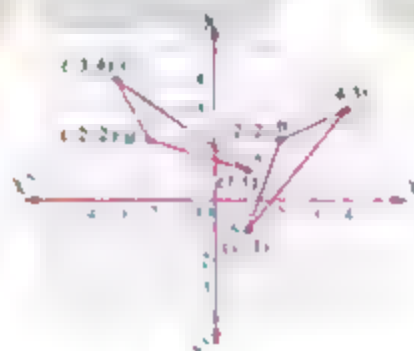
4 (-5, 2) → (5, -2)

5 (0, 6) → (0, -6)

**Example 4** On a square lattice, draw  $\triangle ABC$  where  $A(1, -1)$ ,  $B(2, 2)$  and  $C(4, 3)$  :

- 1 Draw  $\triangle \hat{A}\hat{B}\hat{C}$  which is the image of  $\triangle ABC$  by rotation  $R(O, 90^\circ)$
- 2 Draw  $\triangle \hat{A}\hat{B}\hat{C}$  which is the image of  $\triangle ABC$  by rotation  $R(O, 180^\circ)$

**Solution**





## Lesson Twelve

## TRY 2

In the opposite figure :

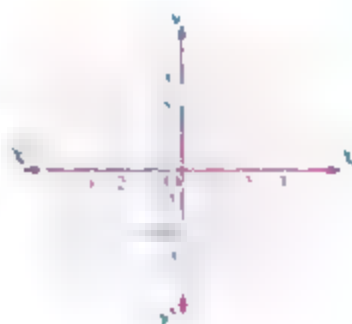
On the square lattice ,

draw  $\overline{AB}$  where A (2 , 1) and B (1 , 3),

then draw its image by rotation :

1 R (O ,  $90^\circ$ )

2 R (O ,  $180^\circ$ )

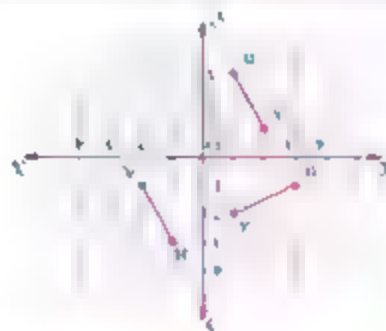


## Optical Illusion :

Look at the picture.

Turn the book with an angle of measure  $180^\circ$  and look at it again.

What do you notice ?



2

3  $\triangle AMB$ 

2 BC

1 Point D

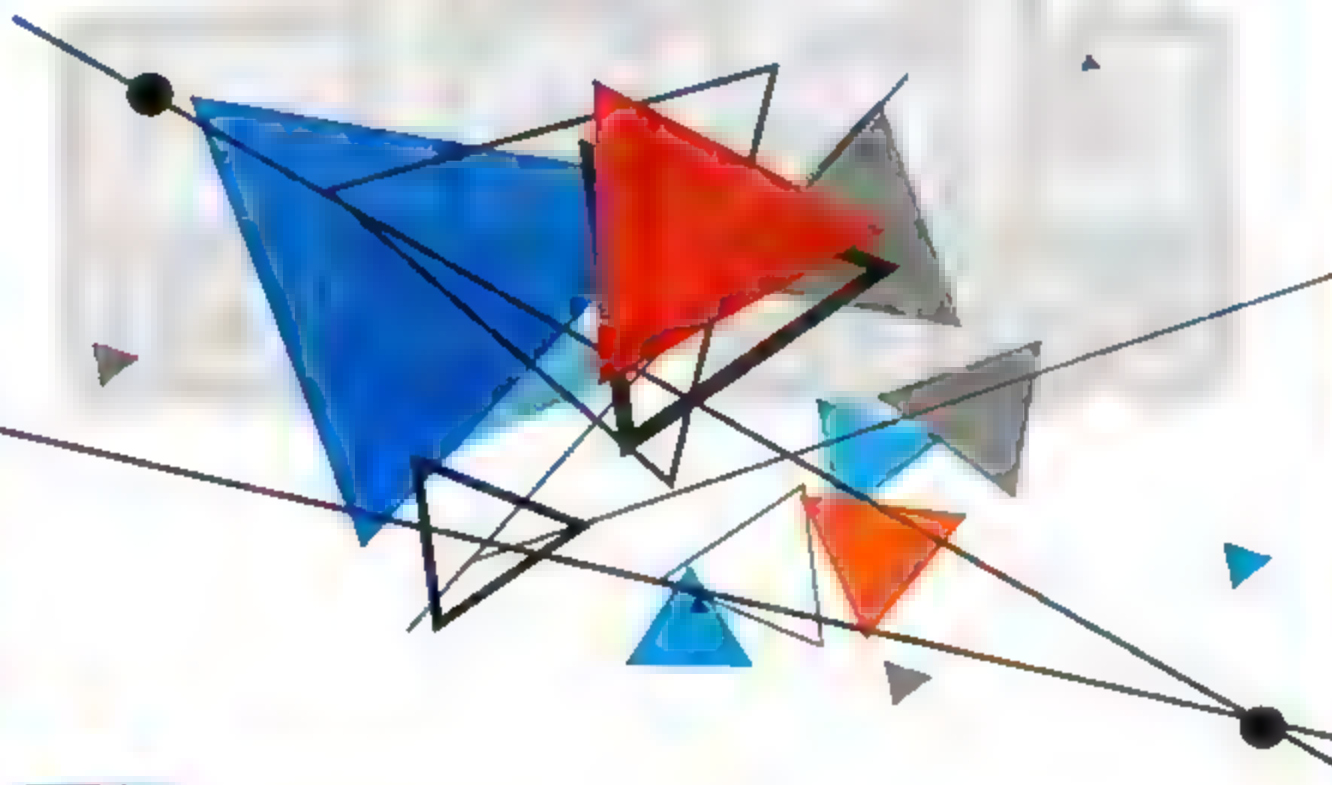
Answers of try by yourself



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# In Mathematics Exercises

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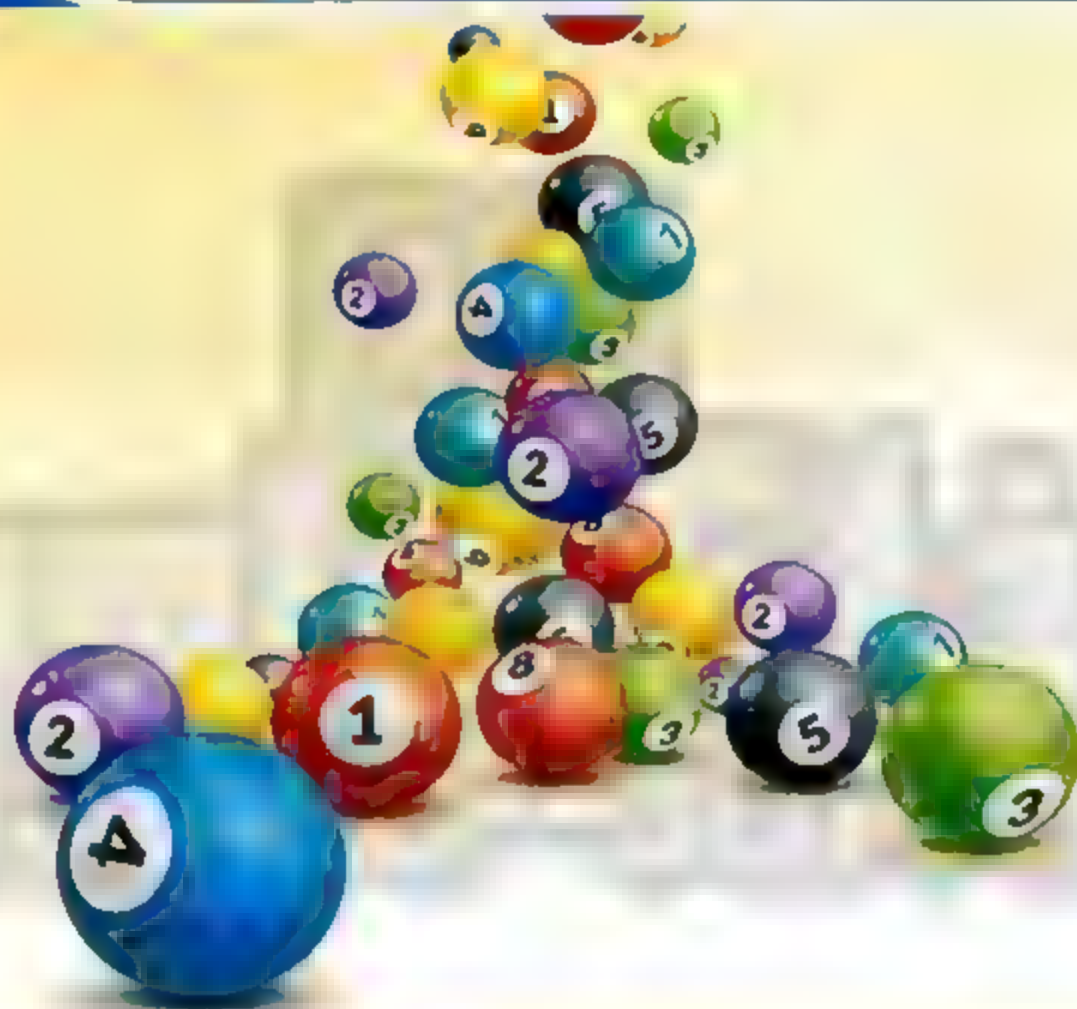


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UNIT

1

## Numbers and Algebra



## Exercises of the unit

- 1 Repeated multiplication
- 2 Non-negative integer powers
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- 5 Order of mathematical operations
- 6 The square root of a perfect square rational number
- 7 Solving equations in  $\mathbb{Q}$
- 8 Solving inequalities in  $\mathbb{Q}$



1

## Repeated multiplication

[...] From the school book

1 Calculate each of the following , then put the result in the simplest form :

1  $(\frac{1}{2})^3$

2  $(\frac{1}{3})^4$

3  $(\frac{3}{5})^2$

4  $(-\frac{1}{7})^3$

5  $(-\frac{3}{4})^4$

6  $(\frac{5}{9})^0$

7  $(1\frac{1}{3})^2$

8  $(-2\frac{1}{2})^3$

9  $(0.04)^2$

10  $(1.5)^3$

11  $(-3.2)^2$

12  $(1 - 1\frac{2}{3})^2$

2 Calculate each of the following , then put the result in the simplest form :

1  $8 \times (\frac{1}{2})^3$

2  $(-\frac{3}{4})^2 \times \frac{8}{27}$

3  $(-\frac{3}{5})^3 \times (-\frac{25}{27})$

4  $(\frac{3}{5})^2 + (-\frac{9}{125})$

5  $(\frac{4}{3})^2 \times (\frac{3}{2})^3$

6  $(-\frac{5}{6})^2 + 3\frac{3}{4}$

7  $(2\frac{1}{2})^2 \times \frac{4}{25}$

8  $2\frac{7}{9} + (-1\frac{2}{3})^2$

3 Calculate each of the following , then put the result in the simplest form :

1  $(\frac{4}{5})^2 \times \frac{5}{16} \times (\frac{2}{3})^0$

2  $\frac{3}{4} \times (-\frac{2}{3})^3 \times (\frac{3}{2})^2$

3  $(-\frac{5}{3})^4 \times (-\frac{3}{5})^3 \times (-1)^7$

4  $(-\frac{2}{3})^3 \times (\frac{1}{3})^3 + (-\frac{2}{9})^2$

5  $[(\frac{5}{2})^3 + (\frac{3}{2})^4] \times (\frac{3}{5})^3$

6  $(-\frac{1}{2})^3 + [8 \times (-\frac{1}{2}) \times \frac{3}{4}]$



## Exercise 1

4 Choose the correct answer from those given :

- 1 The multiplicative inverse of the number  $(\frac{2}{3})^0 =$   
 (a)  $\frac{5}{2}$  (b)  $-\frac{2}{3}$  (c) 1 (d) 0
- 2 The additive inverse of the number  $(-3)^0$  is  
 (a) 1 (b) -3 (c) 3 (d)  $-(3)^0$
- 3 The multiplicative inverse of the number  $(-1)^3$  is  
 (a)  $(-1)^3$  (b)  $(-1)^2$  (c)  $1^3$  (d)  $1^2$
- 4 The additive inverse of the number  $(-\frac{2}{3})^2$  is  
 (a)  $\frac{4}{25}$  (b)  $-\frac{4}{25}$  (c)  $\frac{25}{4}$  (d)  $-\frac{25}{4}$
- 5  $(\frac{1}{4})^0 + \frac{1}{4} = \dots\dots\dots$   
 (a)  $\frac{1}{4}$  (b)  $\frac{3}{4}$  (c)  $\frac{5}{4}$  (d)  $\frac{2}{4}$
- 6  $(\frac{5}{3})^2 \times (\frac{3}{5})^0 = \dots\dots\dots$   
 (a)  $\frac{5}{3}$  (b)  $\frac{25}{9}$  (c) 0 (d) 1
- 7 If  $x = y$ , then  $(\frac{3}{5})^{x-y} =$   
 (a)  $\frac{3}{5}$  (b)  $\frac{5}{3}$  (c) 1 (d) 0
- 8  $(\frac{a}{b})^2 \times \frac{b^2}{a^2} = \dots\dots\dots$  (where  $ab \neq 0$ )  
 (a)  $ab$  (b)  $(\frac{a}{b})^4$  (c)  $(ab)^0$  (d)  $\frac{a}{b}$
- 9 If  $x = -\frac{1}{2}$  and  $y = 3$ , then  $x^y =$   
 (a)  $\frac{1}{8}$  (b)  $-\frac{1}{8}$  (c)  $\frac{1}{6}$  (d)  $-\frac{1}{6}$
- 10 If  $y^{26} + y^{27} = 0$ , then  $y =$   
 (a) 1 (b) -1 (c) 2 (d) -2

5 Complete the following :

- 1  $\frac{8}{27} = (\frac{2}{3})^{\dots\dots\dots}$
- 2  $\frac{9}{16} = (\frac{3}{4})^{\dots\dots\dots}$
- 3  $-\frac{64}{125} = (-\frac{4}{5})^{\dots\dots\dots}$
- 4  $2\frac{1}{4} = (\frac{3}{2})^{\dots\dots\dots}$
- 5  $0.027 = (\frac{3}{10})^{\dots\dots\dots}$
- 6  $64\% = (\frac{4}{5})^{\dots\dots\dots}$
- 7 If  $\frac{x}{y} = -\frac{2}{3}$ , then  $(\frac{x}{y})^3 =$
- 8 If  $c = -3$  and  $d = -5$ , then  $(\frac{c}{d})^2 =$

## Unit 1

9 If  $\frac{a}{b} = 0.2$ , then  $(\frac{a}{b})^3 =$

11  $(-\frac{1}{2})^3 - (-\frac{1}{2})^2 = \dots\dots\dots$

13  $\frac{3}{4}, \frac{9}{16}, \frac{27}{64}, \dots\dots\dots$  (in the same pattern)

14 The greater of the two numbers  $(\frac{1}{4})^2$  and  $(-\frac{8}{3})^3$  is

10 If  $X = \frac{1}{2}$  and  $y = \frac{2}{3}$ , then  $X^2 y^2 =$

12  $2^2 + 2^2 = 2$

6 If  $X = -\frac{2}{3}$  and  $y = -\frac{1}{3}$ , find the value of:  $X^2 + y^2$  =  $\frac{11}{27}$

7 If  $a = \frac{2}{3}$  and  $b = -\frac{4}{3}$ , find the value of  $|a^3 + b^3|$  =  $\frac{1}{8}$

8 If  $X = 0.5$ ,  $y = -\frac{2}{3}$  and  $z = 3$ , find the value of:  $9Xy^2 - z^3$  =  $-29$

9 If  $a = -\frac{1}{2}$ ,  $b = 2$  and  $c = \frac{3}{4}$ , find the numerical value of:  $a^3 b^2 + b^2 c - 8abc$  =  $\frac{1}{2}$

10 If  $X = -\frac{3}{2}$ ,  $y = \frac{1}{2}$  and  $z = -\frac{4}{3}$ , find the numerical value of each of the following in its simplest form:

1  $X^2 y^2 z^2$  =  $-1$

2  $X^2 + z^2$  =  $\frac{81}{64}$

3  $X^2 - yz^2$  =  $-\frac{49}{36}$

4  $\frac{X^2 y^2 z^2}{X+y}$  =  $-1$

## Geometric Application

11 If  $V = l^3$  where  $V$  is the volume of a cube and  $l$  is its edge length, then calculate the volume of the cube whose edge length is  $1\frac{1}{2}$  cm. =  $\frac{27}{8} \text{ cm}^3$

## For excellent pupils

12 Choose the correct answer from those given:

1 If  $y = (\frac{1}{2})^x$  where  $x \in \{0, 1, 2, 3\}$ , then  $y$  takes its maximum value when  $x =$   
(a) 0 (b) 1 (c) 2 (d) 3

2 If  $y = (-\frac{2}{3})^x$  where  $x \in \{0, 1, 3, 4\}$ , then  $y$  takes its minimum value when  $x =$   
(a) 0 (b) 1 (c) 3 (d) 4

13 Arrange the following numbers ascendingly without expanding:

$(\frac{2}{3})^2, (-\frac{2}{3})^3, (-\frac{1}{3})^2, (-\frac{1}{3})^3$



2

## Non-negative integer powers

from the school book

1 Calculate each of the following , then put the result in the simplest form :

1  $\square \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

2  $\square \left(-\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^2$

3  $\frac{1}{3} \times \left(-\frac{1}{3}\right)^4$

4  $\left(\frac{1}{6}\right)^9 + \left(\frac{1}{6}\right)^8$

5  $\square \left(\frac{2}{7}\right)^5 + \left(\frac{2}{7}\right)^3$

6  $\square \left(-\frac{3}{5}\right)^7 + \left(\frac{3}{5}\right)^5$

7  $\left(-\frac{5}{2}\right)^2 + 2\frac{1}{2}$

8  $\left(\frac{1}{2}\right)^2 \times \frac{1}{2} \times \left(\frac{1}{2}\right)^3$

9  $\left(\frac{4}{5}\right)^8 + \left(\frac{4}{5}\right)^6 \times \frac{4}{5}$

2 Calculate each of the following , then put the result in the simplest form :

1  $\frac{3^7 \times 3^3}{3^6}$

2  $\frac{2^6 \times 2}{2^3 \times 2^4}$

3  $\frac{(-5)^4 \times 5^2}{5^3}$

4  $\frac{(-2)^5 \times 2^4}{(-2)^3 \times 2^2}$

5  $\frac{x^2 \times x^3 \times x^4}{x^7 \times x}$

6  $\frac{(-3)^5 \times (-2)^7}{(-3)^3 \times (-2)^5}$

7  $\frac{x^4 \times y^3 \times x^5}{x^6 \times y^2}$

8  $\frac{xy^2 \times x^2y}{x^2y^2}$

9  $\frac{\left(-1\frac{1}{2}\right)^5 \times \left(-1\frac{1}{4}\right)^8}{\left(-\frac{3}{2}\right)^4 \times \left(-\frac{5}{4}\right)^6}$

3 Find each of the following in the simplest form :

1  $\square \left(\frac{ab}{c}\right)^5$

2  $\square \left(\frac{5x}{3y}\right)^2$

3  $\square \left(\frac{2ab}{3c}\right)^4$

4  $\square \left(\frac{x^2}{y^3}\right)^2$

5  $\square \left(\frac{a^3b^2}{c^5}\right)^3$

6  $\square \left(-\frac{c^2}{d}\right)^3$



## Unit 1

7  $\square \left(-\frac{x^3}{y^2}\right)^2$

8  $\frac{(4x^3y^2)^7}{(2x^2y)^7}$

9  $\frac{(2a)^3 \times (2a)^4}{(-2a)^6 \times a}$

4 Calculate each of the following, then put the result in the simplest form :

1  $\left[\left(\frac{1}{2}\right)^2\right]^2$

2  $\square \left[\left(\frac{3}{2}\right)^2\right]^5$

3  $\square \left[\left(2\frac{1}{2}\right)^3\right]^2$

4  $\left[(-1\frac{1}{3})^2\right]^3$

5  $\left(\frac{3}{5}\right)^{10} \times \left(\frac{5}{3}\right)^{10}$

6  $\left[\left(\frac{2}{7}\right)^2\right]^3 \times \left(\frac{7}{2}\right)^6$

7  $\left(2\frac{1}{2}\right)^2 \times \left(-\frac{2}{5}\right)^2$

8  $\left(\frac{x^3}{y^2}\right)^{16} - \left(\frac{x^2}{y^2}\right)^{16}$

5 Match each expression in column (A) with an equivalent expression in column (B) :

Column (A)	Column (B)
(1) $(x^2)^a$	(a) $x^{a^2}$
(2) $(x^a)^a$	(b) $\frac{3m^c}{2n^c}$
(3) $(xy^a)^b$	(c) $27x^{3a}$
(4) $\left(\frac{x}{y^a}\right)^b$	(d) $\frac{3^c m^c}{2^c n^c}$
(5) $(-3x^a)^3$	(e) $x^{2a}$
(6) $(3x^a)^3$	(f) $-27x^{3a}$
(7) $\frac{3}{2} \left(\frac{m}{n}\right)^c$	(g) $\frac{n^b}{y^{ab}}$
(8) $\left(\frac{3m}{2n}\right)^c$	(h) $x^b y^{ab}$
	(i) $\frac{x^b}{y^{ab}}$
	(j) $xy^{ab}$

## Exercise 2

6 Choose the correct answer from those given :

1  $3^2 \times 3^5 =$

(a)  $3^7$

(b)  $3^3$

(c)  $3^{10}$

(d)  $3^{25}$

2  $5^2 + 5^2 =$

(a)  $10^2$

(b)  $10^4$

(c)  $5^4$

(d) 50

3  $3^5 \times 2^5 =$

(a)  $5^{10}$

(b)  $6^{10}$

(c)  $6^5$

(d)  $6^{25}$

4  $(5a)^0 =$  ,  $a \neq 0$

(a) 5

(b) a

(c) 5 a

(d) 1

5  $3^{(2^3)} =$

(a)  $3^6$

(b)  $3^5$

(c)  $3^8$

(d)  $3^{23}$

6  $(5^2)^3 =$

(a)  $5^6$

(b)  $5^5$

(c)  $5^{23}$

(d) 5

7  $3^{10} + 3^{10} + 3^{10} =$

(a)  $3^{30}$

(b)  $3^{30}$

(c)  $9^{10}$

(d)  $3^{11}$

8  $4^x + 4^x + 4^x + 4^x =$

(a)  $4^{x+4}$

(b)  $4^{4x}$

(c)  $4^{x+1}$

(d)  $4x^4$

9  $\frac{(3^2)^5}{(3^3)^2} =$

(a)  $3^{10}$

(b)  $3^{32}$

(c)  $3^{23}$

(d) 1

10  $\frac{(x^2)^3}{x^3} =$  ,  $x \neq 0$

(a)  $x^6$

(b)  $x^3$

(c)  $x^3$

(d) x

11  $(2y)^3 =$

(a)  $2y^3$

(b) 8 y

(c)  $8y^3$

(d) 23 y

12  $(b^3)^4 =$

(a)  $b^{34}$

(b)  $b^7$

(c)  $b^3 \times b^3 \times b^3$

(d)  $b^4 \times b^4 \times b^4$

13 The quarter of the number  $4^{20} =$

(a)  $4^5$

(b)  $4^{10}$

(c)  $4^{19}$

(d)  $2^{10}$

## Unit 1

7 Simplify to the simplest form :

$$\frac{(2y)^4 \times (3y)^2}{12y^5}, \text{ then find the value of the result at } y = -\frac{1}{6}$$

-2-

8 If  $a = \frac{1}{2}$ ,  $b = \frac{3}{4}$  and  $c = -\frac{2}{3}$ , find the numerical value of each of :

1  $(c^2 b)^3$

2  $(4a^3 c)^2$

3  $(a^2 b c^2)^2$

$-\frac{1}{27}, \frac{1}{9}, \frac{1}{144}$

9 If  $a = \frac{5}{3}$ ,  $b = -\frac{3}{2}$  and  $c = \frac{2}{5}$ , find the numerical value of each of :

1  $\frac{(a^2 c^2)^2}{b}$

2  $\left(\frac{2ab}{5c}\right)^3$

$-\frac{32}{243}, -\frac{125}{8}$

10 If  $x = -\frac{1}{2}$ ,  $y = \frac{3}{4}$  and  $z = -\frac{3}{2}$ ,

find the numerical value of each of the following in the simplest form :

1  $x^3 y^2$

2  $y^3 x^2$

3  $\frac{x^3}{y^2 z^2}$

$-\frac{9}{128}, \frac{27}{256}, -\frac{8}{81}$

11 Complete the following :

1  $\left(\left(\frac{7}{9}\right)^3\right)^4 = \frac{7^{12}}{3}$

2 If  $\left(\frac{3}{4}\right)^5 \times X = \left(\frac{3}{4}\right)^7$ , then  $X =$

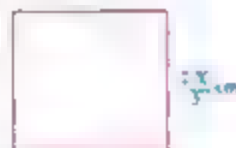
3 The greater number in the two numbers  $((-3)^5)^3$  and  $((-3)^2)^4$  is

4  $((-1)^5)^2 - ((-1)^3)^2 = \frac{4^4}{4^3} + \frac{4^3}{4^2} + \frac{4^2}{4} + 4 = 2$

5  $2^{2X} \times 4^X = 4$

### Geometric Applications

12 Find the area of the square whose side length is  $\frac{2x}{5}$  cm.



13 Find the volume of the cube whose edge length is  $\frac{3a^3}{7}$  cm.

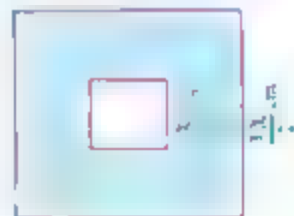




## Exercise 2

14 In the opposite figure :

A square is drawn inside another square  
Find the area of the shaded part.



## Life Application

15 From computer technology , we know that :

1 kilobyte =  $2^{10}$  bytes ,1 megabyte =  $2^{10}$  kilobytes ,1 gigabyte =  $2^{10}$  megabytes ,1 terabyte =  $2^{10}$  gigabyte ,

How many bytes are there in one terabyte ?



## For excellent pupils

16 If four times a number is  $4^3$  , find  $\frac{3}{4}$  this number.

- 12 -

17 If  $x = \left(\left(\frac{2}{3}\right)^3\right)^2$  and  $y = 3\left(\frac{3}{2}\right)^9 \left(\frac{3}{2}\right)^{10}$ , prove that the number  $x$  is the multiplicative inverse of the number  $y$ 18 If  $x = \frac{1}{3}$  and  $y = 5$  , find the value of :  $x^{15} y^{14}$ -  $\frac{1}{3}$  -19 Prove that :  $5^{x+2} - 5^{x+1} = 20 \times 5^x$ \*  $3^{15} + 3^{14}$  is divisible by 4

Free part

## Notebook

- Quizzes.
- Final examinations.



3

## Negative integer powers

from the school book

1 Evaluate each of the following :

1  $4^{-1}$

2  $5^{-2}$

3  $(\frac{1}{2})^{-1}$

4  $(-\frac{2}{3})^{-2}$

5  $(0.2)^{-2}$

6  $(1.2)^{-1}$

2 Evaluate each of the following :

1  $3^7 \times 3^{-3}$

2  $2^{-2} \times 2^{-3}$

3  $\frac{3}{3^2}$

4  $\frac{6 \cdot 2}{6^3}$

3 Evaluate each of the following :

1  $(5^{-1})^{-3}$

2  $(3^{-2})^2$

3  $(0.25)^{-2}$

4  $(2^{-1} \times 2^{-2})^3$

5  $(\frac{3^1}{3})^2$

6  $(\frac{8^4}{8^4})^0$

4 Evaluate each of the following :

1  $\frac{8 \times 8^{-2}}{8^{-3}}$

2  $\frac{7^{-2} \times 7^5}{7^3}$

3  $\frac{2^5 \times 2^{-2}}{2^{-4} \times 2^3}$

4  $\frac{2^3 \times 2^{-3}}{(2^2)^2}$

5  $\frac{(3^{-2})^3}{3^{-2} \times 3^{-6}}$

6  $(\frac{9^3 \times 9}{9^5})^{-3}$

7  $(\frac{4^{-2} \times 3}{4^3})^3$

8  $(\frac{2^5 \times 3^2}{3^4 \times 2^3})^{-1}$

9  $(3^0 \times 2^{-2})^{-2}$

10  $(3^0 - 2^{-2})^{-2}$

11  $\frac{(10)^2 \times (0.01)^3}{(10)^{-3}}$

## Exercise 3

5 Simplify each of the following and write the result in terms of positive exponents, where the denominator does not equal zero :

1  $7x^{-1}$

2  $x^{-1}y^2$

3  $a^{-2}b^{-3}$

4  $x^3 \times x^{-5}$

5  $x^3 \times x^{-2} \times x^{-1}$

6  $\frac{c^{-5}}{c^2}$

7  $x^7 + x^{-5}$

8  $(a^{-2})^3$

9  $(b^{-1})^{-3}$

10  $(a^2 \times a^{-5})^2$

11  $(x^2)^{-3} \times (x^{-3})^{-2}$

12  $\left(\frac{y^3}{y^{-2}}\right)^{-3}$

13  $\frac{x^2 \times x^{-3}}{x^{-4} \times x}$

14  $\frac{(x^2)^{-3} \times (x^{-1})^2}{x^{-3} \times x^{-4}}$

15  $\left(\frac{x^{-2} \times y^{-1}}{y^{-3} \times x}\right)^{-1}$

16  $\frac{a^{-1}}{b^2} \left(\frac{a^{-1}}{2b^2}\right)^{-2}$

17  $(x + x^{-1})^2$

6 Complete the following :

1  $2^{-3} \times c^0 =$

2  $(b^{-1})^{-3} = b$

3  $2x^{-3} = 2$

4  $(3x^{-1})^2 = 9x = 9$

5  $(3y^{-2})^{-2} =$

6  $(3a^2)^{-1} = 1$

7  $2x^{-2}y^{-3} = \frac{2}{}$

8  $\frac{x^{-3}}{y^{-5}} = ( )^5$

9  $\left(\frac{1}{2}\right)^2 + 2^0 - (2)^{-2} =$

10  $(x^2)^{-4} = \frac{1}{x^4}$

11  $2^{10} \times 2^{-10} = 3$

12  $a^{-5} + 1 = a^{-5} (\dots + \dots + \dots)$ , where  $a \neq 0$

13 If  $x = \frac{1}{2}$ ,  $y = \frac{1}{4}$ , then  $(x - y)^{-1} = \dots$

7 Choose the correct answer from those given :

1 If  $a^{-1} = \frac{2}{3}$ , then  $a = \dots$

(a)  $-\frac{2}{3}$

(b)  $\frac{3}{2}$

(c)  $-\frac{3}{2}$

(d) 1

2 If  $a = 7^x$  and  $b = 7^{-x}$ , then  $a \times b =$

(a)  $7^{2x}$

(b)  $49^{2x}$

(c) 1

(d) 0

3  $\frac{5^x}{5^y} =$

(a)  $5^{x+y}$

(b)  $5^{x-y}$

(c)  $5^{x+y}$

(d)  $\frac{x}{y}$

4  $\frac{6a^2x^4}{2a^3x^3} =$

(a)  $3a^2x$

(b)  $3a^5x^7$

(c)  $\frac{3x}{a}$

(d)  $\frac{3}{a^2x}$

5  $\frac{(2s^2t)^3}{(4s^2t)^2} =$

(a)  $\frac{s^3}{2t}$

(b)  $-\frac{s^4}{2t}$

(c)  $\frac{s^3}{2t^2}$

(d)  $\frac{s^4}{t}$



## Unit 1

6.  $\left(\frac{m^2}{n^3}\right)^{-1} \left(\frac{3m^{-2}}{n^2}\right)^2 =$

(a)  $\frac{9m^2}{n^7}$

(b)  $\frac{m^2}{9n^7}$

(c)  $\frac{m^2}{9n}$

(d)  $\frac{9m^6}{n}$

7.  $\frac{(2ab^{-2})^0}{3^0a^{-2}b} =$

(a)  $\frac{a^3}{3b^3}$

(b)  $a^2$

(c) 1

(d)  $\frac{a^2}{b}$

8. If  $a^x = 2$  and  $a^{-y} = 3$ , then  $a^{x-y} = \dots$

(a) 1

(b) -1

(c)  $\frac{2}{3}$

(d) 6

9. If  $xy^{-1} = \frac{1}{2}$ , then  $\frac{y}{x} = \dots$

(a)  $\frac{1}{2}$

(b)  $-\frac{1}{2}$

(c) 1

(d) 2

10.  $3^{-1} + 3^{-1} + 3^{-1} = \dots$

(a)  $3^{-3}$

(b)  $3^3$

(c)  $9^{-3}$

(d) 1

11. The multiplicative inverse of  $5^{-1}$  is

(a)  $\frac{1}{5}$

(b) 5

(c) -5

(d)  $-\frac{1}{5}$

12.  $\left(\frac{3}{5}\right)^2 \times \left(\frac{5}{3}\right)^{-2} = \dots$

(a)  $\left(\frac{3}{5}\right)^4$

(b) 1

(c)  $\left(\frac{3}{5}\right)^{-4}$

(d) 0

B Complete each of the following by the suitable sign of (>) , (<) or (=) :

1.  $2^{10} \dots 2^{-10}$

2.  $3^{-20} \dots 3^2$

3.  $5^{-15} \dots 2^{-15}$

4.  $(-7)^2 \dots (-7)^{19}$

5.  $(-1)^6 \dots (1)^9$

6.  $(-1)^{20} \dots (1)^{-10}$

C Why  $b^{-3}$  is not defined when  $b = 0$  ?

10 Calculate the value of  $\left(-\frac{3}{5}\right)^x \times \left(\frac{3}{5}\right)^y$  in each of the following cases :

1.  $x = -2$  and  $y = 2$

= 1

2.  $x = -1$  and  $y = 2$

=  $\frac{3}{5}$

11 If  $x = -\frac{1}{3}$  ,  $y = \frac{2}{3}$  , then find in the simplest form the numerical value of the

expression :  $\left(\frac{y}{x^2}\right)^{-2}$

=  $\frac{1}{36}$

## Life Applications

- 12 The flea can jump at a height of 200 times of its length.  
If a flea of length  $2^{-4}$  inches can jump at a height of  $2^3$  inches ,  
what does this height represent according to  
the length of the flea ?



- 13 The population of a city has been growing exponentially. It is estimated that  
in (t) years the population (p) will be :  $p = 2(1.03)^t$  million

- 1 What will the population be in 2 years ?      2 What is the population now ?  
3 What was the population last year ?



## For excellent pupils

- 14 Simplify to the simplest form :  $\frac{2^{10} \times 3^4}{(12)^5}$

- 15 Simplify to the simplest form :

$$\frac{6^{2n+1} \times 4^{-n}}{2^n \times 3^{2n+1}} \text{ , then find the value of the result when } n = 3$$

- 16 If  $2^n = 3$  , find the value of :

(1)  $2^{n+1}$       (2)  $4^n$       (3)  $4^{-n}$       (4)  $2^n - 1$        $= 6 \div 9 \div \frac{1}{3} \div \frac{1}{2}$

- 17 If  $a = 5$  and  $b = 5^{-1}$  , find the value of :  $a^{91} b^{50}$

- 18 Without expanding , arrange the following ascendingly by inspection :

$$(-2)^{-15} , (-5)^{20} , (-2)^{15} , 2^{-20} , (-5)^{15} , (-2)^{20}$$

Now at all bookstores



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المعاصر (إصدارات لغات - فزيكا) ٢٠٢١/٢٠٢٢



• *reducing the value of  $\lambda$*

- 
- Hydrogen Atom



## Exercise 4

- 6 The light velocity is about 300 000 km/sec.  
Express this velocity by m/sec.  
in the standard form.



- 7 Dr. Ahmed Zewail discovered the femto second  
which is a millionth of a milliardth of a second.  
Express it by the standard form.



- 8 At writing the number  $2.74 \times 10^{20}$  as a whole number, find the number of zeroes that are on the right of the digit 4

- 9 Write the following numbers in the standard form :

1)  $68 \times 10^5$

2)  $68 \times 10^{-5}$

3)  $720 \times 10^6$

4)  $750 \times 10^{-9}$

5)  $-32.4 \times 10^4$

6)  $-702.5 \times 10^{-8}$

7)  $0.4 \times 10^{-10}$

8)  $0.0005 \times 10^{15}$

9)  $0.0036 \times 10^{-4}$

10)  $0.0020205 \times 10^{12}$

- 10 Put the suitable sign (<) or (>) :

1)  $6.4 \times 10^3$    $4.6 \times 10^3$

2)  $6.2 \times 10^4$    $4.1 \times 10^5$

3)  $0.0041$    $3.2 \times 10^{-2}$

4)  $4370$    $3.41 \times 10^4$

5)  $2.10 \times 10^{-5}$    $1.82 \times 10^{-5}$

6)  $9.1 \times 10^{-4}$    $1.2 \times 10^{-4}$

7)  $6.920 \times 10^5$   96230

8)  $3.69 \times 10^{-4}$   0.0000623

- 11 Arrange the following numbers in a descending order :

$3.6 \times 10^{-3}$ ,  $5.2 \times 10^{-5}$ ,  $1 \times 10^{-2}$ ,  $8.35 \times 10^{-2}$ ,  $6.08 \times 10^{-8}$

12 Choose the correct answer from those given :

1  $3.04 \times 10^7 =$

- (a) 340 000 (b) 304 000 (c) 3 400 000 (d) 30 400 000

2  $2.37 \times 10^{-4} =$

- (a) 0.00237 (b) 0.000237 (c) 23700 (d) 0.0000237

3 If  $0.00079 = 7.9a$ , then  $a =$

- (a)  $10^3$  (b)  $10^{-3}$  (c)  $10^{-4}$  (d)  $10^4$

4 If  $0.0000503 = m \times 10^{-5}$ , then  $m =$

- (a) 503 (b) 5.03 (c) 50.3 (d) 0.503

5 If the thickness of a sheet of paper is 0.012 cm., then a ream of 400 sheets is of height

- (a)  $48 \times 10^{-3}$  cm. (b)  $48 \times 10^{-2}$  cm (c)  $4.8 \times 10^0$  cm. (d) 48 cm.

6 Which of the following equals  $\frac{1}{2}$  milliard ?

- (a)  $50 \times 10^8$  (b)  $5 \times 10^8$  (c)  $0.5 \times 10^8$  (d)  $500 \times 10^7$

7 Which of the following is the greatest ?

- (a)  $6.3 \times 10^5$  (b)  $9.8 \times 10^4$  (c)  $5.2 \times 10^5$  (d)  $7.3 \times 10^4$

8 Which of the following is the smallest ?

- (a)  $0.6 \times 10^5$  (b)  $0.25 \times 10^5$  (c)  $7 \times 10^4$  (d)  $17.5 \times 10^4$

9  $6000 \times 50 =$

- (a)  $300 \times 10^2$  (b)  $30 \times 10^5$  (c)  $3 \times 10^5$  (d)  $30 \times 10^4$

10  $45 \times 900 =$

- (a)  $4.05 \times 10^2$  (b)  $4.05 \times 10^3$  (c)  $4.05 \times 10^4$  (d)  $45 \times 10^2$

11  $0.7 \times 0.005 =$

- (a)  $3.5 \times 10^{-1}$  (b)  $3.5 \times 10^{-2}$  (c)  $3.5 \times 10^2$  (d)  $3.5 \times 10^{-3}$

13 Write the result of each of the following in the standard form :

1  $(6.4 \times 10^8) \times (1.5 \times 10^5)$

2  $(8.2 \times 10^7) \times (2.1 \times 10^{-4})$

3  $(5.02 \times 10^{-4}) \times (0.1 \times 10^{-1})$

4  $(4.4 \times 10^3) \times (2 \times 10)^5$

5  $(3.8 \times 10^8) + (1.9 \times 10^8)$

6  $(125.5 \times 10^{-3}) \div (5 \times 10^4)$

7  $(8.8 \times 10^{25}) + (8.8 \times 10^{22})$

8  $(5 \times 10)^4 + (2.5 \times 10^{-3})$

14 Write the result of each of the following in the standard form :

1  $(3.8 \times 10^5) + (4.6 \times 10^4)$

2  $(4.54 \times 10^4) + (3.76 \times 10^3)$

3  $(5.3 \times 10^8) - (0.8 \times 10^7)$

4  $(2.65 \times 10^{-2}) - (6.34 \times 10^{-3})$

15 Write the result of each of the following in the standard form :

1  $5000 \times 3000$

2  $400 \times 0.00007$

3  $8000 \div 0.004$

4  $0.000033 \div 500$

5  $(20\,000)^3$

6  $(0.002)^2$

7  $(0.1)^{-8}$

16 Find the value of  $n$  in each of the following :

1  $800\,000 = 8 \times 10^n$

2  $0.00000006 = 6 \times 10^n$

3  $0.00052 = 5.2 \times 10^n$

4  $0.000357 = 3.57 \times 10^n$

5  $(0.004)^2 = 1.6 \times 10^n$

6  $76293 = n \times 10^n$

### Life Applications

17 If the diameter of the Earth is about  $1.27 \times 10^4$  km. long and the length of the diameter of Mars is about  $6.79 \times 10^3$  km. Which of the two planets is the greater and what is the difference between the two diameter lengths in the standard form ?



18 If light travels at a speed of  $3 \times 10^8$  m/s :

(a) Calculate the distance from the Sun to the Earth if you know that the light of the Sun takes 8 minutes to reach the Earth

(b) If the distance between planet Venus and the Sun is 108 million kilometres , calculate the elapsed time (in minutes) that light takes to reach Venus from the Sun.



### For excellent pupils

19 Find the result of the following in the standard form :  $\frac{9.02 \times 10^3 + 4.98 \times 10^4}{2.5 \times 10^{-6}}$

20 Without using the calculator , write each of the following numbers in the standard form :

1  $10^{29} - 10^{28}$

2  $2^{19} \times 5^{15}$

21 If  $X = 5 + (3 \times 10) + (4 \times 10^2) + (6 \times 10^3) + (9 \times 10^4) + (4 \times 10^5) + (2 \times 10^6)$

Write  $X$  in the standard form without using the calculator.



## Summary of the first part of unit 1

"From lesson 1 to lesson 4"



If  $a, b$  are two rational numbers,  $n, m$  are two integers, then :

### The repeated multiplication

- $a^n = a \times a \times a \times \dots \times a$ , where  $a$  repeated  $n$  times.
- $(-a)^n = a^n$ , where  $n$  is an even number.
- $(-a)^n = -a^n$ , where  $n$  is an odd number.

### The laws of powers

1.  $a^n \times a^m = a^{n+m}$  (when multiplying like bases, we add their powers)
  2.  $a^n \div a^m = a^{n-m}$  (when dividing like bases, we subtract their powers)
  3.  $(a^n)^m = a^{n \times m}$
  4.  $(a \times b)^n = a^n \times b^n$
  5.  $(a + b)^n = a^n + b^n$  where  $b \neq 0$
- If  $a$  is a rational number,  $a \neq 0$ , then  $a^0 = 1$
  - If  $a$  is a rational number,  $a \neq 0$ ,  $n$  is a positive number, then  $a^{-n} = \frac{1}{a^n}$ ,  $a^n = \frac{1}{a^{-n}}$
  - If  $\frac{a}{b}$  is a rational number not equal to zero,  $n$  is a positive number, then  $\left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{-n}$

### The standard form of a rational number

The number is been written in the standard form as :

$$a \times 10^n, \text{ where } 1 \leq |a| < 10, n \in \mathbb{Z}$$

# Exams on the first part of unit one

from lesson (1) to lesson (4)



## Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 If  $a = b$ , then  $\left(\frac{3}{7}\right)^{a-b} = \dots\dots\dots$

- (a) zero (b) 1 (c)  $\frac{3}{7}$  (d)  $\frac{7}{3}$

2 If  $a^{-1} = \frac{3}{4}$ , then  $a = \dots\dots\dots$

- (a) zero (b) 1 (c)  $\frac{3}{4}$  (d)  $\frac{4}{3}$

3  $\frac{4a^2b^4}{2a^3b^5} = \dots\dots\dots$  where  $ab \neq \text{zero}$

- (a)  $2ab$  (b)  $2a^3b^7$  (c)  $\frac{2b}{a}$  (d)  $\frac{2}{ab}$

4  $2^4 \times 3^4 =$

- (a)  $5^4$  (b)  $6^4$  (c)  $6^8$  (d)  $6^{10}$

5 Quarter of  $4^{20} =$

- (a)  $4^5$  (b)  $4^{10}$  (c)  $4^{19}$  (d)  $2^{10}$

6  $3^{10} + 3^{10} + 3^{10} =$

- (a)  $3^{10}$  (b)  $3^{11}$  (c)  $3^{20}$  (d)  $3^{30}$

2 Complete the following :

1 If  $x = \frac{1}{4}$ ,  $y = \frac{1}{8}$ , then  $(x-y)^{-1} = \dots\dots\dots$

2 The standard form of the number 0.000013 = ...

3  $\frac{8}{27} = \left(\frac{2}{3}\right)^{\dots\dots\dots}$

4  $0.00023 = 2.3 \times n$ , then  $n = \dots\dots\dots$

5  $5y^{-3} = \frac{5}{\dots\dots\dots}$

1

3 [a] Find the value of the following expression in the simplest form :  $\frac{5^{-2} \times 5^5}{5^3}$

[b] Write the result of each of the following in the standard form :

1  $(4.4 \times 10^3) \times (2 \times 10^5)$

2  $(5.3 \times 10^8) - (0.8 \times 10^7)$

4 [a] Find in the simplest form :  $\left(\frac{3^3 \times 3^{-2}}{3^{-1} \times 3^4}\right)^{-2}$

[b] If  $x = -\frac{1}{2}$  ,  $y = \frac{3}{4}$  ,  $z = -\frac{3}{2}$

Find in the simplest form :  $\frac{x^3}{y^2 z^2}$

5 [a] Find the value of n in each of the following :

1  $0.00073 = 7.3 \times 10^n$

2  $4720000 = 4.72 \times 10^n$

[b] If  $a = -\frac{1}{2}$  ,  $b = 2$  ,  $c = \frac{3}{4}$

Find the numerical value of the expression :  $a^3 b^2 + b^2 c - 8 a b c$

### Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

1  $(3 a b^2)^3 = \dots\dots\dots$

(a)  $3 a^3 b^5$

(b)  $9 a^3 b^6$

(c)  $27 a^3 b^6$

(d)  $9 a^3 b^5$

2 Twice the number  $2^5 = \dots\dots\dots$

(a)  $2^{10}$

(b)  $2^6$

(c)  $4^5$

(d)  $4^{10}$

3  $\frac{(3 X^{-2} y)^{2xy}}{4^{2xy} X^{-2} y} = \dots\dots\dots$  where  $X y = \text{zero}$

(a)  $\frac{X^2}{y}$

(b)  $\frac{y}{X^2}$

(c)  $\frac{3 y}{4 X}$

(d) 1

4  $3.35 \times 10^{-3} = \dots\dots\dots$

(a) 0.000335

(b) 0.0335

(c) 335000

(d) 0.00335



1 The additive inverse of  $(-\frac{2}{5})^2$  is \_\_\_\_\_

(a)  $\frac{4}{25}$

(b)  $-\frac{4}{25}$

(c)  $\frac{25}{4}$

(d)  $-\frac{25}{4}$

2  $x^9 \times x^{-4} =$  \_\_\_\_\_

(a)  $x^3$

(b)  $x^5$

(c)  $x^9$

(d)  $x^{-3}$

3 Complete the following :

1 If  $74500000 = 7.45 \times 10^n$ , then  $n =$  \_\_\_\_\_

2  $2\frac{1}{4} = (\frac{2}{3})$  \_\_\_\_\_

3 If  $x^{-1} = \frac{1}{2}$ , then  $x =$  \_\_\_\_\_

4 If  $x = \frac{1}{3}$ ,  $y = \frac{3}{2}$ , then  $x^2 y^2 =$  \_\_\_\_\_

5  $(2x^{-1})^3 = \frac{8}{x}$  \_\_\_\_\_

4 [a] Find in the simplest form :

$$\frac{a^3 \times a^4}{a^3 \times a^2 \times a^4} \text{ where } a \neq 0$$

[b] If  $x = -\frac{1}{4}$ ,  $y = \frac{3}{4}$

, find in the simplest form :  $\frac{x^4 y^3}{x^6 y^2}$

5 [a] Write the result of each of the following in the standard form :

1  $(3.4 \times 10^5) + (1.7 \times 10^5)$

2  $(6.3 \times 10^5) + (2.8 \times 10^4)$

[b] Find in the simplest form :  $\frac{8^2 \times 8^{-4}}{8^{-4} \times 8}$

5 [a] Write each of the following in the standard form :

1 0.00054

2 79400000

3  $46.5 \times 10^3$

[b] Find the volume of a cube of edge length  $\frac{3a^2}{5}$  cm.



Exercises

5

## Order of mathematical operations

from the school book

1 Calculate the value of each of the following :

1  $3 + 12 + 6$

2  $-5 + 2 \times 3$

3  $2 \times 6 - 4 + 2$

4  $4 \times 7 - 3^2$

5  $4 \times 2^3 - 20$

6  $9 + 4 \times 3^2$

7  $144 - 8 + 2^3$

2 Calculate the value of each of the following :

1  $196 \div (7 - 5)^2$

2  $18 \div (9 - 6) \times (1 + 2)$

3  $20 \div 5 + 8 - (4 - 1)$

4  $10 \times 4 - (2 \times 6 - 8)$

5  $(7 - 4) \times 2 + (5 - 3)$

6  $(30 - 6) \div 6 \times 30 \div 3$

7  $7 (6^2 + 2 \times 3)$

8  $12 (2^2) + 24 \div 3^2$

9  $9 (4)^2 + 2^2 \times 3$

10  $9 \times 10 \div 20 + 2 - 3$

11  $6 \div 9 \div 3 + 2 \times 3^2 + 11 - 8$

12  $6 - 5 + 72 \div 9 + 24 \div 2^3 + 4 \times 1 + 10 \times 1 \div 5$

3 Calculate the value of each of the following :

1  $2 - [(7 - 3) - 2]$

2  $[4 - (5 - 2)] - 1$

3  $3 + [5 + 2 (8 + 4)]$

4  $2^3 + [4 + (2 - 1)]$

5  $[(2 + 23 - 7) \times 2] + 4$

6  $10 \times 3 + [4 - (9 - 8)]$

7  $(26 \div 1) + [3 (4 - 3)]$

8  $(15 \times 2) + [5 - (9 - 7)]$

9  $2 + 3 [4 + (6 \times 3 - 8)] \times 2$

10  $2 [(5^2 + 1) - (4^2 - 1)]$

11  $5 [(2^2 - 1) - (2^2 - 2)]$

12  $6 \div 3 + [7 + 20 \div (6 - 2^2)]$

## Exercise 5

4 Calculate the value of each of the following :

1  $[-6 + (-3)] \times [-30 + (-3)]$

2  $(-10 + 3) \div (-8 + 7)$

3  $7 - [10 - (-8)] - 3$

4  $[(11 - (-10)) \times 2 + (-6)]$

5  $2 \div (-1) - (-4)^2$

6  $-6 - [-2 - 5]^2$

5 Calculate the value of each of the following :

1  $\frac{15+7}{15-4}$

2  $\frac{8+20-4}{8-4}$

3  $\frac{-4 \times (-10)}{-9+7}$

4  $\frac{1+15}{8-(2-2)}$

5  $\frac{11-(5-4)}{1+4}$

6  $(3-1)^3 + \frac{7 \times 3}{-1-6} - \frac{2 \times 15}{6}$

7  $\frac{5^2-5 \times 2}{(15+3)+6}$

8  $\frac{5+2 \times 5}{2^2+1} + 5^2 - 5$

9  $\frac{3^2 \times 6 + 3}{2 \times 1 + (3+1)^2}$

6 Calculate the value of each of the following :

1  $(\frac{3}{2} \times 3\frac{1}{2}) + (\frac{6}{3} - 1)$

2  $15 + \frac{1}{3} - \frac{3}{4} \times 10^3 + 27$

3  $16 + 4 + 2 - 3 \times 10^{-2}$

4  $9 + \frac{1}{2} \times 2 - 3 + \frac{1}{3}$

7 12 If  $X = 3$ , what is the numerical value of the expression :  $2 \left( \frac{5X+3}{4X-3} \right)$  « 4 »8 13 Evaluate the expressions when  $t = 2$  and  $s = 5$  :

1  $(t+s)^2$

2  $(s-t)^3$

3  $\left(\frac{s}{t}\right)^3$

4  $\frac{6^2}{s-1}$

5  $\frac{s-t}{s^3}$

6  $\frac{12}{4s^2}$

9 14 Evaluate :  $16t + (4s) + 3st$ , for  $t = 9$  and  $s = 6$  « 168 »10 If  $X = 4(5+6) - 6$  and  $y = 9(36 \div 12) + 3$ ,  
find the value of the expression :  $2X + 4y$  « 112 »11 If  $X = 3(5+7) - 4$  and  $y = 4(8+2) + 5$ ,  
find the numerical value of the expression :  $X - 4y$  « zero »12 If  $X = 18 - 4 \times 2 \div 2 + 1$  and  $y = 8 + 9 \times 3 - 4^2 + 11$ ,  
find the numerical value of the expression :  $\left(\frac{y}{X}\right)^{-3}$  «  $\frac{1}{8}$  »



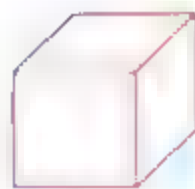
## Unit 1

## Geometric Applications

13 In the opposite figure :

The total area of a cube  
is  $T = 6s^2$ , find T when :

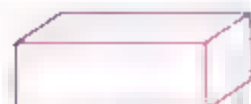
- 1  $s = 3$  m.
- 2  $s = 0.8$  cm.


 $s = 3 \text{ m} \quad \text{or} \quad s = 0.8 \text{ cm}$ 

14 In the opposite figure :

The total area of a cuboid  
is  $T = 2(Xy + yz + zX)$  find T when :

- 1  $X = 2$  cm. ,  $y = 3$  cm. and  $z = 5$  cm.
- 2  $X = \frac{3}{5}$  m. ,  $y = 0.4$  m. and  $z = \frac{1}{5}$  m.


 $X = 2 \text{ cm} \quad y = 3 \text{ cm} \quad z = 5 \text{ cm}$ 

15 In the opposite figure :

The area of a trapezium  
is  $A = \frac{1}{2} h (a + b)$  find A when :

- 1  $h = 2$  metres ,  $a = \frac{3}{4}$  metre and  $b = \frac{1}{4}$  metre.
- 2  $h = 4$  metres ,  $a = \frac{1}{2}$  metre and  $b = \frac{1}{2}$  metre


 $h = 2 \text{ m} \quad a = \frac{3}{4} \text{ m} \quad b = \frac{1}{4} \text{ m}$ 


For excellent pupils

16 Put the parentheses in the place to make each of the following equalities true :

1  $3 + 96 + 12 \times 4 = 5$

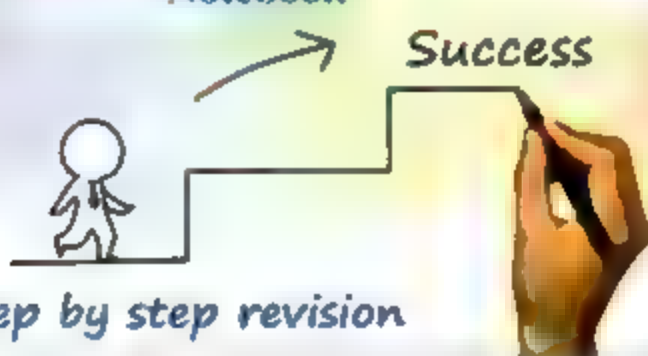
2  $3 + 96 + 12 \times 4 = 35$

3  $3 + 96 + 12 \times 4 = 33$



EL-MOFASSER

Notebook



Step by step revision



16

## The square root of a perfect square rational number

From the school book

1 Find each of the following :

1  $\sqrt{16}$

2  $-\sqrt{25}$

3  $\pm\sqrt{2500}$

4  $\pm\sqrt{40000}$

5  $\sqrt{\frac{9}{49}}$

6  $-\sqrt{\frac{64}{25}}$

7  $\sqrt{0.81}$

8  $\pm\sqrt{1.44}$

9  $\sqrt{6\frac{1}{4}}$

10  $-\sqrt{1\frac{11}{25}}$

11  $-\sqrt{4^2}$

12  $\pm\sqrt{8^2}$

13  $\sqrt{(\frac{81}{100})^2}$

14  $\sqrt{(-\frac{3}{4})^2}$

15  $\pm\sqrt{\frac{576}{1225}}$

16  $-\sqrt{\frac{2.5}{40}}$

17  $-\sqrt{\frac{49a^4}{25b^6}}$

18  $\pm\sqrt{\frac{16b^8}{121h^2}}$

19  $\sqrt{\frac{49a^4b^2}{9}}$

20  $\sqrt{\frac{25x^2y^2}{36}}$

2 Find the two square roots of each of the following numbers :

1 64

2 144

3  $\frac{9}{25}$

4  $6\frac{1}{4}$

5 0.25

6 0.0049

3 Find each of the following :

1  $\sqrt{9} + \sqrt{16}$

2  $\sqrt{36 + 64}$

3  $\sqrt{25 - 9}$

4  $-\sqrt{225 - 81}$

5  $\sqrt{3^2 + 4^2}$

6  $-\sqrt{(10)^2 - 8^2}$

7  $\sqrt{\frac{9}{16} + 1}$

8  $\sqrt{\frac{1}{4}(1 - \frac{3}{4})}$

9  $\sqrt{\frac{5^4 \times 5^3}{5^5}}$

10  $\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2}$

11  $\sqrt{\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{3}\right)^4}$

12  $\sqrt{\left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^3}$

4 Complete the following :

1  $\frac{3}{4} \times \sqrt{\frac{16}{9}} = \dots$

2  $\sqrt{\frac{81}{49}} \times \frac{14}{27} = \dots$

3  $\sqrt{\frac{9}{4}} - \frac{3}{2} + \left(\frac{3}{2}\right)^{200} = \dots$

4  $\sqrt{36} + \sqrt{16} = \sqrt{\dots}$

5  $\sqrt{(81)^2 - 81 \times 2 + 1} = \dots$

6 The multiplicative inverse of  $\sqrt{\frac{4}{25}}$  in the simplest form equals  $\dots$

7 The multiplicative inverse of  $\sqrt{0.49}$  in the simplest form equals  $\dots$

8 The multiplicative inverse of the rational number  $\sqrt{\frac{10}{25}}$  equals  $\dots$

9 The additive inverse of the number  $-\sqrt{\frac{9}{16}}$  in the simplest form equals  $\dots$

10 The rational number  $6\frac{1}{4}$  in the form  $\left(\frac{a}{b}\right)^2$  is  $\dots$

11  $\sqrt{\frac{25}{64}} = \sqrt{\left(\frac{\dots}{\dots}\right)^2} = \dots$

12  $\sqrt{(-3)^2} = \dots$

13  $\sqrt{a^4 b^8} = \dots$

14 If  $a = \sqrt{\frac{1}{4}}$  and  $b = 2$ , then  $ab = \dots$

15 If  $a = -\frac{1}{2}$  and  $b = -\frac{9}{8}$ , then  $\sqrt{ab} = \dots$

16 If  $2x = \sqrt{36}$ , then  $x = \dots$

17 If  $a = 0.000625$ , then  $\sqrt{a} = 2.5 \times 10^{\dots}$

18  $\sqrt{(2009)^2 + 2(2009) \times 213 + (213)^2} = \dots$

5 Choose the correct answer from those given :

1  $\sqrt{1\frac{9}{16}} = \dots$

(a)  $1\frac{3}{4}$

(b)  $-1\frac{3}{4}$

(c)  $1\frac{1}{4}$

(d)  $-1\frac{1}{4}$

2  $\sqrt{10^2 - 6^2} = \dots$

(a) 4

(b) 8

(c)  $\pm 4$

(d)  $\pm 8$

3  $\sqrt{18 \times 10 \times 10 \times 18} = \dots$

(a) 18

(b) 180

(c) 10

(d) 100



## Exercise 6

4  $\sqrt{\sqrt{81}} = \dots$

- (a) 81 (b) 27 (c) 9 (d) 3

5  $\sqrt{2^2 + \sqrt{25}} = \dots$

- (a) 3 (b) -3 (c) 9 (d) -9

6 If  $\frac{X}{2} = \frac{8}{X}$ , then  $X = \dots$

- (a) 4 (b) -4 (c)  $\pm 4$  (d) 16

7 If  $X = \sqrt{\frac{1}{4}}$ , then  $X^3 = \dots$

- (a)  $\frac{3}{8}$  (b)  $\frac{1}{8}$  (c)  $\frac{1}{16}$  (d)  $\frac{1}{64}$

8  $\sqrt{(a+b)^3(a+b)} = \dots$

- (a)  $(a+b)^2$  (b)  $a^4 + b^4$  (c)  $-(a+b)^2$  (d)  $\pm(a+b)^2$

9  $\sqrt{1} + \sqrt{4} + \sqrt{9} + \sqrt{16} + \sqrt{25} + \sqrt{36} + \sqrt{49} + \sqrt{64} =$

- (a) 6 (b)  $\sqrt{204}$  (c)  $\sqrt{81}$  (d)  $6^2$

10 The side length of the square whose area is  $16 X^2 \text{ cm}^2$  equals  $\dots \text{ cm}$ .

- (a)  $8 X$  (b)  $4 X$  (c)  $2 X$  (d)  $8 X^2$

6 Simplify each of the following to the simplest form :

1  $(-\frac{1}{2})^3 \times \sqrt{\frac{64}{9}}$

2  $\sqrt{\frac{49}{4}} \times (\frac{2}{3})^{\text{zero}} \times (-\frac{2}{7})^2$

3  $\frac{2}{3} \times \sqrt{\frac{9}{16}} \div (-\frac{1}{2})^3$

4  $(-\frac{1}{3})^2 + \sqrt{\frac{64}{81}} - (\frac{2}{4})^{\text{zero}}$

5  $\frac{3}{4} \times (-\frac{2}{3})^3 \times (\frac{3}{\sqrt{4}})^2$

6  $\sqrt{(\frac{25}{4})^2 \times (\frac{2}{3})^2}$

7 Simplify each of the following to the simplest form :

1  $\sqrt{16} + \sqrt{25}$

2  $\sqrt{\sqrt{16} + \sqrt{25}}$

3  $\sqrt{(\sqrt{16} + \sqrt{25})^2}$

8 Find two rational numbers lying between :  $\sqrt{\frac{4}{9}}$  and  $\frac{3}{4}$

9 Which is greater :  $\frac{3}{5}$  or  $\sqrt{\frac{4}{9}}$  ? Find the difference between them.

10 Which is smaller :  $\sqrt{2\frac{1}{4}}$  or  $(-\frac{2}{3})^2$  ? Find their difference.

11 Find each of the following :

1  $\sqrt{5^2 - 2 \times 5 + 1}$

2  $\sqrt{(\frac{1}{4})^2 - 2 \times \frac{1}{4} + 1}$

3  $\sqrt{20 + 5 + 8 - (4 - 1)}$

4  $\sqrt{8 \times (5 + 11) + (2 + 6)}$

5  $\sqrt{2 \times 8 + 10 - 3 + 12 + 11 \times 6 + 88 + 2^3 + 99 + 11}$

6  $\sqrt{6 + 3\sqrt{100} - \sqrt{121}}$

### Geometric Applications

12 1  $\overline{XY}$  is a line segment where  $(XY)^2 = 25 \text{ cm}^2$  , E is the midpoint of  $\overline{XY}$

Find the length of :  $\overline{XE}$

= 2.5 cm .

2 .. If  $(AB)^2 = 144 \text{ cm}^2$  ,  $(BC)^2 = 625 \text{ cm}^2$  and  $B \in \overline{AC}$

Find the length of :  $\overline{AC}$

= 37 cm .

3 The area of a square is  $0.49 \text{ cm}^2$  Find its perimeter

= 2.8 cm .

4 The area of a square is equal to the area of a triangle with base = 9 cm. long and its height = 8 cm. Find the side length of the square

= 6 cm .

5 The area of a circle is  $154 \text{ cm}^2$  Calculate its radius length ( $\pi = \frac{22}{7}$ )

= 3.5 cm .

6 The area of a circle is  $78.5 \text{ cm}^2$  Calculate its diameter length ( $\pi = 3.14$ )

= 10 cm .

7 The area of a circle is  $616 \text{ cm}^2$  Calculate its circumference ( $\pi = \frac{22}{7}$ )

= 88 cm .

8 .. If three quarters of the area of a square is  $1\frac{11}{64} \text{ m}^2$

Calculate the side length of the square.

=  $1\frac{1}{4} \text{ m}$  .

9 .. The length of a rectangle is twice its width and its area is  $24.5 \text{ cm}^2$

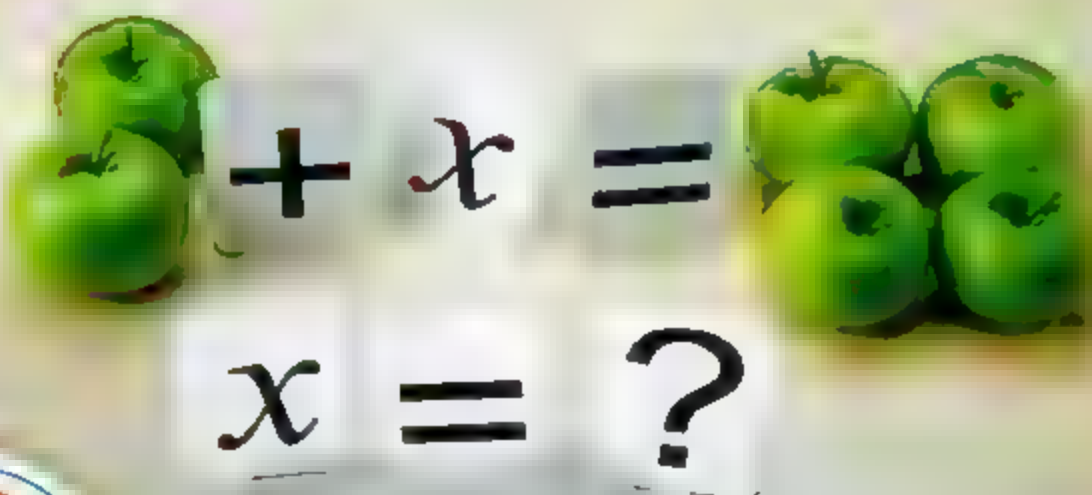
Calculate each of its width and length.

= 3.5 cm , 7 cm .

### For excellent pupils

13 .. If  $\frac{m}{n}$  is a rational number and  $\frac{m^2}{n^2} = 0.16$  , find the value of  $(\frac{m}{n})^3$

= 0.064 .



7

Solving equations in  $\mathbb{Q}$ 

From the school book

1 Find the solution set of each of the following equations :

1)  $x - 7 = 3$  where  $x \in \mathbb{N}$

3)  $5x = 20$  where  $x \in \mathbb{Q}$

5)  $x - 4 + y = 13$  where  $y \in \mathbb{N}$

7)  $x - 7 = 0$  where  $x \in \mathbb{Z}$

9)  $x - 6\frac{1}{4} = 12\frac{1}{2}$  where  $x \in \mathbb{Q}$

2)  $x + 17 = 13$  where  $x \in \mathbb{N}$

4)  $\frac{2}{3}x = \frac{1}{3}$  where  $x \in \mathbb{Q}$

6)  $m - (-3) = 1$  where  $m \in \mathbb{Z}$

8)  $y - (-5) = -3$  where  $y \in \mathbb{Q}$

10)  $8.91 + x = 11.09$  where  $x \in \mathbb{Q}$

2 Solve each of the following equations :

1)  $2x - 1 = 5$  where  $x \in \mathbb{Q}$

3)  $3x - 13 = 26$  where  $x \in \mathbb{N}$

5)  $8 + 2x = 14$  where  $x \in \mathbb{Z}$

7)  $8 - 2x = -2$  where  $x \in \mathbb{Z}$

9)  $2x + 3x + 25 = 5$  where  $x \in \mathbb{Z}$

2)  $8x + 4 = 12$  where  $x \in \mathbb{Q}$

4)  $2x + 14 = 14$  where  $x \in \mathbb{N}$

6)  $\frac{5}{6}x - 4 = 11$  where  $x \in \mathbb{Q}$

8)  $2 - 5x = 0$  where  $x \in \mathbb{Q}$

10)  $6x - 2x + 7 = 4$  where  $x \in \mathbb{Z}$

3 Solve each of the following equations in  $\mathbb{Q}$  :

1)  $2(x - 3) = 4$

3)  $7(x - 2) - 3(x + 1) = 3$

5)  $4(x - 1) - (x + 3) = 0$

7)  $2(x - 3) + 3(x - 2) - 4x = -3$

2)  $3x + 2(5x - 3) = 7$

4)  $3(x + 2) + 7(x - 1) = 12$

6)  $5(x - 2) + 2(x + 4) = -16$

8)  $3y + 6(y + 3) - (8y - 16) = 60$



## Unit 1

4 Find in  $\mathbb{Q}$  the solution set of each of the following equations :

1.  $2x + 5 = x + 9$

2.  $x + 3 = 18 - 3x$

3.  $4(x + 1) = 2(x - 1)$

4.  $a + 5a - 2 = 2(3 - a)$

5.  $\frac{x+1}{3} = \frac{x-1}{4}$

6.  $5x - 4 = 2x + 11$

7.  $3x + 6 = 30 - 5x$

8.  $3(x - 2) = 5x - 10$

9.  $3(2x - 8) - (2x + 2) = x - 3$

10.  $\frac{5}{4+4x} = \frac{3}{1-2x}$

5 Complete the following :

1 If  $x + 5 = 7$ , then  $x =$

2 If  $3t = 6$ , then the value of  $6t =$

3 If  $2x = 5$ , then the value of  $4x =$

4 If  $x + 9 = 11$ , then the value of  $7x =$

5 If  $2t + 3 = 15$ , then the value of  $\frac{1}{3}t =$

6 If  $z - 1\frac{1}{4} = 5\frac{1}{2}$ , then the value of  $4z - 18 =$

7 If  $\frac{p}{4} = \frac{2}{3}$ , then the value of  $\frac{p}{2} =$

8 If the age of a man now is  $x$  years, then his age 5 years ago is

9 If the age of a man now is  $y$  years, then his age after 4 years is

10 If the age of a man after 5 years is  $x$  years, then his age now is

11 If the age of Youssef after 4 years is  $x$  years, then his age 2 years ago is

12 A rectangle with length equals triple its width. If the length =  $x$  cm, then its width = ..... cm.

13 The rectangle whose width =  $x$  cm, and its length is twice its width, then its perimeter = ..... cm.

14 Two integers, their sum is 5, if one of them is  $x$ , then the other one is

15 Two integers, the difference between them is 2, if the small one is  $x$ , then the great one is

6 Choose the correct answer from those given :

1 If  $2x = 2$ , then  $3x - 1 =$

(a) 2

(b) 3

(c) 4

(d) 5

2 If  $2x = 0$ , then  $x =$

(a) 2

(b) 3

(c) 5

(d) zero

## Exercise 7

- 3 If  $2a + b = 10$ , then  $3a + b =$   
 (a) 5 (b) 6 (c) 15 (d) 30
- 4 If  $0.2 + a = 5$ , then  $\frac{a}{4} = \dots$   
 (a) 4.8 (b) 1.3 (c) 1.2 (d) 19.2
- 5 If  $5x + 8x + 2x + 4x = 114$ , then  $5x + 3 =$   
 (a) 33 (b) 35 (c) 47 (d)  $8x$
- 6 The S.S. of the equation  $\frac{2a}{3} = 8 + 4a$  in  $\mathbb{Q}$  is  
 (a)  $\{-2.4\}$  (b)  $\{2.4\}$  (c)  $\{-3\frac{1}{3}\}$  (d)  $\{0\}$
- 7 Which of the following equations is equivalent to the equation  $x + 3 = 12$ ?  
 (a)  $x - 3 = -12$  (b)  $x + (-3) = 12$   
 (c)  $x - (-3) = 12$  (d)  $x - (-3) = -12$
- 8 Which of the following equations is equivalent to the equation  $x - 12 = 15$ ?  
 (a)  $x + 12 = -15$  (b)  $\frac{1}{3}x - 4 = 5$  (c)  $x - 4 = -5$  (d)  $x + 4 = 5$

## Geometric Applications

- 7 Find the measure of each angle in each of the following triangles :



Fig. (1)

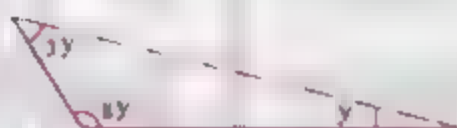
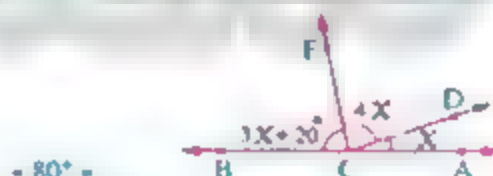


Fig. (2)



Fig. (3)

- 8 In the opposite figure :  
 If  $C \in \overline{AB}$ , find  $m(\angle DCE)$



- 9 In each of the following figures find the value of  $X$  :



Fig. (1)

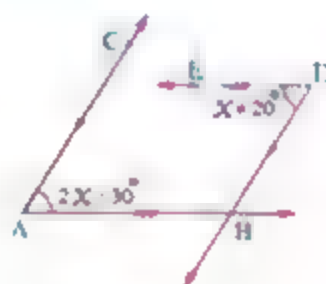


Fig. (2)

## Unit 1

- 10 The length of a rectangle exceeds its width by 4 metres and its perimeter is 68 metres.  
Find the dimensions of the rectangle.  $\rightarrow 19 \text{ m. } \rightarrow 15 \text{ m.}$

- 11 The length of a rectangle is less than twice its width by 4 cm. , if its perimeter equals the perimeter of a square of side length equals 7 cm.  
Find the dimensions of the rectangle.  $\rightarrow 6 \text{ cm. } \rightarrow 8 \text{ cm.}$

- 12 The length of a rectangle is twice its width. If the length decreases by 5 cm. and the width increases by 6 cm. , then the rectangle becomes a square.  
Find the area of the rectangle.  $\rightarrow 242 \text{ cm}^2$

## Life Applications

- 13 Two integers , the smaller number is  $2X$  and the greater number is  $7X$  , if the difference between them is 25 , find the two integers.  $\rightarrow 10 , 15$

- 14 Two natural numbers , one of them is twice the other and their sum is 108  
Find the two numbers.  $\rightarrow 36 , 72$

- 15 The difference between two natural numbers is 5 and their sum is 21  
What are the two numbers ?  $\rightarrow 13 , 8$

- 16 Find the number which if added to its triple the result is 32  $\rightarrow 8$

- 17 Find the number which if we subtract 9 from its triple , the result will be 6  $\rightarrow 9$

- 18 Three consecutive natural numbers their sum is 213  
What are these numbers ?  $\rightarrow 70 , 71 , 72$

- 19 The sum of three consecutive even numbers is 966 , find them.  $\rightarrow 320 , 322 , 324$


- 20 Find three consecutive odd numbers if their sum is 357  $\rightarrow 117 , 119 , 121$

- 21 A man's age now is three times his son's age and after two years , the sum of their ages will be 52 years. What is the age of each now ?  $\rightarrow 12 \text{ years } , 36 \text{ years}$

- 22 Three brothers , Amgad , Bassim and Ayman , the sum of their ages is 89 years  
If Amgad was born before Bassim by 2 years and Bassim was born before Ayman by 6 years , what is the age of each of them now ?  $\rightarrow 25 \text{ years } , 31 \text{ years } , 33 \text{ years}$



## Exercice 7

- 23  The price of one metre of wool exceeds by 2 pounds than the price of one metre of silk. If the price of 3 metres of wool and 4 metres of silk is 671 pounds. Find the price of one metre of each kind. • 97 pounds • 95 pounds •



For excellent pupils:

- 24 Find in  $\mathbb{Q}$  the S.S. of each of the following equations :

1  $5 - \frac{6}{x} = -1$

2  $-\frac{3}{5} + \frac{x}{10} = -\frac{1}{5} - \frac{x}{5}$

- 25 Find in  $\mathbb{Q}$  the S.S. of each of the following equations :

1  $(x+3)^2 - (x-2)^2 = 15$

2  $(2x+3)(2x-1) - (2x-1)^2 = 14$

- 26 If the S.S. of the equation  $12x + 3 = 39$  in  $\mathbb{Q}$  equals the S.S. of the equation  $ax - 12 = a$  in  $\mathbb{Q}$ , find the value of  $a$ . • 6 •

- 27 If  $a+1$  is a solution of the equation  $(x+a)(x-a) = x^2 - ax + 3$  in  $\mathbb{Q}$ , find the value of  $a$ . • 1 •

- 28 Three brothers were born in 1980, 1984 and 1986, the required is finding the year in which the sum of their ages became 41 years. • 1997 •



8

Solving inequalities in  $\mathbb{Q}$ 

From the school book

- 1 Which number would you add to each side of the inequality to obtain  $x$  in one side of it ?

1  $x + 5 > 9$

2  $x - 4 < 6$

3  $x - 7 < 3$

4  $x + 9 > 12$

5  $x - 1.5 \leq 32$

6  $4.8 \leq x + 0.6$

7  $1\frac{1}{2} > x - 2\frac{1}{2}$

8  $x + \frac{1}{3} > -\frac{1}{6}$

- 2 Find the solution set of the inequality  $x + 3 \leq 6$  in each of the following cases :

a  $x \in \mathbb{Z}$

b  $x \in \mathbb{N}$

then represent the solution set on the number line.

- 3 Find the solution set of each of the following inequalities in  $\mathbb{Q}$  :

1  $x + 2 > 5$

2  $x + 4 > 1$

3  $y - 5 > 7$

4  $19 < y + 14$

5  $-1 \geq x - 3$

6  $-5\frac{1}{2} > a + 1\frac{1}{4}$

7  $-2x < 12$

8  $\frac{2}{3}x \geq 1$

9  $-\frac{1}{4}x \leq \frac{1}{4}$

- 4 Solve each of the following inequalities in  $\mathbb{Q}$  :

1  $3x - 2 < 1$

2  $2x + 3 < 9$

3  $4x + 2 \geq -10$

4  $3x - 2 \geq 5$

5  $3x - 9 < 0$

6  $1 + 2x \leq -3$

7  $9 - 6x < 15$

8  $2 - 3x \leq 4$

9  $\frac{3x - 2}{5} \geq \frac{1}{2}$

10  $8x - 3x + 1 \leq 29$

11  $4n - 2(n - 1) \geq 0$

12  $-3m + 6(m - 4) > 9$

5 Solve each of the following inequalities in  $\mathbb{Q}$  :

1  $6d + 1 \leq 5d - 3$

3  $3x - 2 < 5x - 8$

5  $5x + 1 \geq 2(x + 2)$

7  $3(x + 2) \geq -2(x + 1)$

9  $3(7y - \frac{1}{3}) \leq 20y - 1$

2  $6x + 2 \geq 14 + 5x$

4  $8 - 2x \leq 5x$

6  $3(x + 2) < -x + 4$

8  $2 - 3(x - 5) \geq x + 7$

10  $\frac{x}{2} + 3 \leq 2x + 1$

6 Find the S.S. of each of the following inequalities :

1  $9 \leq 4x + 1 \leq 17$  ,  $x \in \mathbb{Z}$

3  $9 > x + 6 > 2$  ,  $x \in \mathbb{N}$

2  $9 \leq 3x + 2 < 12$  ,  $x \in \mathbb{Q}$

7 Complete :

1 If  $x > y$  , then  $x + z$   $y + z$

3 If  $x < y$  and  $y < z$  , then  $x <$

5 If  $a - 3 < 0$  , then  $\dots > \dots$

7 If  $b < 0$  , then  $b + 3 \dots 3$

9 If  $x > y$  and  $z$  is positive ( $z > 0$ ) , then  $xz$   $yz$

8 If  $x < y$  and  $z$  is negative ( $z < 0$ ) , then  $xz$   $yz$

2 If  $x < y$  , then  $x + z$   $y + z$

4 If  $z > y$  and  $y > x$  , then  $z >$

6 If  $a + 5 > 0$  , then  $\dots >$

8 Choose the correct answer from those given :

1 If  $-x < 5$  , then ..

(a)  $x > 5$

(b)  $x > -5$

(c)  $x < 5$

(d)  $x < -5$

2 If  $x \in \mathbb{N}$  , then the S.S. of the inequality  $x > 3$  is

(a)  $\{4, 5, \dots\}$

(b)  $\{-4, -5, \dots\}$

(c)  $\{-3\}$

(d)  $\emptyset$

3  $\frac{x}{3} < 4$  is equivalent to ...

(a)  $x > \frac{4}{3}$

(b)  $x < \frac{4}{3}$

(c)  $x > 12$

(d)  $x < 12$

4 If  $x \in \mathbb{Z}$  , then the S.S. of the inequality  $20 < 5x < 25$  is

(a)  $\{4\}$

(b)  $\{5\}$

(c)  $\{4, 5\}$

(d)  $\emptyset$

5 The S.S. of the inequality  $-2x < \text{zero}$  in  $\mathbb{Q}$  is

(a)  $\emptyset$

(b)  $\mathbb{Q}_+$

(c)  $\mathbb{Q}_-$

(d)  $\mathbb{Z}_+$

6 The number of solutions of the inequality  $\frac{1}{3} < x < \frac{2}{3}$  , where  $x \in \mathbb{Q}$  is

(a) zero

(b) 1

(c) 2

(d) an infinite number.



## Unit 1

7 If  $X > y$ , then  $\frac{1}{X}$   $\frac{1}{y}$ , where  $X \neq 0, y \neq 0$

- (a)  $>$  (b)  $<$  (c)  $=$  (d)  $\geq$

8 The number 2 belongs to the S.S. of the inequality where  $X$  is an integer

- (a)  $X > 2$  (b)  $X < 2$  (c)  $-X > -3$  (d)  $-X > 3$

9 If  $X > 5$ , then  $-X$

- (a)  $< -9$  (b)  $\geq -5$  (c)  $< -5$  (d)  $> -5$

10 Show by using examples that if  $a > b$  and  $c > d$ , then it is not always correct that  $a - c > b - d$

11 Put ( $\checkmark$ ) for the correct statement and ( $\times$ ) for the incorrect statement, when a statement is false, give an example that shows why it is false (given that  $X > y$ ):

- 1  $y < X$  ( ) 2  $X > 0$  ( ) 3  $y^2 \geq 0$  ( ) 4  $y^2 > y$  ( )  
 5  $XY > 0$  ( ) 6  $X + y > y$  ( ) 7  $y^2 > X$  ( ) 8  $y^2 < XY$  ( )  
 9  $XY < X^2$  ( ) 10  $X^3 < y^2$  ( )

## Life Application

11 Hany wants to buy a pair of shoes and some shirts. if Hany has L.E. 200, the price of the pair of shoes is L.E. 70 and the price of one shirt is L.E. 40. What is the greatest number of shirts Hany can buy?

- 3 -



## For excellent pupils

12 If the S.S. of the inequality  $a \leq 3X - 5 \leq b$  in  $\mathbb{Q}$  is  $\{X : X \in \mathbb{Q}, 2 \leq X \leq 5\}$ , find the values of  $a$  and  $b$

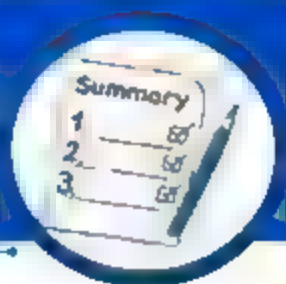
- 1, 10 -

13 If  $-4 \leq X \leq 5$  and  $2 \leq y \leq 7$ , where  $X \in \mathbb{Q}$  and  $y \in \mathbb{Q}$ , find:

- 1 The greatest possible value of the expression  $X + y$  - 12 -  
 2 The greatest possible value of the expression  $y - X$  - 11 -  
 3 The smallest possible value of the expression  $Xy$  - 28 -  
 4 The smallest possible value of the expression  $X^2 + y^2$  - 0 -

# Summary of the second part of unit 1

"From lesson 5 to lesson 8"



## • The order of performing the mathematical operations :

- 1 Perform the operations within parentheses (interior parentheses then exterior ones).
- 2 Evaluate the powers.
- 3 Perform multiplications and divisions in order from left to right
- 4 Perform additions and subtractions in order from left to right.

## • The square root of the perfect square rational number "a" is the number whose square equals "a"

•  $\sqrt{\text{zero}} = \text{zero}$

## • In the set of rational numbers it is meaningless to find $\sqrt{a}$ if a is a negative rational number because there is no rational number if it is multiplied by itself, the result will be negative.

•  $\sqrt{a^2} = |a|$

• If  $X^2 = a$  where  $a \geq 0$ , then  $X = \pm \sqrt{a}$

## • Solving the equation or the inequality is to find the values of the variable which satisfies the equation or the inequality.

## • Let a , b and c are three rational numbers :

- If  $a < b$ , then  $a + c < b + c$
- If  $a < b$ , then  $a - c < b - c$
- If  $a < b$ , c is a positive number, then  $a \cdot c < b \cdot c$
- If  $a < b$ , c is a negative number, then  $\frac{a}{c} > \frac{b}{c}$
- If  $a < b$ , c is a negative number, then  $a \cdot c > b \cdot c$
- If  $a < b$ , c is a negative number, then  $\frac{a}{c} > \frac{b}{c}$
- If  $a > b$ ,  $b > c$ , then  $a > c$

Find the value of  $x$  in the second part of unit one  
 (For example:  $15 \times 10 = 150$ )



### Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1  $\sqrt{10^2 - 6^2} =$

- (a) 4 (b) 8 (c) -4 (d)  $\pm 8$

2  $4 \times 2^3 - 20 =$

- (a) -48 (b) 4 (c) 12 (d) 16

3 If  $2x = 4$ , then  $3x + 1 =$

- (a) 13 (b) 4 (c) 15 (d) 7

4 If  $-x < 7$ , then

- (a)  $x > 7$  (b)  $x > -7$  (c)  $x < 7$  (d)  $x < -7$

5 The multiplicative inverse of the number  $\sqrt{\frac{9}{25}}$  is

- (a)  $\frac{5}{3}$  (b)  $\frac{3}{5}$  (c)  $\frac{25}{9}$  (d)  $\frac{9}{25}$

6 The S.S. of the equation :  $3x = -9$  in  $\mathbb{N}$  is ...

- (a)  $\{-3\}$  (b)  $\{-6\}$  (c) zero (d)  $\emptyset$

2 Complete the following :

1 If the age of a man now is  $x$  years, then his age 9 years ago is \_\_\_\_\_ years.

2  $\sqrt{\frac{49a^2b^2}{9}} =$  (in the simplest form)

3  $144 - 8 \div 2^3 =$

4 The S.S. of the inequality :  $-3x < 0$  in  $\mathbb{Q}$  is \_\_\_\_\_

5 If  $x + 9 = 11$ , then  $7x =$  \_\_\_\_\_

3 [a] Find the S.S. of the following inequality in  $\mathbb{Q}$  :  $3 - 2x \geq 1$

[b] Simplify :  $\sqrt{\frac{49}{4}} \times \left(\frac{3}{11}\right)^{-100} \times \left(-\frac{2}{7}\right)^2$



4 [a] Find :

1  $\frac{5+2 \times 5}{2^2+1} + 5^2 - 5$

2  $2^3 \times [4 + (2 - 1)]$

[b] A rectangle in which the length is twice the width and its perimeter is 24 cm.

Find the dimensions of the rectangle.

5 [a] Find in  $\mathbb{Q}$  the S.S. of the equation :  $X + 3 = 18 - 3X$ [b] Find in  $\mathbb{Q}$  the S.S. of the inequality :  $9 \leq 3X + 2 < 12$ 

## Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

1 If  $3y = 15$ , then  $5y =$ 

(a) 5

(b) 15

(c) 25

(d) 125

2 The S.S. of the inequality :  $3 < X \leq 4$  in  $\mathbb{N}$  is(a)  $\{3\}$ (b)  $\{4\}$ (c)  $\{3, 4\}$ (d)  $\emptyset$ 3 The side length of a square whose area :  $9X^2 \text{ cm}^2$  is(a)  $3X$ (b)  $3X^2$ (c)  $9X$ (d)  $9X^2$ 4 If  $X > y$ ,  $z > \text{zero}$ , then  $Xz \dots yz$ (a)  $>$ (b)  $<$ (c)  $=$ (d)  $\leq$ 5  $\sqrt{9+16} = 3 + \dots$ 

(a) 4

(b) 2

(c) 25

(d) 22

6 If  $\frac{26}{X} + 1 = 14$ , then  $X =$ 

(a) 2

(b) 10

(c) 13

(d) 15

2 Complete the following :

1  $20 + 5 - 2^2 =$

2 If  $X = 0.0009$ , then  $\sqrt{X} = \dots$

3 If  $3X + 1 \geq 10$ , then  $X \geq$

## Unit 1

4. If  $\frac{6X}{5} = -2$ , then  $3X =$  \_\_\_\_\_

5. An equilateral triangle of side length  $X$  cm.  
then its perimeter = ..... cm.

3 [a] Find :  $2[(5^2 + 1)] - (4^2 - 1)$

[b] Simplify :  $(-\frac{3}{7})^{\text{zero}} \times (-\frac{2}{5})^2 \times \sqrt{6\frac{1}{4}}$

4 [a] Find in  $\mathbb{Q}$  the S.S. of the equation :

$$(3X + 2) + 5 = 12$$

[b] Find in  $\mathbb{Q}$  the S.S. of the inequality :

$$2X - 7 \leq 3$$

5 [a] Find in  $\mathbb{Q}$  the S.S. of the inequality :

$$6X + 2 \geq 14 + 4X$$

[b] Find the number if added to its three times the result will be 28

For the next year,

Ask for



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For 2<sup>nd</sup> prep. \_\_\_\_\_

UNIT

2

## Statistics and Probability



## Exercises of the unit

## 1- Sample

- Systematic sample
- Random sample

## 2- Probability

- Experimental probability
- Theoretical probability



19

## Samples

From the school book

- 1 A factory's canteen service wanted to find the preferences of their 427 employees during their 15 minute break. Each employee was given a number from 1 to 427. A 10% sample of the 427 were to be surveyed and asked to select a preference from :



• Hot beverage. • Hot soup with bread. • Cold drink with biscuits. • Fruit with fresh water.

The sample were determined by selecting 43 sample numbers in the range using calculator. Identify the sample numbers using a calculator.

- 2 A school makes a study about how the pupils come to school. If the number of pupils in the school is 320 , each pupil is given a number from 1 to 320 A sample of 10% from this number is selected as a sample to ask them how they come to school :



• On foot • Public transport • Taxi • Bike • Private car

Determine the numbers of the sample using the calculator.

## Exercise 9

- 3 A company makes a study about the best places which the workers in the company prefer to spend their annual holiday among .

- Port Said
- Alexandria
- Matrouh
- The North Coast
- Ismailia

If the number of the workers in the company is 250 workers and a sample of 10% from the number of workers is selected to make a survey on it , determine the numbers of the sample using the calculator



- 4 It is noticed that 230 persons use a public bus daily and the public transport authority wanted to collect some informations concerning with the using daily of this service. It is necessary to form a random sample representing 10% from the users of this bus to make a survey on them Determine the numbers of this sample using the calculator.





10

## Probability

From the school book

## First Problems on experimental probability:

- 1 In the experiment of spinning game, roll the disc 50 times. In each time, record the number at which the pointer stops in the following table.

	1	2	3	4	5	Total
The tally sign						
Frequency						50



- Calculate :
- 1 The probability that The pointer stops at the number 2
  - 2 The probability that : The pointer stops at the number 5

- 2
- (a) Draw six parallel lines with a distance of 2 cm. between each of them on an A<sub>4</sub> sheet of paper.
  - (b) Bring a piece of wood of length 2 cm.
  - (c) Slightly toss the piece of wood in the air so that it falls from a suitable height onto the A<sub>4</sub> sheet.
  - (d) Repeat the trial 50 times
  - (e) Record the number of times that the piece of wood falls across the line and also between the lines.
  - (f) Deduce the probability of the piece of wood falling between the lines.



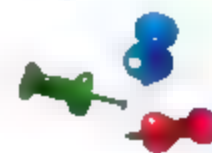
	Across	Between	Total
Tally			
Frequency			50



## Exercise 10

- 3 (a) Drop a drawing pin 100 times from a suitable height.  
 (b) Record the number of times it lands with its point up and its point down.

	Up	Down	Total
Tally			
Frequency			100



- (c) Deduce the probability of the drawing pin landing point "UP" and point "Down"

- 4 In the spinner game :

The disc is divided into two unequal parts X and Y The pointer is rolled 800 times.  
 It stands 197 times at the X-zone

In which of the following figures the pointer points to X-zone ?



(a)



(b)



(c)



(d)

### Second Problems on theoretical probability:

- 1 As throwing a fair die and observing the upper face , complete the following :

- The probability of appearance a number greater than 2 =
- The probability of appearance a number less than 3 =
- The probability of appearance an even number =
- The probability of appearance the number 4 = .....
- The probability of appearance the number 7 =
- The probability of appearance a number less than or equal to 6 =
- The probability of appearance a prime number =
- The probability of appearance a prime even number =
- The probability of appearance a number divisible by 5 =
- The probability of appearance the number 5 or the number 6 =



- 2 Complete the following :

- The probability of occurring the impossible event = ... and the probability of occurring the certain event =
- If a coin is flipped once , then the probability of appearance of a head =

## Unit 2

- 3 10 cards numbered from 1 to 10. If a card is drawn randomly , then the probability that the card is numbered by an odd number =
- 4 A box has 5 white balls , 7 red balls , 3 blue balls. If a ball is drawn randomly from the box , then the probability that the ball is blue =
- 5 In the experiment of throwing a fair die once and observing the upper face , the probability that the apparent number is less than 1 =
- 6 If one of the digits of the number 867742231 is selected randomly , then the probability that the selected number is even equals
- 7 A box contains 48 oranges and 4 oranges of them are bad. If an orange is drawn randomly , then the probability that the drawn orange is bad = and the probability that the drawn orange is good = ...
- 8 If the probability of occurring an event is  $\frac{5}{8}$  , then the probability that the event doesn't occur =
- 9 An activity room has 3 doors numbered from 1 to 3. If a student went out using one of them , then the probability that the student went out using the door number 2 is
- 10 If the probability that a person get infected (in a city whose number of inhabitants is 200000) with a disease is 0.003 , then the expected number of infected persons with the disease in this city is ..... persons.

## 3 Choose the correct answer from those given :

- 1 Which of the following is the probability of occurrence of an event ?  
(a) 1.2 (b) - 0.4 (c) 315% (d) 75%
- 2 As throwing a fair die once , the probability of appearance of a number greater than 4 is  
(a)  $\frac{1}{6}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{2}$  (d) 1
- 3 A basket contains cards numbered from 1 to 20 If a card is drawn randomly , what is the probability that the number written on it is divisible by 6 ?  
(a)  $\frac{3}{20}$  (b)  $\frac{4}{20}$  (c)  $\frac{5}{20}$  (d)  $\frac{6}{20}$
- 4 A bag has 5 red balls and 3 white balls. If the balls are similar and a person draws a ball randomly , then the probability that the drawn ball is white =  
(a)  $\frac{3}{8}$  (b)  $\frac{3}{5}$  (c)  $\frac{5}{8}$  (d)  $\frac{5}{3}$
- 5 A letter is selected randomly from the name "ZAMALEK". The probability of selecting the letter A is  
(a)  $\frac{1}{7}$  (b)  $\frac{2}{7}$  (c)  $\frac{3}{7}$  (d)  $\frac{4}{7}$

## Exercise 10

- 8 Rashad is in grade 7 in a class of 36 students. 16 of them are girls. If a student is selected randomly from the class, what is the probability that the student is a boy?

(a)  $\frac{4}{9}$  (b)  $\frac{1}{2}$  (c)  $\frac{5}{9}$  (d)  $\frac{1}{36}$

- 7 A class has 25 boys and 20 girls. A pupil of them is selected randomly, then the probability that the pupil is a girl =

(a)  $\frac{1}{20}$  (b)  $\frac{4}{9}$  (c)  $\frac{1}{25}$  (d)  $\frac{5}{9}$

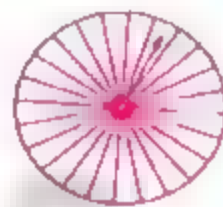
- 6 If a die is tossed once, then the probability of getting a number satisfying the inequality  $2 < X < 3$  equals .....

(a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{4}$  (d) zero

- 5 The opposite figure shows a spinner with 24 sectors.

When someone spins the arrow it is likely equal to stop on any sector.  $\frac{1}{8}$  of sectors are blue,  $\frac{1}{3}$  are red,  $\frac{1}{2}$  are orange and  $\frac{1}{24}$  are purple. If a person spins the arrow, at which colour of sectors does the arrow least likely to stop?

(a) Blue. (b) Purple. (c) Orange. (d) Red.



- 4 If the probability of success of a student is 70%, then the probability of his failure = ....

(a) 0.7 (b) 0.07 (c) 0.3 (d) 0.03

- 3 A card is drawn from a bag of 25 cards numbered from 1 to 25

Calculate the probability that the drawn card carries :

- 1) A number divisible by 5      2) A number  $\geq 20$   
3) A perfect square number.

- 2 One card is selected randomly from 8 cards numbered from 1 to 8

Write down the sample space. Then find the probability of each of the following events :

- 1) Getting an even number.  
2) Getting an odd number.  
3) Getting a number greater than or equal to 6  
4) Getting a number divisible by 3

- 1 A letter is selected randomly from the word "SAMEH"

Calculate the probability of selecting the letter :

- 1 S      2 E      3 R



## Unit 2

- 7 A bag contains 5 red balls , 3 yellow balls and 2 black balls. If all balls are alike and a ball is drawn from the bag randomly , find :

- 1 The probability that the drawn ball is yellow.
- 2 The probability that the drawn ball is yellow or red.
- 3 The probability that the drawn ball is not yellow.



- 8 A card is chosen randomly from ten cards numbered from 1 to 10. What is the probability that the chosen card shows

- 1 An odd number.
- 2 A prime number.
- 3 An even number.
- 4 An odd number greater than 3

- 9 If a fair die is tossed once , what is the probability of each of the following events :

- 1 Appearance of an even number less than or equal to 4
- 2 Appearance of a number between zero and 10
- 3 Appearance of a number divisible by 7
- 4 Appearance of a number not divisible by 2



- 10 A fair die is rolled once and the number of dots on the upper face is observed. Write down the sample space , then find the probability of each of the following events :

- 1 Getting a number greater than 6
- 2 Getting a number satisfying the inequality  $1 \leq X \leq 6$
- 3 Getting a number satisfying the inequality  $2 < X < 4$



- 11 8 cards , numbered by the opposite numbers , are put in a bag. Bassim drew a card from these cards randomly. Find :

- 1 The probability that the card carries a number whose tens digit is even.
- 2 The probability that the card carries a number whose units digit is odd.
- 3 The probability that the card carries a number multiple of 4



- 12 A cube is designed such that each two opposite faces carry one of the digits 1 , 2 and 3. The cube is rolled and the apparent face is observed :

- 1 Write down the sample space.
- 2 What is the probability such that the number on the upper face is 2 ?
- 3 What is the probability such that the number on the upper face is odd ?



## Exercise 10

- 13 A bag contains 30 similar marbles. Hani drew a marble randomly and he found it red. If the probability of drawing a red marble =  $\frac{2}{3}$ , find the number of red marbles in the bag.
- 14 A box contains 80 similar balls. Some of them are red and the rest is blue. If the probability of drawing a red ball is  $\frac{1}{4}$ , find the number of blue balls.
- 15 The set  $\{2, 3, 5\}$  is used in writing a 2-digit number. Find the probability of each of the following events:
- |                                  |                                       |
|----------------------------------|---------------------------------------|
| 1 The tens digit is odd.         | 2 The units digit is odd.             |
| 3 The sum of the two digits is 7 | 4 The product of the two digits is 15 |
- 16 Wael has a bag containing 22 marbles, 12 of them are black and the rest is red. If two marbles of them are drawn without returning them to the bag and they were red. Then he drew a third marble without looking at it. What is the probability that the last marble is black?
- 17 A class has 50 students. The number of girls is less than the number of boys by 10. If one student is chosen randomly, find the probability that the student is a boy.
- 18 Choose the correct answer from those given:
- A bowl contains 32 coloured beads. All beads are of the same size. Some of them are blue, some are green, some are red and the rest is yellow. The probability of drawing a blue bead is  $\frac{3}{8}$ , how many blue beads are in the bowl?  
(a) 4 (b) 8 (c) 12 (d) 16
  - A bag contains 3 white balls, 2 black and one red. If a ball is drawn randomly from the bag, then the probability that the drawn ball is not black equals  
(a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{2}{3}$  (d)  $\frac{1}{6}$
  - A bag has a number of similar balls. Half of them is red, the third is black and the rest is white. A ball is drawn randomly, then the probability that the drawn ball is white =  
(a)  $\frac{1}{2}$  (b)  $\frac{1}{6}$  (c)  $\frac{1}{3}$  (d) zero
  - A box contains coloured balls (red, green, blue and yellow). If the box contains 20 yellow balls and the probability of drawing a yellow ball from the box randomly is  $\frac{1}{4}$ . What is the number of all balls in the box?  
(a) 5 (b) 25 (c) 60 (d) 80
  - The number of pupils in a class (7 grade) is 36 pupils. The probability of choosing a pupil whose age is less than or equal to 13 is  $\frac{1}{6}$ . What is the number of pupils whose ages are more than 13 years old?  
(a) 23 (b) 24 (c) 30 (d) 32

Unit 2

- 6 In a mixed school, if the ratio between the number of boys to the number of girls is 7 : 9 A student is selected randomly from the students of this school.

The probability that the selected student is a boy equals

- (a) zero (b)  $\frac{7}{16}$  (c)  $\frac{9}{16}$  (d) 7

- 7 A small box contains 25 tickets numbered from 1 to 25 A large box contains 50 tickets numbered from 1 to 50 Without looking at them, a ticket is picked from one of the two boxes. Which box would give the larger chance of picking a ticket with the number 17 ?

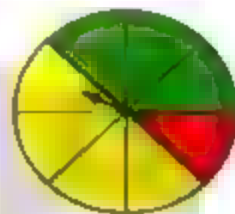
- (a) The larger box. (b) The smaller box.  
(c) Both would give the same chance. (d) The given information is not enough.

- 19 The opposite figure represents the spinning game. Find :

- 1 The probability that the pointer stops at :

- (a) Red colour.  
(b) Green colour.  
(c) Yellow colour.

- 2 The probability that the pointer does not stop at the red colour.



- 20 The opposite spinning game is divided into 8 sectors of the same area.  $\frac{1}{8}$  of the sectors is coloured in red, and  $\frac{1}{4}$  of the sectors is coloured in green,  $\frac{3}{8}$  of the sectors is coloured in blue and the rest in yellow.

If the pointer of the spinner is spinned, what is the probability that the pointer stops at the yellow or the red colour ?



- 21 A class contains 40 students, 30 of them succeeded in maths, 24 succeeded in science and 20 succeeded in both.

A student is chosen randomly.

Find the probability that this student :

- 1 Succeeded in maths.  
2 Succeeded in science.  
3 Failed in science.  
4 Failed in both maths and science.





## Exercise 10

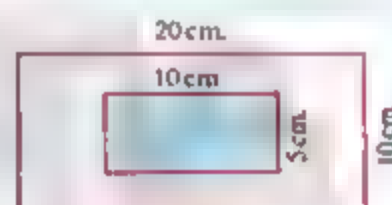
- 22 Two players play in a football team.  
During training, one of them shot 21 penalty kicks and he scored 18 goals and the other shot 32 penalty kicks and he scored 25 goals. Which of them should you choose to shoot a penalty kick? Why?



- 23 Maryam and Souad played together with two dice.  
If the product of the two apparent numbers on the upper face is even, then Souad wins the game.  
If the product of those numbers is odd, then Maryam wins:
- 1 On your opinion, is this system of the game fair? Why?
  - 2 If it is not fair, determine which one of the two girls has the greater chance to win? Why?



- 24 In the opposite figure:  
If a person shot towards the drawn board, find the probability of shooting the shaded part



## For excellent pupils

- 25 In the game of the target and the arrow, the target was in the shape of a rectangle divided into parts as shown in the opposite figure:
- 1 Find the probability of shooting the part E
  - 2 Find the probability of shooting the part formed from A, B and C together.
- 26 A bag contains a number of similar balls, 5 of them are white and the rest are red.  
If the probability of drawing a red ball is  $\frac{2}{3}$   
Find the total number of balls.



- 27 A card is drawn from a group of cards numbered from 1 to n.  
If the probability that the drawn card carries a number greater than 8 is  $\frac{1}{3}$ , then find the value of n
- 28 When you throw a regular die two successive times and notice the upper face.  
Find the probability of appearance of the number 3 in the two times.

## Summary of unit 2



- The sample is a small part from a large society that looks like this society and represents it well and is selected randomly.

### 1 Systematic sample :

It is a sample whose elements are selected from the elements of a society distributed randomly by following a certain system or method in selection.

### • Random sample :

It is a sample whose elements are selected from the elements of a society distributed randomly by following a random and irregular method or system of selecting.

- The experimental probability . depends on performing an experiment , then we record the results and use them to calculate the value of probability of an event occurrence using the rule :

$$\text{Experimental probability} = \frac{\text{Number of trials in which the outcome occurs}}{\text{Total number of trials}}$$

### • Random experiment :

It is an experiment in which we can specify all its possible outcomes before carrying it out but we cannot determine certainly which of them will occur

### • Sample space :

It is the set of all possible outcomes of a random experiment and it is denoted by S

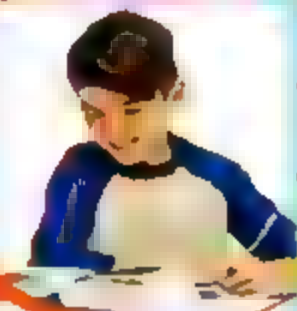
### • Event : The event is a subset of the sample space.

- The probability of any event occurrence  $A \subset S$  is denoted by  $P(A)$  and it is given by using the relation :

$$P(A) = \frac{\text{The number of elements of the event «A»}}{\text{The number of elements of sample space «S»}} = \frac{n(A)}{n(S)}$$

- The probability of the impossible event = zero
- The probability of the certain event = 1
- $0 \leq$  The probability of an event occurrence  $\leq 1$

## Exam on unit two



Answer the following questions :

1 Choose the correct answer from those given :

- 1 In the experiment of throwing a fair die once , the probability of appearing a number greater than 4 is  
 (a)  $\frac{1}{3}$  (b)  $\frac{2}{3}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{4}$
- 2 Which of the following is the probability of occurrence of an event ?  
 (a) 1.25 (b) - 0.3 (c) 215 % (d) 23 %
- 3 A class has 15 boys , 20 girls. A pupil is selected randomly , then the probability that the pupil is a girl =  
 (a)  $\frac{3}{4}$  (b)  $\frac{4}{3}$  (c)  $\frac{3}{7}$  (d)  $\frac{4}{7}$
- 4 If the probability of success of a student is 75% , then the probability of his failure =  
 (a) 0.75 % (b) 0.25 (c) 75 (d) 25
- 5 As throwing a fair die once , then the probability of having a number satisfies the inequality :  $3 < X < 6$  is  
 (a)  $\frac{2}{3}$  (b)  $\frac{1}{2}$  (c)  $\frac{1}{3}$  (d)  $\frac{1}{6}$
- 6 If a letter is selected randomly from the word "NOVEMBER" , then the probability of selecting the letter E is  
 (a)  $\frac{1}{8}$  (b)  $\frac{3}{8}$  (c)  $\frac{1}{4}$  (d) 1

2 Complete the following :

- 1 The probability of the impossible event =
- 2 If a coin is flipped once , then the probability of appearance of a tail =
- 3 If the probability of occurrence of an event is  $\frac{3}{7}$  , then the probability of non occurrence of this event =



2

- 4 A garden has 5 doors numbered from 1 to 5. A man got out from one of these doors , then the probability that he got out of door number 5 is \_\_\_\_\_
- 5 As throwing a fair die once , then the probability of appearing a number divisible by 5 = \_\_\_\_\_

3 A bag contains 5 red balls , 7 yellow balls and 3 black balls. If all balls are alike and a ball is drawn randomly from the bag , find :

- 1 The probability that the drawn ball is red.
- 2 The probability that the drawn ball is not yellow.
- 3 The probability that the drawn ball is not green.
- 4 The probability that the drawn ball is black or yellow.

4 If a fair die is rolled once , then find the probability of appearing :

- 1 A prime number.
- 2 An even prime number.
- 3 A number less than 6
- 4 A number greater than 6
- 5 A number less than 10

5 From the set of numbers  $\{1, 3, 4\}$  a 2-digit number is formed

What is the probability of each of the following events :

- 1 The event of the tens digit is odd.
- 2 The event of the units digit is even.
- 3 The event of the sum of the two numbers is 5

# SKILLS

TIMSS Problems

## Accumulative basic skills

1 Choose the correct answer from the given ones :

1  $3x^2 + 2x + 2x^2 - x =$

- (a)  $6x$  (b)  $6x^2$  (c)  $5x^2 + x$  (d)  $7x^2 - x$

2 If  $y = \frac{a+b}{c}$ ,  $a = 8$ ,  $b = -6$  and  $c = -2$ , then  $y =$

- (a)  $-1$  (b)  $1$  (c)  $-7$  (d)  $7$

3 At dividing  $113 + 113 + 113 + 113$  by  $4$ , then the remainder =

- (a) zero (b)  $1$  (c)  $4$  (d)  $13$

4  $4(3 + x) =$

- (a)  $12 + x$  (b)  $7 + x$  (c)  $12 + 4x$  (d)  $12x$

5  $\frac{4}{10} + \frac{3}{100} =$

- (a)  $0.34$  (b)  $0.43$  (c)  $4.3$  (d)  $3.4$

6 If three times of a number equals  $27$ , then  $\frac{1}{9}$  of this number is

- (a)  $1$  (b)  $3$  (c)  $9$  (d)  $27$

7 Which of the following is equal to  $\frac{3}{5}$ ?

- (a)  $6\%$  (b)  $60\%$  (c)  $\frac{9}{10}$  (d)  $0.53$

8 If the fractions  $\frac{4}{14}$  and  $\frac{x}{21}$  are equal, then  $x =$

- (a)  $6$  (b)  $7$  (c)  $11$  (d)  $14$

9  $2 \times 4 \times 6 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{6} = \dots\dots\dots$

- (a)  $48$  (b)  $2304$  (c)  $1$  (d) zero

10 A worker cut a part of a pipe equals  $\frac{1}{3}$  of this pipe, if the length of the cut part equals  $3\text{ m.}$ , then the length of the pipe completely =

- (a)  $8\text{ m.}$  (b)  $12\text{ m.}$  (c)  $15\text{ m.}$  (d)  $18\text{ m.}$

## Basic skills

- 11 Which of the following does express the number 36 as the product of its prime factors ?  
 (a)  $6 \times 6$  (b)  $4 \times 9$  (c)  $4 \times 3 \times 3$  (d)  $2 \times 2 \times 3 \times 3$
- 12  $5 \times 4 \times 3 \times 2 \times 1 \times 0 =$   
 (a) 120 (b) 60 (c) 20 (d) zero
- 13 Double of the square of the number (half) is  
 (a)  $\frac{1}{4}$  (b)  $\frac{1}{8}$  (c)  $\frac{1}{2}$  (d) 2
- 14 If the number of boys in one of the parties is  $m$  and the number of girls is  $n$ , if each person has 2 balloons, then which of the following expression does express the number of balloons in this party ?  
 (a)  $2(m + n)$  (b)  $2 + (m + n)$  (c)  $2m + n$  (d)  $m + 2n$
- 15 The smallest number of the following numbers is  
 (a) 0.52 (b) 0.5 (c) 0.056 (d) 0.562

## 2 Complete the following :

- 1  $24.65 + 5.748 =$
- 2  $-2\frac{3}{4} + 1\frac{2}{8} = \dots$
- 3 Third of the third =
- 4  $\frac{(19)^2 - 9 \times 19 + 19}{19} =$
- 5  $\frac{3x}{8} + \frac{x}{4} + \frac{x}{2} =$  (in the simplest form)
- 6 If  $y = 100 - \frac{100}{1+m}$ , at  $m = 9$ , then  $y = \dots$
- 7 If  $a + b = 25$ , then  $2a + 2b = \dots$
- 8  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$  (in the same pattern)
- 9 If  $3y = 6$ , then  $5y = \dots$
- 10 If  $\frac{1}{2}x = 5$   $y = 10$ , then  $xy =$
- 11 If  $x + y = yx = 5$ , then  $x^2y + y^2x =$
- 12 If  $x - y = 3$ ,  $x + y = 5$ , then  $x^2 - y^2 =$
- 13 If  $x^2 = 16$ ,  $y^2 = 9$ ,  $xy = 12$ , then  $(x - y)^2 =$
- 14 If the degree of the algebraic term  $5x^n y^2$  is 5, then  $n =$
- 15 A piece of wood of length 40 cm, it is cut into three parts of lengths  $2x - 5$ ,  $x + 7$  and  $x + 6$  in centimetres, then the length of the longest part = cm.



Second

## Geometry and Measurement



UNIT

3

Geometry and Measurement.

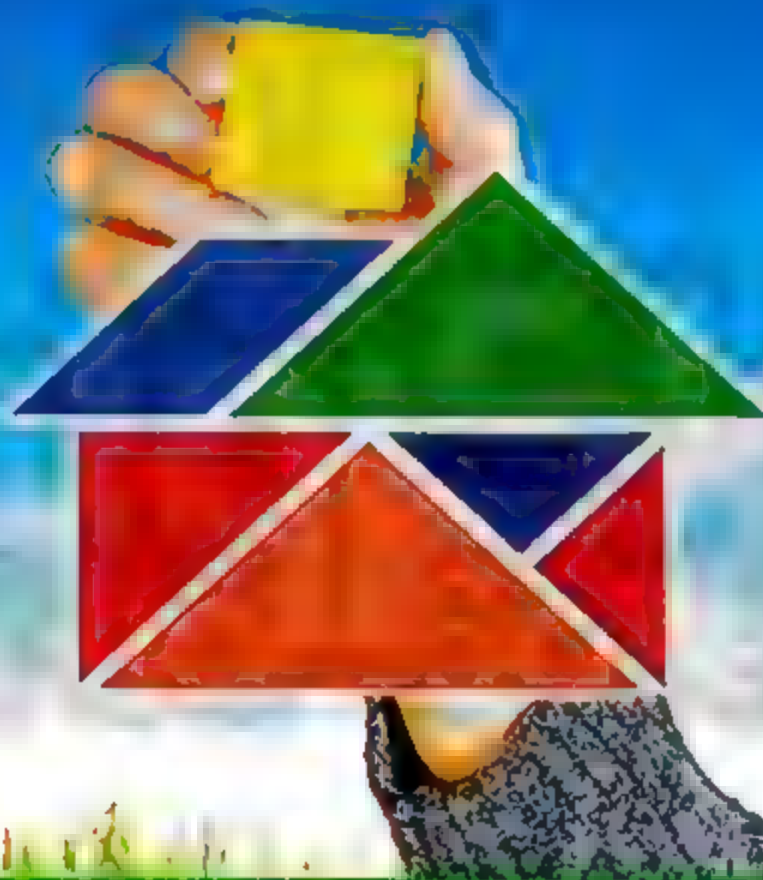
62

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UNIT

3

## Geometry and Measurement



## Exercises of the unit

1. Deductive proof
2. The polygon
3. The parallelogram and its properties
4. The special cases of the parallelogram
5. The triangle Theorem (1) exterior angle of the triangle
6. Theorem (2) , theorem (3)
7. Pythagorean theorem
8. Geometric transformation
9. Reflection in a straight line
10. Reflection in a point
11. Translation
12. Rotation



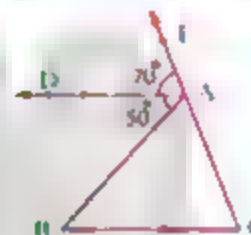
## Deductive proof

From the school book

1 In the opposite figure :

 $\overline{AD} \parallel \overline{BC}$  ,  $m(\angle DAB) = 50^\circ$  and  $m(\angle DAE) = 70^\circ$ Find the measures of the angles of  $\triangle ABC$ 

Complete the following table by writing the reason of each step of the solution steps :



## Mathematical Statements

## The reason

$m(\angle DAB) = 50^\circ$  ,  $m(\angle DAE) = 70^\circ$

[1]

$m(\angle CAB) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$

[2]

$\overline{AD} \parallel \overline{BC}$

[3]

$m(\angle C) = m(\angle DAE) = 70^\circ$

[4]

$m(\angle B) = m(\angle DAB) = 50^\circ$

[5]

2 In the opposite figure :

 $m(\angle AMB) = 50^\circ$  ,  $m(\angle EMD) = 80^\circ$  ,  $\overline{MC}$  bisects  $\angle BMD$  and  $m(\angle CMD) = 65^\circ$ Complete the following proof to find  $m(\angle AME)$ 

Given

R.T.F.

Proof

 $\therefore \overline{MC}$  bisects  $\angle$  ..... (given)

$\therefore m(\angle BMC) = m(\angle$  ..... ) = .....

$\therefore m(\angle AMB) + m(\angle BMC) + m(\angle CMD) + m(\angle DME) + m(\angle AME) =$  .....

$\therefore m(\angle AME) =$  ..... - ..... = ..... (The req.)

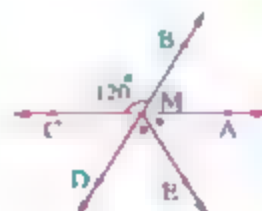




Unit 3

3 In the opposite figure :

$$\overline{AC} \cap \overline{BD} = \{M\}, m(\angle BMC) = 120^\circ$$

and  $\overline{ME}$  bisects  $\angle AMD$ Complete the following solution steps to find  $m(\angle EMC)$ 

Given

R.T.F.

Proof

$$\because \overline{AC} \cap \overline{BD} = \{M\}$$

$$\therefore m(\angle BMC) = m(\angle \dots) \text{ (V.O.A.)}$$

$$\therefore m(\angle \dots) = 120^\circ$$

$$\because \overline{ME} \text{ bisects } \angle AMD$$

$$\therefore m(\angle \dots) = m(\angle \dots)$$

$$\therefore m(\angle EMC) = \dots = \dots$$

$$\because M \in \overline{BD}$$

$$\therefore m(\angle BMC) + m(\angle \dots) = 180^\circ$$

$$\therefore m(\angle DMC) = \dots^\circ - \dots^\circ = \dots^\circ$$

$$\therefore m(\angle EMC) = m(\angle \dots) + m(\angle \dots)$$

$$\therefore m(\angle EMC) = \dots^\circ + \dots^\circ = \dots^\circ$$

(The req.)

4 In the opposite figure :

$$AB = AC, BD = CD$$

Complete the following proof to prove that  $\overline{AD}$  bisects  $\angle BAC$ 

Given

R.T.P.

Proof

.....

.....

$$\therefore \text{In } \triangle ADB, \dots :$$

$$\begin{cases} AB = \dots & \text{(given)} \\ \dots = CD & \text{(given)} \\ \overline{AD} \end{cases}$$

$$\therefore \triangle ADB = \triangle \dots$$

$$\therefore \triangle ADB = \triangle \dots, \text{ then we deduce that :}$$

$$m(\angle \dots) = m(\angle \dots)$$

$$\therefore \overline{AD} \text{ bisects } \angle \dots$$

$$\therefore \overline{AD} \text{ bisects } \angle \dots$$

(Q.E.D.)



Exercice 50

1

5 In the opposite figure :

$$m(\angle ABC) = 110^\circ, m(\angle CBD) = 35^\circ$$

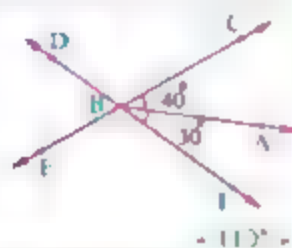
$$\text{and } m(\angle ABE) = 140^\circ$$

Find :  $m(\angle EBD)$ 

6 In the opposite figure :

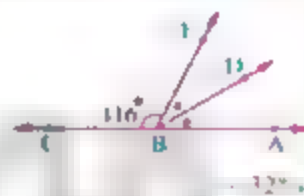
$$\overline{CE} \cap \overline{FD} = \{B\}$$

$$m(\angle ABC) = 40^\circ \text{ and } m(\angle ABF) = 30^\circ$$

Find :  $m(\angle DBC)$ 

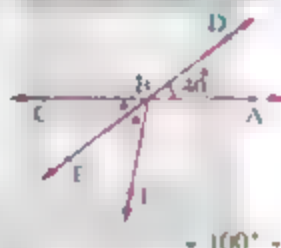
7 In the opposite figure :

$$B \in \overline{AC}, m(\angle CBE) = 116^\circ$$

and  $\overline{BD}$  bisects  $\angle ABE$ Find :  $m(\angle ABD)$ 

8 In the opposite figure :

$$\overline{AC} \cap \overline{DE} = \{B\}, m(\angle ABD) = 40^\circ$$

and  $\overline{BE}$  bisects  $\angle CBF$ Find :  $m(\angle ABF)$ 

9 In the opposite figure :

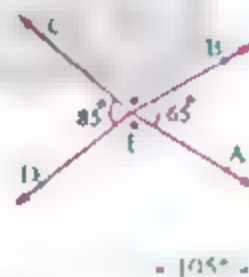
$$\overline{EA} \cap \overline{EB} \cap \overline{EC} \cap \overline{ED} = \{E\}$$

$$\text{If } m(\angle BEC) = m(\angle AED)$$

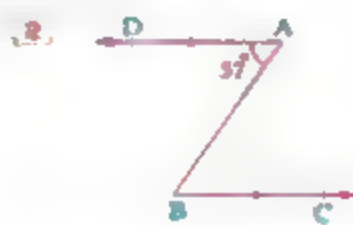
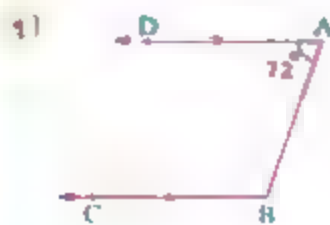
$$, m(\angle AEB) = 65^\circ, m(\angle CED) = 85^\circ$$

Find :  $m(\angle BEC)$ 

Are A, E, and C on the same straight line ? Why ?



10 In each of the following figures ,

If  $\overline{AD} \parallel \overline{BC}$  , Find :  $m(\angle ABC)$  , giving reason.

3

- 11 In the opposite figure :

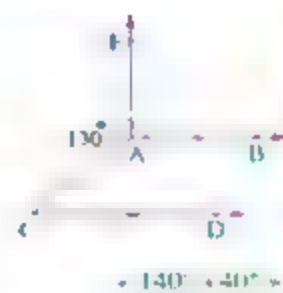
$$\overline{AB} \parallel \overline{CD}$$

$$m(\angle EAC) = 130^\circ$$

$$\text{and } m(\angle EAB) = 90^\circ$$

Find : 1.  $m(\angle BAC)$

$$2. m(\angle C)$$

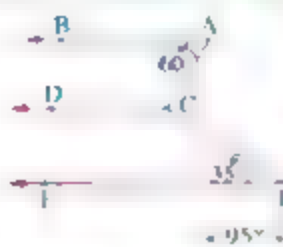


- 12 In the opposite figure :

$$\overline{AB} \parallel \overline{CD}, \overline{AB} \parallel \overline{EF}$$

$$m(\angle A) = 60^\circ \text{ and } m(\angle E) = 35^\circ$$

Find :  $m(\angle ACE)$



- 13 In the opposite figure :

$$\overline{DE} \parallel \overline{BC}, A \in \overline{DE}, m(\angle DAB) = 80^\circ$$

$$\text{and } m(\angle EAC) = 50^\circ$$

Find the measures of the angles of  $\triangle ABC$



$$m(\angle BAC) = 50^\circ, m(\angle B) = 80^\circ, m(\angle C) = 50^\circ$$

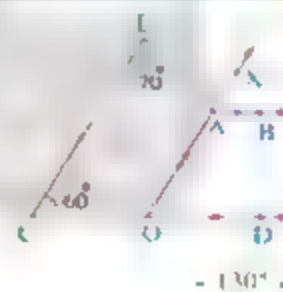
- 14 In the opposite figure :

$$\overline{AB} \parallel \overline{OD}, \overline{OA} \parallel \overline{CE}, X \in \overline{OA}$$

$$O \in \overline{CD}, m(\angle B) = 70^\circ$$

$$\text{and } m(\angle C) = 60^\circ$$

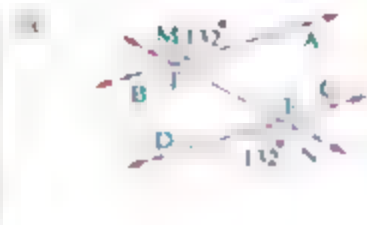
Find :  $m(\angle BAE)$



- 15 In each of the following figures,

If  $\overline{MN}$  intersects  $\overline{AB}$ ,  $\overline{CD}$  at E and F respectively,

Prove that :  $\overline{AB} \parallel \overline{CD}$





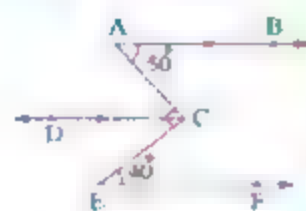
## Exercise 1

16 In the opposite figure :

$$\overline{AB} \parallel \overline{CD}, m(\angle A) = 50^\circ,$$

$$\angle ACE \text{ is right and } m(\angle E) = 40^\circ$$

Prove that :  $\overline{AB} \parallel \overline{EF}$



17 In the opposite figure :

$$\overline{EF} \parallel \overline{CD}, m(\angle CEF) = 95^\circ,$$

$$m(\angle ACE) = 30^\circ, m(\angle BAC) = 115^\circ$$

Prove that :  $\overline{AB} \parallel \overline{EF}$



18 In the opposite figure :

The two triangles are congruent

Prove that :  $\overline{BC} \parallel \overline{EF}$



19 In the opposite figure :

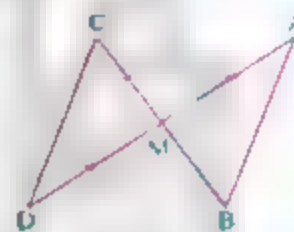
$$\overline{AD} \cap \overline{BC} = \{M\},$$

$$MA = MD \text{ and } MB = MC$$

Prove that :

1  $AB = CD$

2  $\overline{AB} \parallel \overline{CD}$



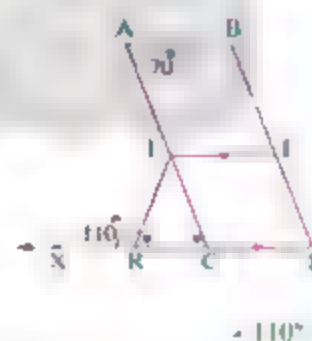
20 In the opposite figure :

$$\overline{FE} \parallel \overline{CD}, m(\angle FRC) = m(\angle FCR),$$

$$m(\angle A) = 70^\circ \text{ and } m(\angle FRX) = 110^\circ$$

1 Prove that :  $\overline{CD} \parallel \overline{AB}$

2 Find in three different ways :  $m(\angle AFE)$



21 Prove that :

1 A straight line which is perpendicular to one of two parallel lines is also perpendicular to the other.

2 A straight line that is parallel to one of two parallel lines is also parallel to the other.

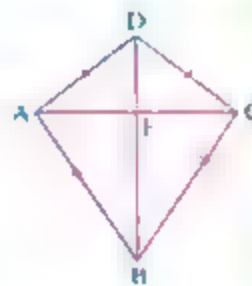
## Unit 3

22 In the opposite figure :

$AD = CD$  and  $AB = BC$

Use the properties of congruent triangles to show that :

- 1  $\overline{DB}$  bisects  $\angle ADC$
- 2  $\overline{AC}$  and  $\overline{DB}$  are perpendicular to each other.



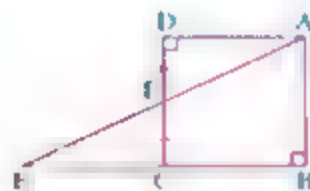
23 In the opposite figure :

ABCD is a square in which F

is the midpoint of  $\overline{CD}$

and  $\overline{AF} \cap \overline{BC} = \{E\}$

Prove that :  $CE = CB$

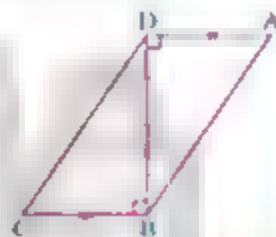


24 In the opposite figure :

$AD = BC$  and  $m(\angle ADB) = m(\angle DBC) = 90^\circ$

Prove that :

- 1  $AB = CD$
- 2  $\overline{AB} \parallel \overline{CD}$

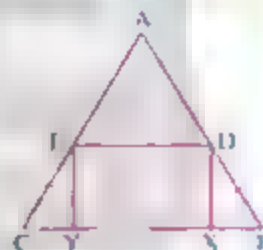


25 In the opposite figure :

$EC = DB$  and

DXYE is a rectangle.

Prove that :  $m(\angle ADE) = m(\angle AED)$

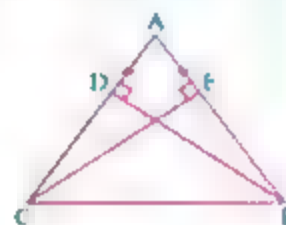


26 In the opposite figure :

$\overline{BD} \perp \overline{AC}$ ,  $\overline{CE} \perp \overline{AB}$  and

$AD = AE$

Prove that :  $BD = CE$



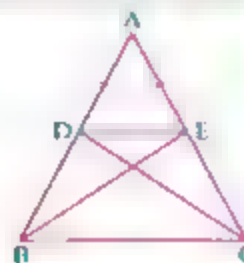
27 In the opposite figure :

$AD = AE$  and

$m(\angle ADC) = m(\angle AEB)$

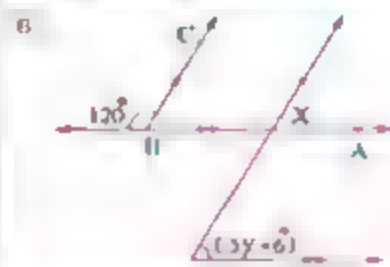
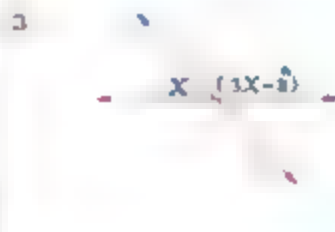
Show that :

- 1  $BE = CD$
- 2  $BD = CE$



Exercise 1

28 Find the values of  $X$  and  $y$  in each of the following :



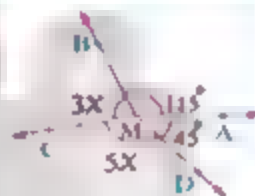
29 In the opposite figure :

$m(\angle AMB) = 115^\circ$  ,  $m(\angle AMD) = 45^\circ$  ,

$m(\angle BMC) = 3x$  and  $m(\angle CMD) = 5x$

What is the value of  $x$ ?

Are A , M and C collinear ? Why ?



For excellent pupils

30 In the opposite figure :

1 Is  $\triangle ADE$  congruent to  $\triangle CBF$  ? Give your reason(s).

2 Prove that :

First :  $\triangle DEF = \triangle BFE$  Second :  $\triangle ABE = \triangle CDF$

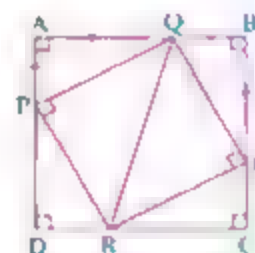


31 In the opposite figure :

1 Is  $\triangle PAQ$  congruent to  $\triangle QBO$  ? Give your reason (s)

2 Show that :

First :  $\triangle PQR = \triangle OQR$  Second :  $\triangle PDR = \triangle RCO$





12

## The polygon

From the school book

## 1 Complete the following :

- 1 The regular polygon is the one in which :  
(a) (b)
- 2 The sum of measures of the interior angles of the quadrilateral =
- 3 The sum of measures of the interior angles of the pentagon =
- 4 The sum of measures of the interior angles of the hexagon =
- 5 The sum of measures of the interior angles of the heptagon =
- 6 The measure of the interior angle of the regular pentagon =  
and the measure of the interior angle of the regular heptagon =
- 7 The sum of measures of the exterior angles of the hexagon equals
- 8 If the perimeter of a regular hexagon is 30 cm. , then its side length = \_\_\_\_\_ cm.  
and the measure of each interior angle in it = \_\_\_\_\_
- 9 If the perimeter of a regular polygon = 80 cm. and its side length = 10 cm. ,  
then the measure of each interior angle in it = \_\_\_\_\_

## 2 Choose the correct answer from those given :

- 1 The sum of measures of the interior angles of a polygon of  $n$  sides =  
(a)  $n \times 180^\circ$  (b)  $(n - 2) \times 180^\circ$  (c)  $\frac{(n - 2) \times 180^\circ}{2}$  (d)  $\frac{(n - 2) \times 180^\circ}{2n}$
- 2 The measure of the interior angle of a regular polygon of  $n$  sides equals  
(a)  $\frac{(n - 2) \times 90^\circ}{n}$  (b)  $\frac{(n - 2) \times 180^\circ}{2}$  (c)  $\frac{(n - 2) \times 180^\circ}{n}$  (d)  $180^\circ \times (n - 1)$
- 3 The measure of the interior angle of the regular polygon of 10 sides equals  
(a)  $72^\circ$  (b)  $108^\circ$  (c)  $144^\circ$  (d)  $150^\circ$

## Exercise 2

- 4 The measure of the interior angle of a regular polygon of 18 sides equals  
 (a)  $130^\circ$  (b)  $140^\circ$  (c)  $150^\circ$  (d)  $160^\circ$
- 5 If the measure of an interior angle of a regular polygon is  $135^\circ$ , then the number of its sides is  
 (a) 6 (b) 4 (c) 7 (d) 8
- 6 The sum of measures of the exterior angles of the triangle equals  
 (a)  $90^\circ$  (b)  $180^\circ$  (c)  $360^\circ$  (d)  $720^\circ$
- 7 In the quadrilateral ABCD, if  $m(\angle A) = 2m(\angle B) = m(\angle C) = 96^\circ$ , then  $m(\angle D) = \dots$   
 (a)  $96^\circ$  (b)  $48^\circ$  (c)  $120^\circ$  (d)  $144^\circ$

3 Find the number of the diagonals of each of the following figures :

1 Triangle.

2 Quadrilateral.

3 Pentagon.

Hint : The number of diagonals of the polygon of  $n$  sides =  $\frac{n(n-3)}{2}$

4 In each of the following, find the measure of the angle marked by ( ? ) :

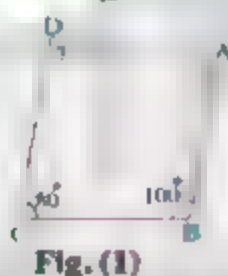


Fig. (1)

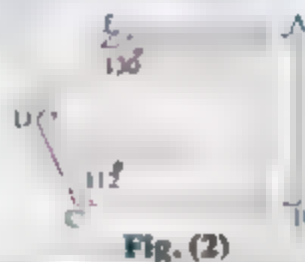


Fig. (2)

5 In the opposite figure :

$\overline{AE} \parallel \overline{BC}$ ,  $m(\angle B) = 70^\circ$ ,  $m(\angle C) = 150^\circ$  and  $m(\angle D) = 80^\circ$

Complete the following proof to find  $m(\angle E)$

Given

R.T.F.

Proof

$\therefore \overline{AE} \parallel \overline{BC}$  and  $\overline{AC}$  is a transversal to them.

$\therefore m(\angle A) + m(\angle C) = 180^\circ$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle C) = 150^\circ$

$\therefore m(\angle A) = 180^\circ - 150^\circ = 30^\circ$

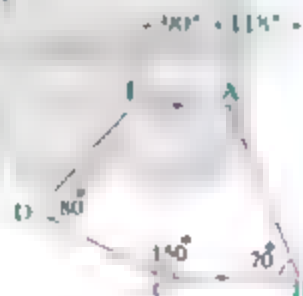
$\therefore ABCDE$  is a pentagon.

$\therefore$  The sum of measures of its interior angles

$= (5 - 2) \times 180^\circ = 540^\circ$

$\therefore m(\angle E) = 540^\circ - (70^\circ + 150^\circ + 80^\circ + 110^\circ) = 140^\circ$

(The req.)



Unit 3

6 In the opposite figure :

$$\overline{CE} \cap \overline{AD} = \{B\}, m(\angle A) = 50^\circ$$

$$m(\angle C) = 70^\circ, m(\angle D) = 130^\circ \text{ and}$$

$$m(\angle F) = 90^\circ$$

Complete the following proof to find  $m(\angle E)$ 

Given

R.T.F.

Proof

 $\therefore$  The sum of measures of the interior angles of the triangle =  $\dots\dots\dots$ 

$$\therefore \text{In } \triangle ABC : m(\angle ABC) = 180^\circ - (\dots\dots\dots + \dots\dots\dots) = \dots\dots\dots$$

$$\therefore \overline{CE} \cap \overline{AD} = \{B\}$$

$$\therefore m(\angle \dots\dots\dots) = m(\angle \dots\dots\dots) = \dots\dots\dots^\circ \text{ (V.O.A.)}$$

 $\therefore$  The sum of measures of the interior angles of the quadrilateral =  $\dots\dots\dots$ 

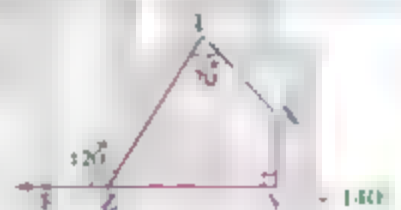
$$\therefore m(\angle E) = \dots\dots\dots - (\dots\dots\dots + \dots\dots\dots + \dots\dots\dots) = \dots\dots\dots^\circ \quad \text{(The req.)}$$



7 In the opposite figure :

$$F \in \overline{YZ}, m(\angle L) = 70^\circ,$$

$$m(\angle Y) = 90^\circ \text{ and } m(\angle LZF) = 120^\circ$$

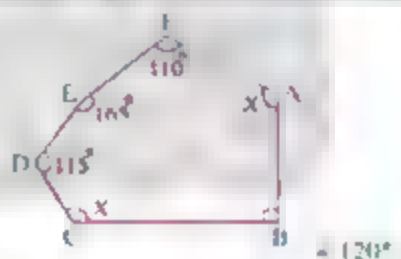
Find :  $m(\angle X)$ 

8 In the opposite figure :

ABCDEF is a hexagon.

$$m(\angle FAB) = m(\angle DCB)$$

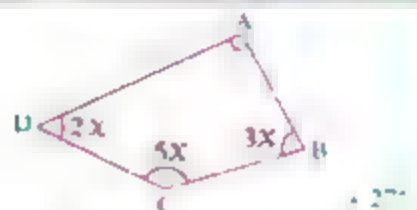
Find :

The value of  $X$ 

9 In the opposite figure :

ABCD is a quadrilateral

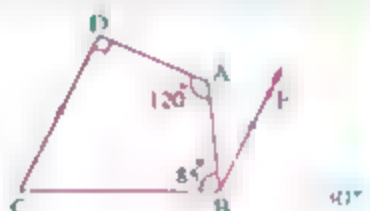
$$\text{in which : } m(\angle A) = 90^\circ$$

Find : The value of  $X$ 

10 In the opposite figure :

$$m(\angle A) = 120^\circ, m(\angle D) = 90^\circ,$$

$$m(\angle ABC) = 85^\circ \text{ and } \overline{BE} \parallel \overline{CD}$$

Find :  $m(\angle ABE)$ 



## Exercise 2

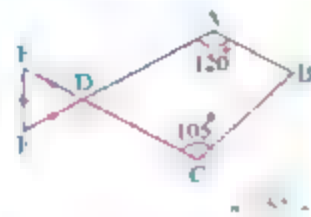
- 11 In the opposite figure :

$$\overline{AE} \cap \overline{CF} = \{D\},$$

$\triangle DEF$  is an equilateral triangle ,

$$m(\angle A) = 120^\circ \text{ and } m(\angle C) = 105^\circ$$

Find :  $m(\angle B)$



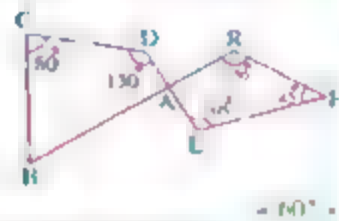
- 12 In the opposite figure :

$$\overline{ED} \cap \overline{RB} = \{A\}, m(\angle F) = 45^\circ,$$

$$m(\angle R) = 120^\circ, m(\angle E) = 105^\circ,$$

$$m(\angle D) = 130^\circ \text{ and } m(\angle C) = 80^\circ$$

Find :  $m(\angle B)$



- 13 In the opposite figure :

$$\overline{BD} \cap \overline{AF} = \{C\}, \overline{AB} \parallel \overline{ED},$$

$$m(\angle A) = 30^\circ \text{ and } m(\angle ACB) = 58^\circ,$$

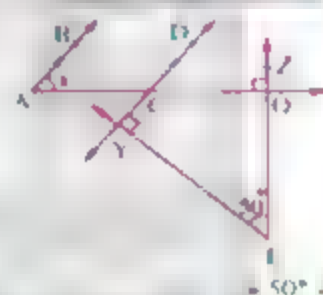
$$m(\angle CFE) = 125^\circ$$

Find :  $m(\angle E)$



- 14 In the opposite figure :

Find : The value of  $x$

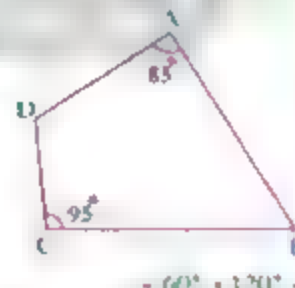


- 15 In the opposite figure :

$$m(\angle A) = 85^\circ, m(\angle C) = 95^\circ$$

$$\text{and } m(\angle B) = \frac{1}{2} m(\angle D)$$

Find the measure of each of them.



- 16 In each of the following , if the polygon is regular , find the measures of the unknown angles :

1



2



## Unit 3



17 In each of the following, find the values of the unknown symbols :

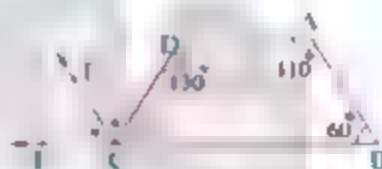


18 In the opposite figure :

$$m(\angle A) = 110^\circ, m(\angle B) = 60^\circ,$$

$$m(\angle D) = 130^\circ, \overline{CF} \text{ bisects } \angle DCE \text{ and } C \in \overline{BE}$$

Prove that :  $\overline{CF} \parallel \overline{AB}$



19 In the opposite figure :

ABCDEF is a regular hexagon,

$$Y \in \overline{CB}, \overline{XY} \perp \overline{YB} \text{ and } m(\angle X) = 60^\circ$$

Prove that :  $\overline{BX}$  bisects  $\angle ABY$



20 In the opposite figure :

ABC is a triangle in which :  $m(\angle A) = 30^\circ,$

$$\overline{DE} \cap \overline{CB} = \{F\}, m(\angle F) = 60^\circ \text{ and}$$

$$m(\angle BEF) = 45^\circ$$

Find the measure of each angle of the figure DEBC



$$= m(\angle EBC) = 105^\circ, m(\angle C) = 45^\circ, m(\angle BED) = 135^\circ, m(\angle EDC) = 75^\circ$$

21 If the ratio among the measures of the angles of a pentagon is  $3 : 3 : 2 : 3 : 4$

find the greatest measure of the angles of this pentagon

$$= 144^\circ$$

22 If the measure of the exterior angle of a regular polygon is  $30^\circ$ , how many sides does it have ? What is the sum of the measures of its interior angles ?

$$= 12, 1800^\circ$$

## Exercise 2

23 Is it possible that a regular polygon has an interior angle of measure  $100^\circ$  ? Why ?

24 A polygon of 9 sides. The sum of measures of eight angles of it is  $1140^\circ$

1 Find the measure of the remained angle.

-  $120^\circ$  -

2 Is it possible that this polygon is regular ? Explain your answer.

25 A polygon has 15 sides :

1 Calculate the sum of the measures of its interior angles.

-  $2340^\circ$  -

2 If the sum of the measures of five of its exterior angles is  $200^\circ$ , calculate the sum of the measures of the ten interior angles which are not adjacent to the five exterior angles.

-  $1640^\circ$  -

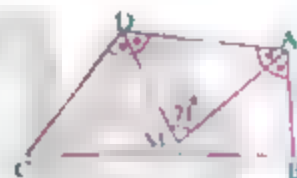


For excellent pupils:

26 In the opposite figure :

$\overline{AM}$  bisects  $\angle BAD$ ,  $\overline{DM}$  bisects  $\angle ADC$  and  
 $m(\angle AMD) = 71^\circ$

Prove that :  $m(\angle B) + m(\angle C) = 142^\circ$

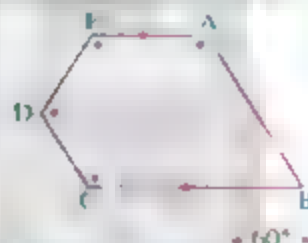


27 In the opposite figure :

$\overline{AE} \parallel \overline{BC}$ ,

$m(\angle A) = m(\angle E) = m(\angle D) = m(\angle C)$

Find :  $m(\angle B)$



EL-MONASSER

Notebook

Free part

- Quizzes.
- Final examinations.







13

## The parallelogram and its properties

from the school book

### 1 Complete the following :

- 1 In a parallelogram , every two opposite sides are .....
- 2 In a parallelogram , every two opposite angles are .....
- 3 In a parallelogram , every two consecutive angles are .....
- 4 In a parallelogram , the two diagonals .....
- 5 The quadrilateral in which two sides only are parallel is called .....
- 6 A quadrilateral represents a parallelogram if ..... (Write only one answer)
- 7 ... ABCD is a parallelogram in which  $m(\angle A) = 50^\circ$  , then  $m(\angle B) = \dots^\circ$
- 8 ... In the parallelogram XYZL , if  $m(\angle X) = \frac{1}{2} m(\angle Y)$  , then  $m(\angle Y) = \dots^\circ$

### 2 In the opposite figure :

ABCD is a parallelogram in which  $AB = 2 \text{ cm}$  ,  
 $AD = 6 \text{ cm}$  and  $m(\angle B) = 105^\circ$

Complete the following :

- 1  $BC = \dots \text{ cm}$  ,  $DC = \dots \text{ cm}$ .
- 2  $m(\angle D) = \dots^\circ$  ,  $m(\angle A) = \dots^\circ$  and  $m(\angle C) = \dots^\circ$
- 3 The perimeter of the parallelogram ABCD = ..... cm.



### 3 In the opposite figure :

XYZL is a parallelogram in which :  
 $XY = 3 \text{ cm}$  ,  $YZ = 4 \text{ cm}$  ,  $m(\angle LXZ) = 31^\circ$   
 and  $m(\angle LZX) = 43^\circ$



## Exercise 3

Complete the following proof to find :

1  $m(\angle Y)$

2 The perimeter of the parallelogram XYZL

Given

R.T.F.

Proof

$\therefore \Delta XLZ$  in which :  $m(\angle LXZ) = \dots^\circ$  ,  $m(\angle LZX) = \dots^\circ$

$\therefore m(\angle L) = 180^\circ - (\dots^\circ + \dots^\circ) = \dots^\circ$

$\therefore$  The figure XYZL is a parallelogram.

$\therefore m(\angle Y) = m(\angle \dots) = \dots^\circ$

(First req.)

$\therefore$  the perimeter of the parallelogram XYZL

$= (XY + \dots) \times 2 = (3 + \dots) \times \dots = \dots \times \dots = \dots \text{ cm.}$

(Second req.)

4 In the opposite figure :

ABCD is a quadrilateral whose diagonals intersect

at M ,  $MA = MC$  ,  $MB = MD$  ,  $m(\angle AMB) = 110^\circ$  and  $m(\angle MBA) = 25^\circ$

Complete the following proof :

1 To prove that the figure ABCD is a parallelogram.

2 To find  $m(\angle ACD)$

Given

R.T.P.

R.T.F.

Proof

In the figure ABCD :

$\therefore MA = \dots$  (given) ,  $MB = \dots$  (given)

$\therefore$  Its diagonals  $\dots$  each other.

$\therefore$  The figure ABCD is  $\dots$

(First req.)

In  $\Delta MBA$  :

$\therefore m(\angle AMB) = \dots^\circ$  ,  $m(\angle MBA) = \dots^\circ$

$\therefore m(\angle MAB) = 180^\circ - (\dots^\circ + \dots^\circ) = \dots^\circ$

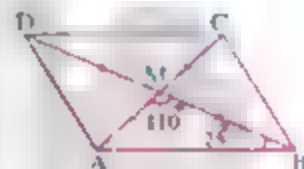
$\therefore$  The figure ABCD is  $\dots$

$\therefore \overline{AB} \parallel \dots$

$\therefore \dots$  is a transversal to them.

$\therefore m(\angle ACD) = m(\angle \dots) = \dots^\circ$  ( angles)

(Second req.)



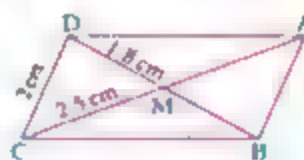
5 In the opposite figure :

ABCD is a parallelogram such that :

$\overline{AC} \cap \overline{BD} = \{M\}$  If  $CD = 2 \text{ cm.}$  ,

$MC = 2.5 \text{ cm.}$  and  $MD = 1.8 \text{ cm.}$

Calculate the perimeter of  $\Delta AMB$



$= 6.3 \text{ cm.}$

[ 77 ]

## Unit 3

## 6 In the opposite figure :

XYZL is a parallelogram in which :

$$m(\angle Y) = 118^\circ, m(\angle XZY) = 27^\circ$$

Find :

1  $m(\angle YXZ)$

2  $m(\angle LXZ)$

3  $m(\angle LXZ)$

4  $m(\angle L)$



$$= 35^\circ, 35^\circ, 27^\circ, 118^\circ$$

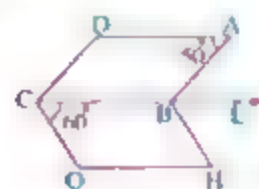
## 7 In the opposite figure :

ABCD is a parallelogram in which :

$$m(\angle BAD) = 50^\circ \text{ and } BHOC \text{ is a parallelogram}$$

$$\text{in which } m(\angle BCO) = 60^\circ, L \in \overline{CB}$$

Find with proof :  $m(\angle ABH)$



$$= 110^\circ$$

## 8 In the opposite figure :

ABCD is a quadrilateral in which :

$$\overline{AD} \parallel \overline{BC}, E \in \overline{DA}, m(\angle BAE) = 110^\circ$$

$$\text{and } m(\angle DCB) = 70^\circ$$

Prove that : ABCD is a parallelogram.



## 9 In the opposite figure :

ABCD is a parallelogram in which :

$$m(\angle B) = 120^\circ \text{ and } \overline{DH} \perp \overline{BC}$$

$$\text{where } \overline{DH} \cap \overline{BC} = \{H\}$$

Find :  $m(\angle HDC)$



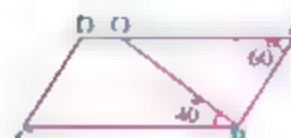
$$= 30^\circ$$

## 10 In the opposite figure :

ABCD is a parallelogram , in which :

$$m(\angle A) = 60^\circ, m(\angle OBC) = 40^\circ \text{ where } O \in \overline{AD}$$

Find :  $m(\angle ABO)$



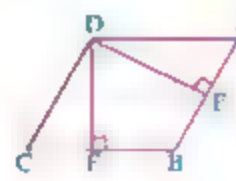
$$= 80^\circ$$

## 11 In the opposite figure :

ABCD is a parallelogram in which :

$$\overline{DE} \perp \overline{AB} \text{ and } \overline{DF} \perp \overline{BC}$$

Prove that :  $m(\angle EDF) = m(\angle A)$



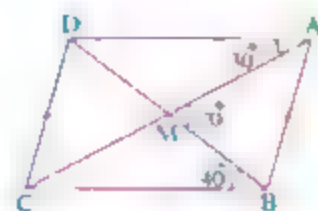


## Exercice 3

12 In the opposite figure :

$ABCD$  is a quadrilateral where  $AC \cap BD = \{M\}$ ,  
 $\overline{AB} \parallel \overline{DC}$ ,  $m(\angle AMB) = 70^\circ$ ,  $m(\angle MBC) = 40^\circ$   
 and  $m(\angle MAD) = 30^\circ$

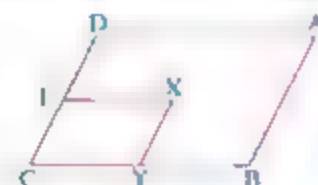
Prove that :  $ABCD$  is a parallelogram.



13 In the opposite figure :

If  $ABCD$  and  $XYCL$  are two parallelograms,

Prove that :  $m(\angle A) = m(\angle X)$



14 In the opposite figure :

Each of  $ABCD$

and  $HBCO$  is a parallelogram

Prove that :  $AD = HO$

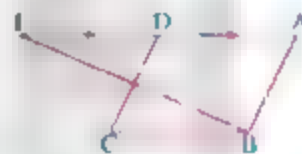


15 In the opposite figure :

$ABCD$  is a parallelogram,

$E \in \overline{AD}$  where :  $AD = DE$

Prove that :  $\overline{DC}$  and  $\overline{BE}$  bisect each other.

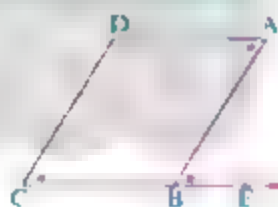


16 In the opposite figure :

$ABCD$  is a quadrilateral,

$E \in \overline{CB}$  and  $m(\angle BCD) = m(\angle EBA) = m(\angle A)$

Prove that :  $ABCD$  is a parallelogram.

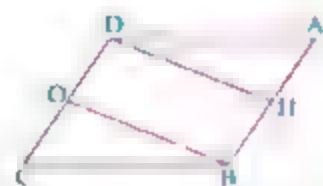


17 In the opposite figure :

$ABCD$  is a parallelogram,  $H$  is the midpoint of  $\overline{AB}$

and  $O$  is the midpoint of  $\overline{DC}$

Prove that :  $HBOD$  is a parallelogram.

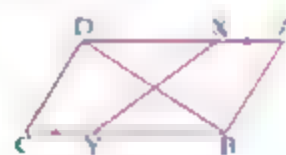


18 In the opposite figure :

$ABCD$  is a parallelogram,

$X \in \overline{AD}$ ,  $Y \in \overline{BC}$ , if  $AX = CY$

Prove that :  $\overline{XY}$  and  $\overline{BD}$  bisect each other.

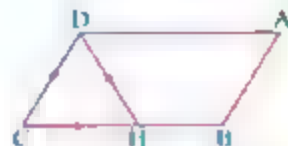


## Unit 3

19 In the opposite figure :

ABCD is a parallelogram ,  $H \in \overline{BC}$  where :  
 $\triangle DHC$  is an equilateral triangle.

- 1 Prove that :  $HC = AB$
- 2 Find :  $m(\angle B)$  and  $m(\angle HDA)$

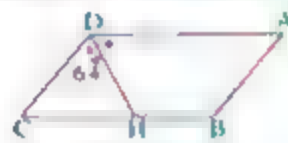


$$= 120^\circ + 60^\circ =$$

20 In the opposite figure :

ABCD is a parallelogram ,  $H \in \overline{BC}$   
 $\overline{DH}$  bisects  $\angle ADC$  and  $m(\angle HDC) = 64^\circ$

Find : 1  $m(\angle DHB)$  2  $m(\angle ABC)$

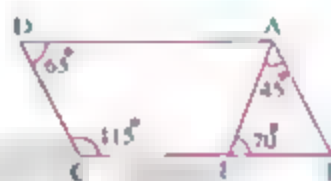


$$= 116^\circ + 128^\circ =$$

21 In the opposite figure :

$E \in \overline{BC}$  ,  $m(\angle BAE) = 45^\circ$  ,  
 $m(\angle AEB) = 70^\circ$  ,  $m(\angle D) = 65^\circ$   
 and  $m(\angle C) = 115^\circ$

Prove that : ABCD is a parallelogram.



22 In the opposite figure :

ABCD and HBCO are two parallelograms  
 such that  $m(\angle O) = 50^\circ$   
 and  $m(\angle ADC) = 115^\circ$

Find :  $m(\angle ABH)$



$$= 65^\circ =$$

23 In the opposite figure :

$\overline{AB} \parallel \overline{CD}$  ,  $\overline{AE}$  bisects  $\angle BAC$  ,  
 $\overline{CF}$  bisects  $\angle ACD$

If  $AE = CF$  ,

Prove that : AECF is a parallelogram.

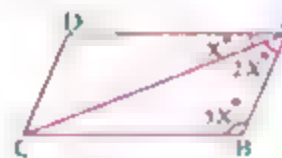


24 In the opposite figure :

ABCD is a parallelogram in which :

$m(\angle DAC) = x^\circ$  ,  $m(\angle BAC) = 2x^\circ$   
 and  $m(\angle ABC) = 5x^\circ$

Find :  $m(\angle BCD)$  and  $m(\angle ADC)$  in degrees.



$$= 67.5^\circ + 112.5^\circ =$$

25 Choose the correct answer from the given ones :

- 1 ABCD is a parallelogram in which :  $m(\angle A) = 50^\circ$  , then  $m(\angle C) =$ 
  - (a)  $50^\circ$
  - (b)  $60^\circ$
  - (c)  $130^\circ$
  - (d)  $150^\circ$

## Exercise 3

- 2 ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 140^\circ$   
 , then  $m(\angle B) = \dots\dots\dots$   
 (a)  $70^\circ$  (b)  $40^\circ$  (c)  $110^\circ$  (d)  $220^\circ$
- 3 If the lengths of two consecutive sides of a parallelogram are 3 cm and 5 cm, , then its perimeter equals  $\dots\dots\dots$  cm.  
 (a) 12 (b) 14 (c) 16 (d) 18
- 4 If the perimeter of a parallelogram is 25 cm. and if one of its sides is of length 7 cm. , then the consecutive side is of length  $\dots\dots\dots$  cm.  
 (a) 7 (b) 18 (c) 12.5 (d) 5.5

5 In the opposite figure :

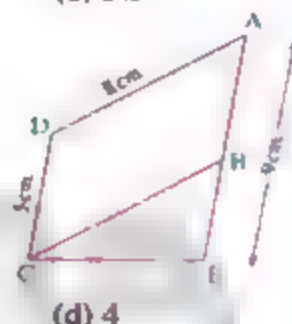
If ABCD is a parallelogram

,  $E \in \overline{AB}$  ,  $CD = 5$  cm. ,  $AB = 9$  cm.

,  $AD = 8$  cm. , the perimeter of  $\triangle BEC = 18$  cm.

, then the length of  $\overline{EC} = \dots\dots\dots$  cm.

- (a) 8 (b) 6 (c) 5 (d) 4

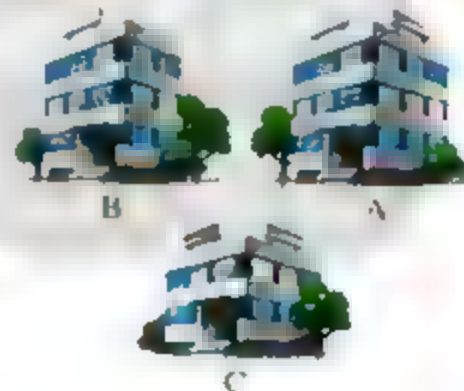


## Life Application

- 26 In one of the new cities , each four buildings are put like A , B , C and D according to the following .

- The distance between the two buildings A and B equals the distance between the two buildings C and D
- $\overline{AB} \parallel \overline{CD}$

Explain how we can decide the place where the building D will be built.



## For excellent pupils

- 27 ABCD is a parallelogram in which : E is the midpoint of  $\overline{AB}$  ,  
 F is the midpoint of  $\overline{CD}$  , if  $\overline{AF} \cap \overline{DE} = \{M\}$  ,  $\overline{BF} \cap \overline{CE} = \{N\}$   
 Prove that : 1  $\overline{ED} \parallel \overline{FB}$  2 FMEN is a parallelogram.

- 28 XYZL is a parallelogram in which :  $m(\angle Y) = 3 m(\angle X)$

Find the measures of the interior angles of XYZL

$$m(\angle Y) = m(\angle L) = 135^\circ \text{ and } m(\angle X) = m(\angle Z) = 45^\circ$$



4

## The special cases of the parallelogram

From the school book

## 1 Complete the following :

- 1 The parallelogram whose two diagonals are perpendicular is called
- 2 ☐ The parallelogram whose two diagonals are \_\_\_\_\_ is called a rectangle.
- 3 The parallelogram whose two diagonals are equal in length and perpendicular is called
- 4 ☐ The quadrilateral whose sides are equal in length is called
- 5 ☐ The quadrilateral whose diagonals bisect each other is called \_\_\_\_\_
- 6 ☐ The rectangle is a \_\_\_\_\_ with a right angle
- 7 The rhombus is a \_\_\_\_\_ in which its diagonals are perpendicular.
- 8 ☐ The square is a \_\_\_\_\_ with a right angle.
- 9 The rhombus whose two diagonals are equal in length is called
- 10 The rectangle whose two diagonals are perpendicular is called
- 11 The rectangle whose two adjacent sides have the same length is called
- 12 If  $XY \parallel ZL$ ,  $XY = ZL$ , then the quadrilateral  $XYZL$  is called
- 13 ☐ If  $ABCD$  is a rhombus, then  $\perp$
- 14 The perimeter of the square = \_\_\_\_\_  
 , the perimeter of the rectangle = \_\_\_\_\_  
 and the perimeter of the rhombus = \_\_\_\_\_
- 15 ☐ The rhombus whose perimeter is 42 cm, its side length = \_\_\_\_\_ cm.

## Exercise

2 Choose the correct answer from the given ones :

- 1 The two diagonals of a rectangle
  - (a) are perpendicular.
  - (b) are equal in length.
  - (c) are perpendicular and equal in length.
  - (d) bisect its interior angles.
- 2 The two diagonals of a rhombus are
  - (a) perpendicular and not equal in length
  - (b) equal in length and not perpendicular.
  - (c) perpendicular and equal in length.
  - (d) not equal in length and not perpendicular.
- 3 The two diagonals of the square are
  - (a) just perpendicular.
  - (b) just equal in length.
  - (c) perpendicular and equal in length.
  - (d) not equal in length and not perpendicular.
- 4 The adjacent sides are equal in length in a parallelogram, then the figure is a
  - (a) square.
  - (b) rhombus.
  - (c) rectangle
  - (d) trapezium.
- 5 If ABCD is a rectangle in which  $AC = 5$  cm, then  $BD =$  ..... cm
  - (a) 2.5
  - (b) 5
  - (c) 10
  - (d) 20
- 6 If ABCD is a square, then :  $m(\angle CAB) =$  .....
  - (a)  $90^\circ$
  - (b)  $45^\circ$
  - (c)  $60^\circ$
  - (d)  $30^\circ$
- 7 If ABCD is a parallelogram in which  $m(\angle A) = m(\angle B)$ , then ABCD is a .....
  - (a) rectangle.
  - (b) rhombus.
  - (c) square.
  - (d) trapezium.
- 8 If ABCD is a rhombus in which  $m(\angle ACB) = 32^\circ$ , then :  $m(\angle D) =$  .....
  - (a)  $32^\circ$
  - (b)  $64^\circ$
  - (c)  $116^\circ$
  - (d)  $26^\circ$

3 In the opposite figure :

ABCD is a rectangle in which :

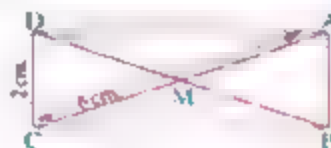
$AC = 6$  cm,  $CD = 2$  cm.

and  $\overline{AC} \cap \overline{BD} = \{M\}$

Complete : 1  $AB =$  ..... cm.

2  $DM =$  ..... cm.

3 The perimeter of  $\triangle ABM =$  ..... cm.



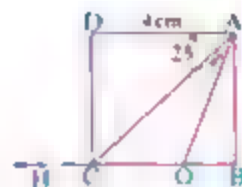
## Unit 3

## 4 In the opposite figure :

ABCD is a square in which  $AD = 4 \text{ cm}$  ,

$O \in \overline{BC}$  such that :  $m(\angle OAC) = 25^\circ$

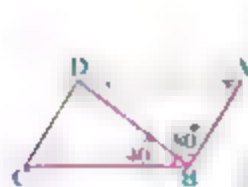
and  $H \in \overline{BC}$



Complete the following :

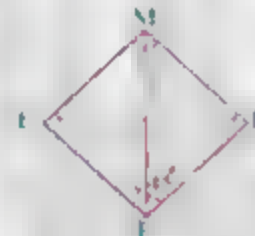
- 1 The perimeter of the square = ..... cm
- 2  $m(\angle ACH) = \dots\dots\dots^\circ$
- 3  $m(\angle AOC) = \dots\dots\dots^\circ$

## 5 Find the measures of the angles marked by (?) in each of the following figures :



a parallelogram

Fig. (1)



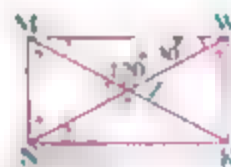
a square

Fig. (2)



a rhombus

Fig. (3)



a rectangle

Fig. (4)

## 6 In the opposite figure :

ABCD is a square whose side length = 5 cm.,

$E \in \overline{AC}$  where  $m(\angle EDC) = 30^\circ$

Complete the following proof to find :

- 1 The perimeter of the square ABCD
- 2  $m(\angle AED)$



Given

R.T.F.

Proof

$\therefore$  The perimeter of the square = side length  $\times$

$\therefore$  The perimeter of the square ABCD

= .....  $\times$  ..... = ..... cm.

(First req.)

$\therefore$  ABCD is a square  $\therefore$  AC is a diagonal.

$\therefore m(\angle ACD) = \dots\dots\dots^\circ$

In  $\triangle DEC$  :

$m(\angle DEC) = 180^\circ - (\dots\dots\dots^\circ + \dots\dots\dots^\circ) = \dots\dots\dots^\circ$

$\therefore E \in \overline{AC}$

$\therefore m(\angle AED) = \dots\dots\dots^\circ - \dots\dots\dots^\circ = \dots\dots\dots^\circ$

(Second req.)



## Exercise 4

## 7 In the opposite figure :

ABCD is a parallelogram,

$E \in \overline{CB}$  where  $BC = BE$ , if  $DE = DC$

Complete the following proof to prove that :

The figure AEBD is a rectangle.

Given

R.T.P.

Proof

$\because$  ABCD is a parallelogram

$\therefore AD = \dots\dots\dots$  ,  $\dots\dots\dots \parallel \overline{BC}$

$\because EB = \dots\dots\dots$  ,  $E \in \overline{CB}$  (given)

$\therefore AD = \dots\dots\dots$  ,  $\dots\dots\dots \parallel \overline{EB}$

$\therefore$  The figure AEBD is a parallelogram.

$\because DE = \dots\dots\dots$  (given)

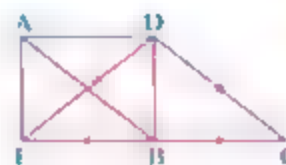
$\therefore AB = \dots\dots\dots$  (properties of parallelogram)

$\therefore DE = \dots\dots\dots$

$\therefore$  The two diagonals of the parallelogram AEBD are

$\therefore$  The figure AEBD is a

(Q.E.D.)

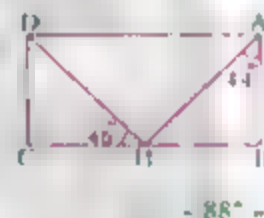


## 8 In the opposite figure :

ABCD is a rectangle,  $H \in \overline{BC}$  such that :

$m(\angle DHC) = 46^\circ$  and  $m(\angle BAH) = 44^\circ$

Calculate :  $m(\angle AHD)$



## 9 In the opposite figure :

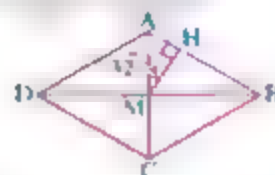
ABCD is a rhombus in which :

$\overline{AC} \cap \overline{BD} = \{M\}$ ,

$H \in \overline{AB}$  where  $\overline{MH} \perp \overline{AB}$

If  $m(\angle AMH) = 32^\circ$ ,

then calculate the measures of the angles of the rhombus ABCD



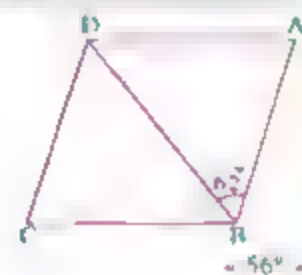
## 10 In the opposite figure :

ABCD is a rhombus,

$\overline{BD}$  is a diagonal in it,

$m(\angle ABD) = 62^\circ$

Find with proof :  $m(\angle A)$



## Unit 3

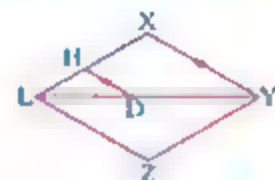
11 In the opposite figure :

$XYZL$  is a rhombus ,  $D \in \overline{YL}$

Draw  $\overline{DH} \parallel \overline{YX}$  such that :

$$\overline{DH} \cap \overline{XL} = \{H\}$$

Prove that :  $m(\angle HDL) = m(\angle HLD)$

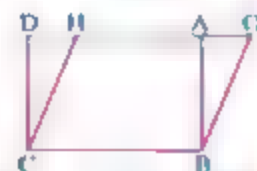


12 In the opposite figure :

$ABCD$  is a rectangle

and  $OBCH$  is a parallelogram.

Prove that :  $DH = AO$

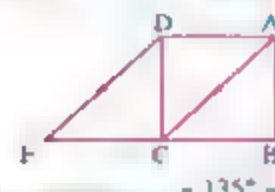


13 In the opposite figure :

$ABCD$  is a square ,  $E \in \overline{BC}$  ,  $\overline{AC} \parallel \overline{DE}$

1 Prove that :  $ACED$  is a parallelogram.

2 Find :  $m(\angle ACE)$



14  $ABCD$  is a rhombus in which  $m(\angle BAC) = 45^\circ$

Prove that :  $ABCD$  is a square.

15 In the opposite figure :

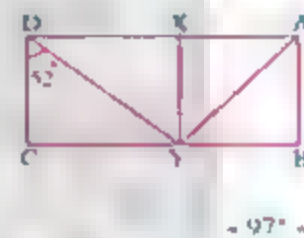
$ABCD$  is a rectangle ,  $X \in \overline{AD}$

and  $Y \in \overline{BC}$  such that :

$AXYB$  is a square.

If  $m(\angle YDC) = 52^\circ$  ,

then find with proof :  $m(\angle AYD)$

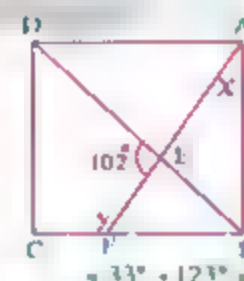


16 In the opposite figure :

$ABCD$  is a square

Find in degrees the value

of each of  $x$  and  $y$

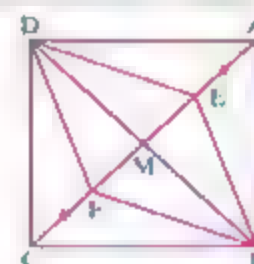


17 In the opposite figure :

$ABCD$  is a square , its diagonals intersect at  $M$  ,

$E \in \overline{AC}$  ,  $F \in \overline{AC}$  such that :  $AE = CF$

Prove that :  $EBFD$  is a rhombus



## Exercise

18 In the opposite figure :

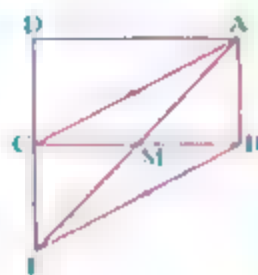
ABCD is a rectangle,

 $\overline{BE} \parallel \overline{AC}$ ,  $\overline{BE} \cap \overline{DC} = \{E\}$ , $\overline{BC} \cap \overline{AE} = \{M\}$ 

Prove that :

1  $DC = CE$

2  $MC = \frac{1}{2} AD$



## Life Application

19 Eslam drew a parallelogram, a rhombus, a rectangle and a square, then he hid parts of them as in the opposite figure and he asked his friend Bassem to recognize each figure.

Help Bassem to name each drawn figure.



## For excellent pupils

20 Use "some" or "all" to get a correct statement :

- 1 squares are rectangles.
- 2 quadrilaterals are parallelograms.
- 3 ... squares are rhombuses.
- 4 ... parallelograms are rectangles.
- 5 ... rectangles are parallelograms.
- 6 rhombuses are squares.

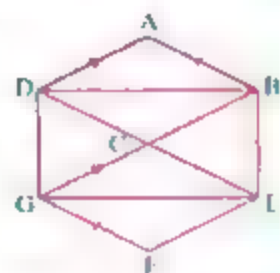
21 In the opposite figure :

ABCD is a parallelogram,

CEFG is a rhombus,

if  $AB = GF$ ,  $AD = CG$ 

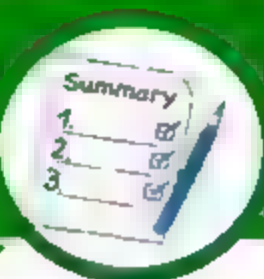
Prove that : BEGD is a rectangle.





# Summary of the first part of unit 3

From lesson 1 to lesson 4



- ✧ If two straight lines intersect , then the measures of each two vertically opposite angles are equal.
- ✧ The sum of the measures of the accumulative angles at a point is equal to  $360^\circ$
- ✧ The polygon is a simple closed line that consists of three or more line segments , and the polygon is named according to the number of its sides.
- ✧ The polygon is convex if the measure of any of its interior angles is less than  $180^\circ$
- ✧ The polygon is concave if the measure of one of its interior angles at least is greater than  $180^\circ$  (reflex angle)
- ✧ The sum of measures of the interior angles of a polygon of  $n$  sides equals  $(n - 2) \times 180^\circ$
- ✧ The sum of measures of the exterior angles of a convex polygon of  $n$  sides  $= 360^\circ$
- ✧ The polygon is regular if :
  - 1 All its sides are equal in length.
  - 2 All its angles are equal in measure.
- ✧ The measure of each interior angle of the regular polygon of  $n$  sides  $= \frac{(n - 2) \times 180^\circ}{n}$
- ✧ The number of sides of the regular polygon in which the measure of one of its interior angles is  $x^\circ = \frac{360^\circ}{180^\circ - x^\circ}$
- ✧ The number of diagonals of a polygon of  $n$  sides  $= \frac{n(n - 3)}{2}$
- ✧ Parallelogram is a quadrilateral , in which each two opposite sides are parallel.
- ✧ Properties of parallelogram :
  - 1 Each two opposite sides are equal in length.
  - 2 Each two opposite angles are equal in measure.
  - 3 The sum of measures of each two consecutive angles is  $180^\circ$
  - 4 The two diagonals bisect each other

• A quadrilateral represents a parallelogram if one of the following conditions take place :

- 1 Each two opposite sides are parallel.
- 2 Each two opposite sides are equal in length
- 3 Two opposite sides are parallel and equal in length.
- 4 Each two opposite angles are equal in measure
- 5 The two diagonals bisect each other

• The rectangle is a parallelogram with a right angle.

• Properties of the rectangle :

The rectangle has the same properties as the parallelogram and more additional properties as the following :

- 1 Its four angles are all equal in measure and the measure of each is  $90^\circ$
- 2 Its two diagonals are equal in length.

• The rhombus is a parallelogram in which two adjacent sides are equal in length.

• Properties of the rhombus :

The rhombus has the same properties as the parallelogram and more additional properties as the following

- 1 Its four sides are all equal in length
- 2 Its two diagonals are perpendicular and bisect its interior angles.

• The square is a parallelogram with a right angle and two adjacent sides are equal in length.

• Properties of the square :

The square has the same properties as the parallelogram and more additional properties as the following :

- 1 Its four sides are all equal in length
- 2 Its four angles are all equal in measure and each of them is of measure  $90^\circ$
- 3 Its two diagonals are equal in length, perpendicular and each diagonal bisects the vertices angles which this diagonal joins and the measure of each is  $45^\circ$

# Exams on the first part of unit three

from lesson (1) to lesson (4)



## Model 1

Answer the following questions :

1 Choose the correct answer from those given :

- 1 The measure of the interior angle of the regular hexagon is  
(a)  $720^\circ$  (b)  $120^\circ$  (c)  $150^\circ$  (d)  $108^\circ$
- 2 If two adjacent sides are equal in length in a parallelogram , then the figure is a  
(a) square. (b) rhombus. (c) rectangle. (d) trapezium.
- 3 ABCD is a parallelogram in which ,  $m(\angle A) = 70^\circ$  , then  $m(\angle C) =$   
(a)  $110^\circ$  (b)  $180^\circ$  (c)  $100^\circ$  (d)  $70^\circ$
- 4 The sum of measures of the exterior angles of a triangle =  
(a)  $180^\circ$  (b)  $100^\circ$  (c)  $360^\circ$  (d)  $90^\circ$
- 5 The parallelogram which has a right angle is called . . .  
(a) rhombus. (b) rectangle. (c) square (d) trapezium
- 6 If the measure of an interior angle of a regular polygon is  $150^\circ$  , then the number of its sides is .....  
(a) 6 (b) 8 (c) 10 (d) 12

2 Complete the following :

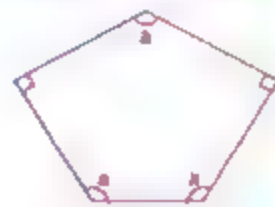
- 1 The sum of measures of the interior angles of a heptagon is .....
- 2 The sum of measures of the accumulative angles at a point is .....
- 3 ABCD is a parallelogram in which  $m(\angle A) = \frac{1}{2} m(\angle B)$  , then  $m(\angle B) =$  .....
- 4 The square is a ..... with a right angle.
- 5 The quadrilateral in which only two sides are parallel is called .....



## Unit Exams

- 3 (a) In the opposite figure :

Find : The value of  $a$



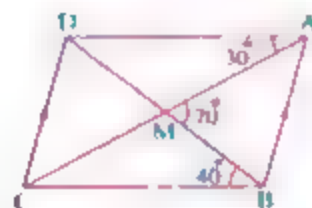
- (b) In the opposite figure :

$$\overline{AC} \cap \overline{BD} = \{M\}, \overline{AB} \parallel \overline{DC}$$

$$m(\angle AMB) = 70^\circ, m(\angle MBC) = 40^\circ$$

$$\text{and } m(\angle MAD) = 30^\circ$$

Prove that : ABCD is a parallelogram.



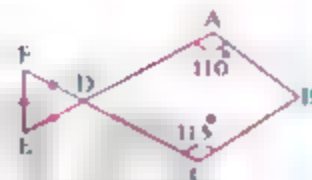
- 4 (a) In the opposite figure :

$$\overline{AE} \cap \overline{CF} = \{D\}$$

$\triangle DEF$  is an equilateral triangle

$$m(\angle A) = 110^\circ \text{ and } m(\angle C) = 115^\circ$$

Find :  $m(\angle B)$



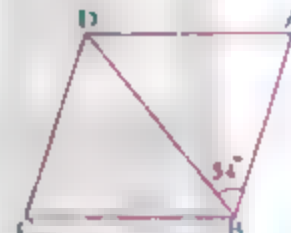
- (b) In the opposite figure :

ABCD is a rhombus

$\overline{BD}$  is a diagonal in it

$$m(\angle ABD) = 54^\circ$$

Find with proof :  $m(\angle A)$



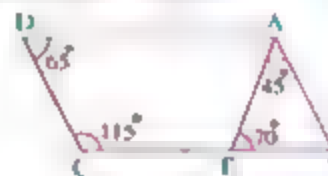
- 5 (a) In the opposite figure :

$$E \in \overline{BC}, m(\angle BAE) = 45^\circ$$

$$m(\angle AEB) = 70^\circ, m(\angle D) = 65^\circ$$

$$\text{and } m(\angle C) = 115^\circ$$

Prove that : ABCD is a parallelogram.

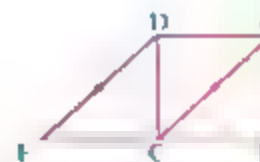


- (b) In the opposite figure :

ABCD is a square,  $E \in \overline{BC}$ ,  $\overline{AC} \parallel \overline{DE}$

1 Prove that : ACED is a parallelogram.

2 Find :  $m(\angle ACE)$



## Unit 3

## Model 2

Answer the following questions :

1 Choose the correct answer from the given ones :

- A rhombus of perimeter 60 cm , then its side length = cm.  
(a) 20 (b) 18 (c) 15 (d) 10
- The sum of measures of the interior angles of a polygon of  $n$  sides =  
(a)  $n \times 180^\circ$  (b)  $(n - 2) \times 180^\circ$   
(c)  $\frac{(n - 2) \times 180^\circ}{n}$  (d)  $\frac{(n - 2) \times 180^\circ}{2n}$
- If ABCD is a square , then  $m(\angle CAB) =$   
(a)  $90^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $30^\circ$
- The sum of measures of the exterior angles of a regular pentagon =  
(a)  $108^\circ$  (b)  $120^\circ$  (c)  $360^\circ$  (d)  $540^\circ$
- In the parallelogram , the sum of measures of each two consecutive angles equals .  
(a)  $90^\circ$  (b)  $360^\circ$  (c)  $120^\circ$  (d)  $180^\circ$
- The parallelogram in which the two diagonals are equal in length and perpendicular is called a .....  
(a) rectangle. (b) square. (c) rhombus. (d) trapezium.

2 Complete the following :

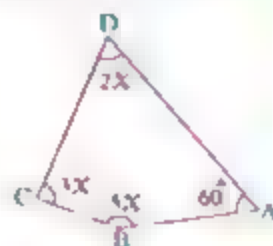
- The measure of an interior angle of a regular polygon of 10 sides =  $^\circ$
- In the parallelogram , the two diagonals
- The rectangle is a ..... with a right angle.
- ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 110^\circ$  , then  $m(\angle B) =$   $^\circ$
- If ABCD is a rhombus ,  $m(\angle ACB) = 31^\circ$  , then  $m(\angle D) =$   $^\circ$

3 {a} In the opposite figure :

ABCD is a quadrilateral in which

$m(\angle A) = 60^\circ$

Find : The value of  $X$



[b] In the opposite figure :

$$E \in \overline{DA}, m(\angle EAB) = 130^\circ$$

$$, m(\angle C) = 50^\circ, \overline{DA} \parallel \overline{CB}$$

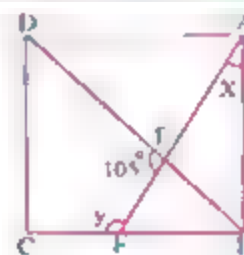
Prove that : The figure ABCD is a parallelogram



4 [a] In the opposite figure :

ABCD is a square

Find in degrees the value of :  $x, y$

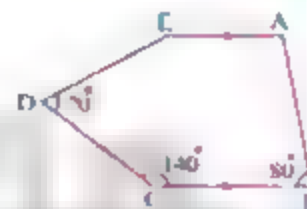


[b] In the opposite figure :

$$\overline{AE} \parallel \overline{BC}, m(\angle B) = 80^\circ$$

$$, m(\angle C) = 140^\circ, m(\angle D) = 70^\circ$$

Find with proof :  $m(\angle E)$



5 [a] In the opposite figure :

ABCD is a rectangle

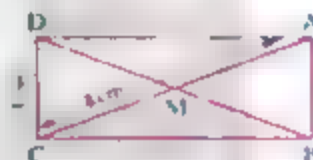
$$, AC = 8 \text{ cm}, CD = 3 \text{ cm}$$

, its two diagonals intersect at M

Find :

1  $MD, AB$

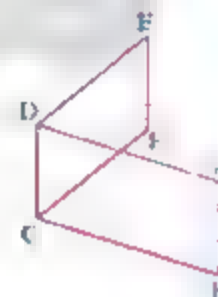
2 The perimeter of  $\triangle ABM$



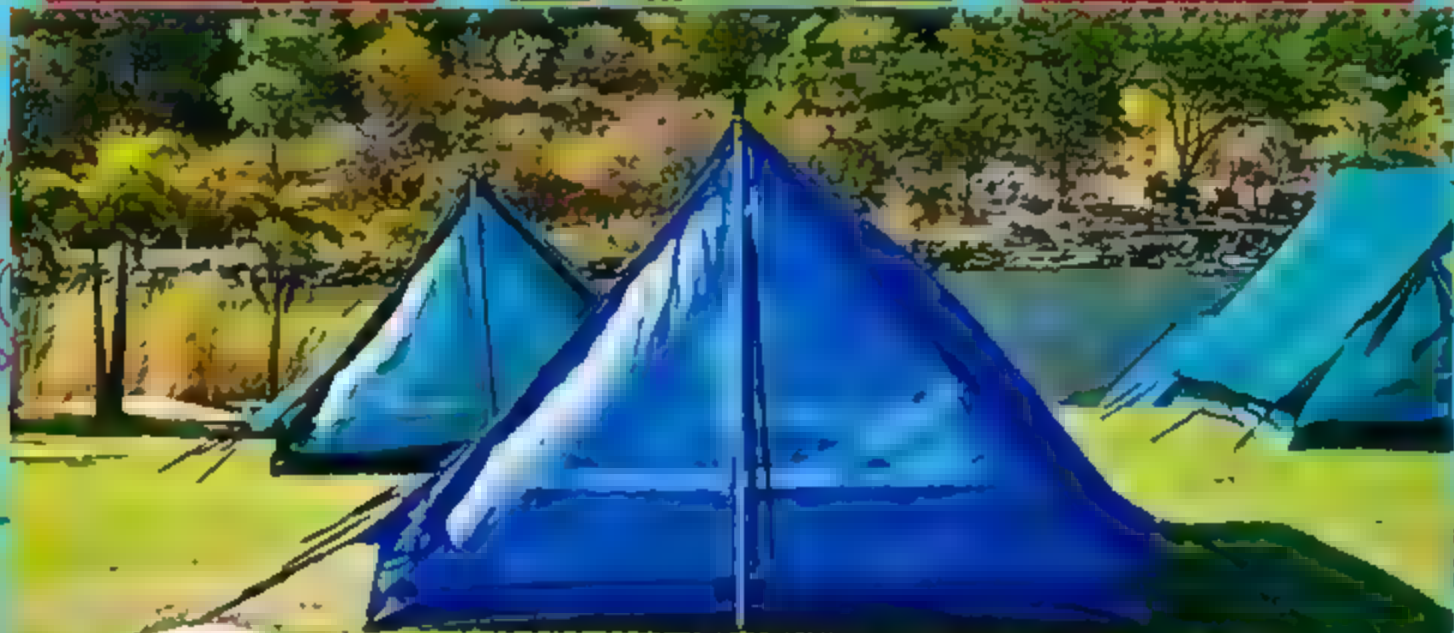
[b] In the opposite figure :

ABCD, EFCB are two parallelograms.

Prove that :  $AB = EF$







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## The triangle

From the school book

### 1 Complete the following :

- The sum of measures of the interior angles of a triangle =
- The measure of the exterior angle of a triangle is equal to the sum of
- If the measure of an angle in a triangle equals the sum of measures of the other two angles in the triangle , then the triangle is
- If the measure of an angle in a triangle is greater than the sum of measures of the other two angles , then the triangle is
- In  $\triangle ABC$  : If  $m(\angle A) + m(\angle C) = m(\angle B)$  , then  $m(\angle B) =$
- In  $\triangle ABC$  : If  $m(\angle B) > m(\angle A) + m(\angle C)$  , then  $\angle B$  is
- It is possible to find a triangle each of its interior angles is of measure

### 2 Choose the correct answer from the given ones :

- The triangle contains two ..... angles at least.  
(a) acute (b) obtuse (c) right (d) reflex
- The sum of measures of the interior angles of a triangle equals the measure of ..... angle.  
(a) a right (b) a straight (c) an acute (d) a reflex
- In  $\triangle XYZ$  : If  $m(\angle X) = 50^\circ$  ,  $m(\angle Y) = 100^\circ$  , then  $m(\angle Z) =$   
(a)  $30^\circ$  (b)  $50^\circ$  (c)  $80^\circ$  (d)  $100^\circ$
- In  $\triangle ABC$  : If  $m(\angle A) + m(\angle B) = 110^\circ$  , then  $m(\angle C) =$   
(a)  $110^\circ$  (b)  $90^\circ$  (c)  $70^\circ$  (d)  $55^\circ$
- If the measures of two angles in a triangle are  $35^\circ$  and  $45^\circ$  , then the triangle is  
(a) acute-angled. (b) right-angled. (c) obtuse-angled. (d) equilateral

## Exercise 5

6 The measure of the exterior angle of the equilateral triangle at any one of its vertices equals .....

(a)  $60^\circ$ (b)  $120^\circ$ (c)  $150^\circ$ (d)  $30^\circ$ 

3 In each of the following figures, find the measure of the angle marked by (?) :



Fig. (1)

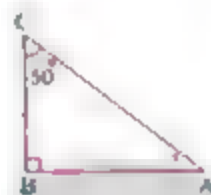


Fig. (2)

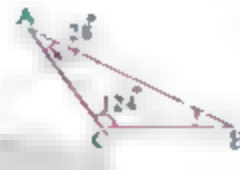


Fig. (3)

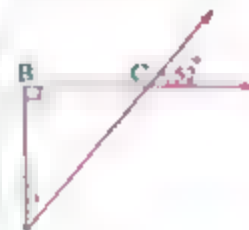


Fig. (4)

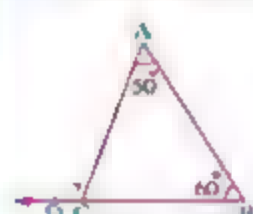


Fig. (5)

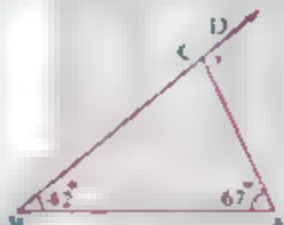


Fig. (6)

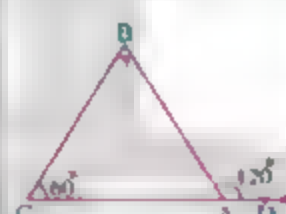


Fig. (7)



Fig. (8)

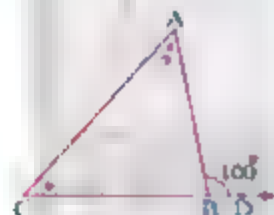


Fig. (9)



Fig. (10)

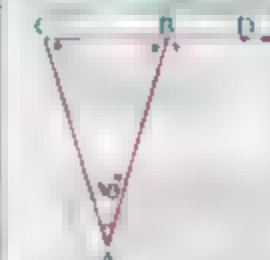


Fig. (11)

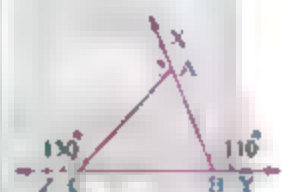


Fig. (12)

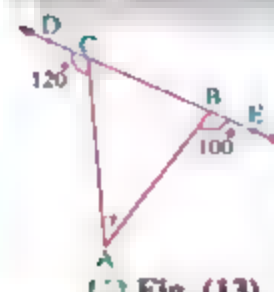


Fig. (13)

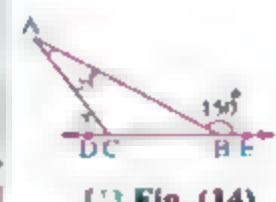


Fig. (14)

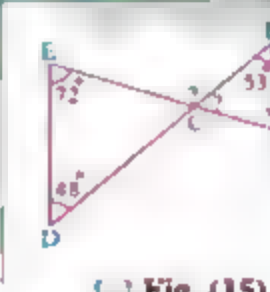


Fig. (15)

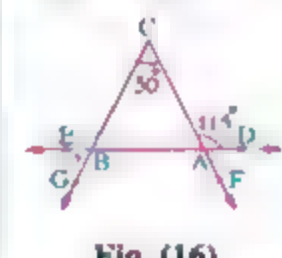


Fig. (16)

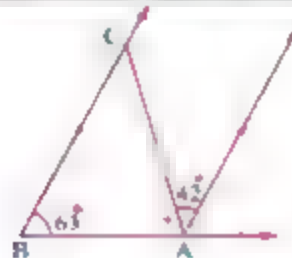


Fig. (17)

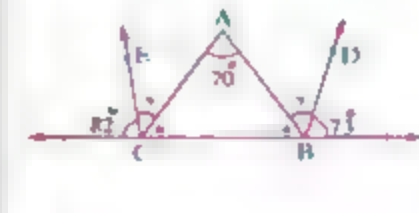


Fig. (18)

## Unit 3

4 In the opposite figure :

$$\overline{BD} \cap \overline{AE} = \{C\}, \overline{AF} \parallel \overline{BC}$$

$$m(\angle BAF) = 65^\circ \text{ and } m(\angle DCE) = 55^\circ$$

Complete the following proof to find :

The measures of the interior angles of  $\triangle ABC$ 

Given

R.T.F.

Proof

$$\because \overline{BD} \cap \overline{AE} = \{C\} \quad (\text{given})$$

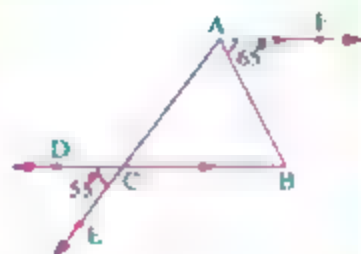
$$\therefore m(\angle ACB) = m(\angle \dots) = \dots \quad (\dots)$$

$$\because \overline{AF} \parallel \overline{BC}, \overline{AB} \text{ is a transversal to them}$$

$$\therefore m(\angle FAB) = m(\angle \dots) = \dots \quad (\dots \text{ angles})$$

$$\because \text{the sum of measures of the interior angles of the triangle} = \dots$$

$$\therefore m(\angle BAC) = \dots - (\dots + \dots) = \dots \quad (\text{The req.})$$



5 In the opposite figure :

$$A \in \overline{DC}, \overline{DE} \parallel \overline{CB},$$

$$m(\angle D) = 100^\circ \text{ and } m(\angle B) = 40^\circ$$

Complete the following proof to find :  $m(\angle BAD)$ 

Given

R.T.F.

Proof

$$\because \overline{DE} \parallel \dots, \dots \text{ is a transversal}$$

$$\therefore m(\angle D) + m(\angle C) = \dots$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle D) = \dots$$

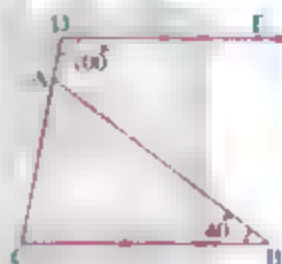
$$\therefore m(\angle C) = \dots = \dots$$

$$\because \angle BAD \text{ is an exterior angle of } \triangle$$

$$\therefore m(\angle BAD) = m(\angle \dots) + m(\angle \dots)$$

$$= \dots + \dots = \dots$$

(The req.)



6 In the opposite figure :

$$\overline{BD} \parallel \overline{CA}, m(\angle C) = 45^\circ$$

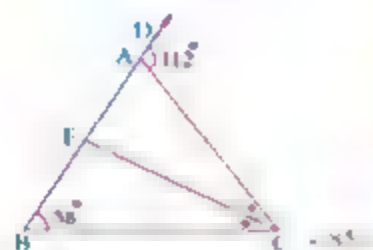
$$\text{and } m(\angle ABD) = 75^\circ$$

Find :  $m(\angle ABC)$ 

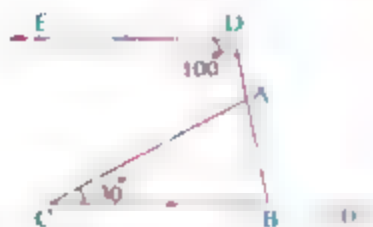


## Exercise 5

7 In the opposite figure :

ABC is a triangle in which :  $m(\angle B) = 58^\circ$  , $E \in \overline{AB}$  such that  $\overline{CE}$  bisects  $\angle ACB$  , $D \in \overline{BA}$  and  $m(\angle CAD) = 112^\circ$ Find :  $m(\angle AEC)$ 

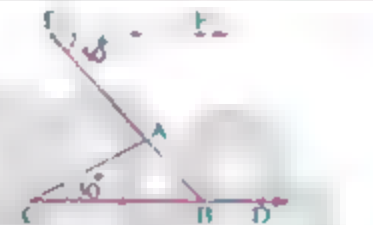
8 In the opposite figure :

 $\overline{DE} \parallel \overline{BC}$  ,  $m(\angle D) = 100^\circ$  , $m(\angle C) = 30^\circ$ and  $A \in \overline{DB}$ Find :  $m(\angle BAC)$ 

9 In the opposite figure :

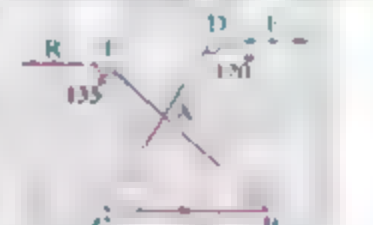
 $\overline{EF} \parallel \overline{CD}$  , $m(\angle E) = 50^\circ$  and  $m(\angle C) = 30^\circ$ 

Find the measures of the angles

of  $\triangle ABC$  and  $m(\angle ABD)$ 

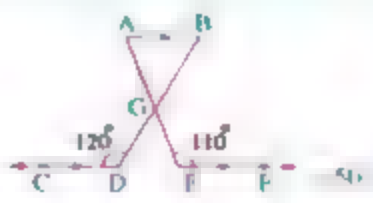
$$m(\angle ABC) = 50^\circ + m(\angle HAC) = 100^\circ + m(\angle ABD) = 130^\circ$$

10 In the opposite figure :

 $\overline{DE} \parallel \overline{FR} \parallel \overline{BC}$  , $m(\angle CDE) = 120^\circ$  and  $m(\angle RFB) = 135^\circ$ Calculate the measures of the angles of  $\triangle ABC$ 

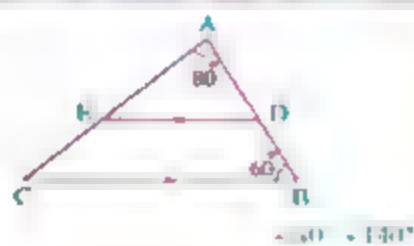
$$m(\angle B) = 45^\circ + m(\angle C) = 60^\circ + m(\angle A) = 75^\circ$$

11 In the opposite figure :

 $\overline{AB} \parallel \overline{DC} \parallel \overline{EF}$  , $m(\angle E) = 110^\circ$  and  $m(\angle D) = 120^\circ$ Find :  $m(\angle EGD)$ 

12 In the opposite figure :

ABC is a triangle in which :

 $m(\angle A) = 80^\circ$  and  $m(\angle B) = 60^\circ$  $\overline{DE} \parallel \overline{BC}$  where :  $D \in \overline{AB}$  and  $E \in \overline{AC}$ Find :  $m(\angle AED)$  and  $m(\angle DEC)$ 

$$m(\angle AED) = 140^\circ$$

33

## 13 In the opposite figure :

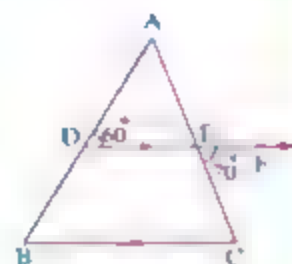
ABC is a triangle ,  $m(\angle ADE) = 60^\circ$  ,

$m(\angle FEC) = 70^\circ$  ,

$D \in \overline{AB}$  ,  $\overline{DF} \parallel \overline{BC}$

and  $\overline{AC} \cap \overline{DF} = \{E\}$

Find the measures of the interior angles of  $\triangle ABC$



$$m(\angle C) = 70^\circ + m(\angle B) = 60^\circ + m(\angle A) = 50^\circ$$

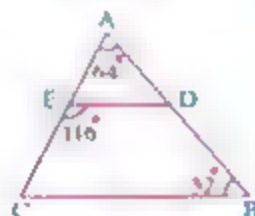
## 14 In the opposite figure :

ABC is a triangle in which  $m(\angle A) = 64^\circ$  ,

$m(\angle B) = 52^\circ$  ,  $m(\angle DEC) = 116^\circ$  ,

$E \in \overline{AC}$  and  $D \in \overline{AB}$

Prove that :  $\overline{DE} \parallel \overline{BC}$



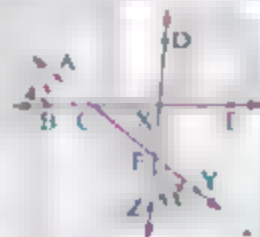
## 15 In the opposite figure :

Prove that :

$m(\angle DXE) = 85^\circ$  ,

then calculate  $m(\angle DXC)$

and  $m(\angle EXF)$



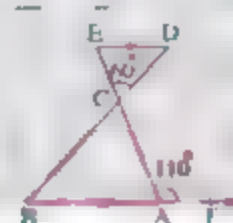
## 16 In the opposite figure :

$\overline{ED} \parallel \overline{BA}$  ,  $m(\angle CAF) = 110^\circ$  ,

$\overline{DB} \cap \overline{AE} = \{C\}$  ,

$m(\angle DCE) = 60^\circ$  and  $F \in \overline{BA}$

Find the measures of each of the angles of the two triangles DCE and ABC



$$m(\angle E) = 70^\circ + m(\angle D) = 50^\circ + m(\angle B) = 50^\circ + m(\angle ACB) = 60^\circ + m(\angle BAC) = 70^\circ$$

## 17 In the opposite figure :

ABCD is a rectangle ,  $E \in \overline{BC}$  where :

$m(\angle BAE) = 30^\circ$  and  $m(\angle EDC) = 60^\circ$

Find :  $m(\angle AED)$

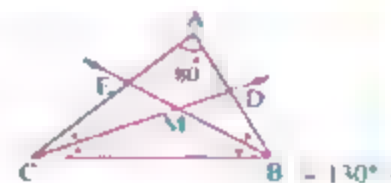


## 18 In the opposite figure :

$\overline{BM}$  bisects  $\angle ABC$  and  $\overline{CM}$  bisects  $\angle ACB$

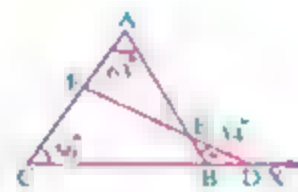
If  $m(\angle A) = 80^\circ$  ,

Find :  $m(\angle EMD)$

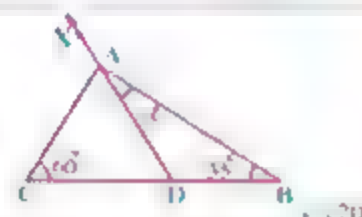


## Exo 50

19 In the opposite figure :

ABC is a triangle ,  $D \in \overline{CB}$  ,  $X \in \overline{CB}$  , $m(\angle A) = 63^\circ$  ,  $m(\angle C) = 56^\circ$ and  $m(\angle DEB) = 34^\circ$ Find :  $m(\angle EDX)$ 

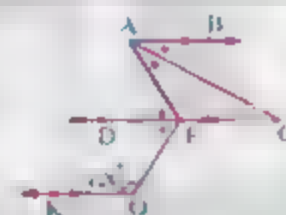
20 In the opposite figure :

ABC is a triangle ,  $m(\angle B) = 35^\circ$  ,  $m(\angle C) = 60^\circ$  , $m(\angle BAD) = 25^\circ$  ,  $D \in \overline{BC}$  and  $X \in \overline{DA}$ Find :  $m(\angle XAC)$ 

21 In the opposite figure :

ABC is a triangle ,  $\overline{AE} \parallel \overline{BC}$  ,  $D \in \overline{BC}$  ,  $F \in \overline{BA}$  , $m(\angle FAE) = 130^\circ$  and  $m(\angle FAC) = 110^\circ$ Find :  $m(\angle ACD)$ 

22 In the opposite figure :

 $\overline{AB} \parallel \overline{CD} \parallel \overline{OK}$  ,  $F \in \overline{CD}$  ,  $\overline{AC}$  bisects  $\angle BAE$  $\overline{ED}$  bisects  $\angle AEO$  and  $m(\angle O) = 120^\circ$ Find :  $m(\angle C)$ 

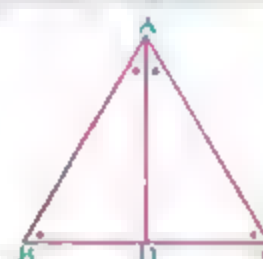
23 In the opposite figure :

The points F , C , E and B are collinear ,

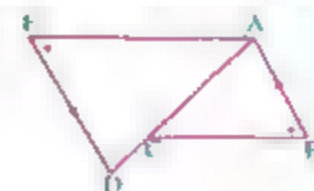
 $m(\angle B) = m(\angle DEC) = 90^\circ$  and  $\overline{AC} \parallel \overline{DF}$ Prove that :  $m(\angle A) = m(\angle D)$ 

24 In the opposite figure :

ABC is a triangle ,

 $m(\angle B) = m(\angle C)$ and  $\overline{AD}$  is the bisector of  $\angle A$ Prove that :  $AB = AC$ 

25 In the opposite figure :

 $\overline{AB} \parallel \overline{ED}$ and  $m(\angle ABC) = m(\angle AED)$ Prove that :  $\overline{BC} \parallel \overline{AE}$ 



## Unit 3

26 In the opposite figure :

XYZ is a triangle in which  $m(\angle Z) = 52^\circ$ ,  $L \in \overline{XY}$

and  $M \in \overline{XZ}$  such that :  $m(\angle Y) = m(\angle XML)$

Find :  $m(\angle XLM)$



27 In the opposite figure :

ABC is a triangle in which :

$m(\angle B) = m(\angle ACB)$  and  $D \in \overline{BC}$

Prove that :  $m(\angle B) > m(\angle D)$

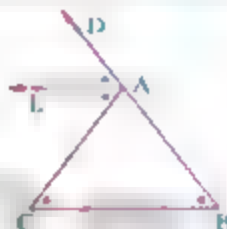


28 In the opposite figure :

ABC is a triangle in which :  $m(\angle B) = m(\angle C)$ ,

$D \in \overline{BA}$  and  $\overline{AE}$  bisects  $\angle DAC$

Prove that :  $\overline{AE} \parallel \overline{BC}$



29 In the opposite figure :

ABC is a triangle in which :  $D \in \overline{AC}$ ,

$m(\angle 1) = m(\angle A)$  and  $m(\angle 2) = m(\angle C)$

Prove that :  $\angle ABC$  is a right angle.



For excellent pupils

30 ABC is a triangle in which :  $m(\angle A) = 2m(\angle C)$  and  $m(\angle B) = 4m(\angle C)$

Prove that :  $\angle B$  is an obtuse angle.

31 ABC is a triangle in which :  $m(\angle C) = 28^\circ$ ,  $m(\angle A) = 4x^\circ$

,  $m(\angle B) = (2x + 2)^\circ$

Find :  $m(\angle A)$  and  $m(\angle B)$

$= 100^\circ, 52^\circ$

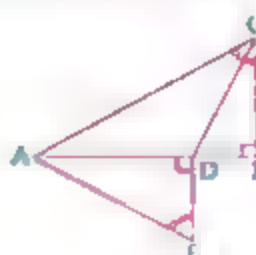
32 In the opposite figure :

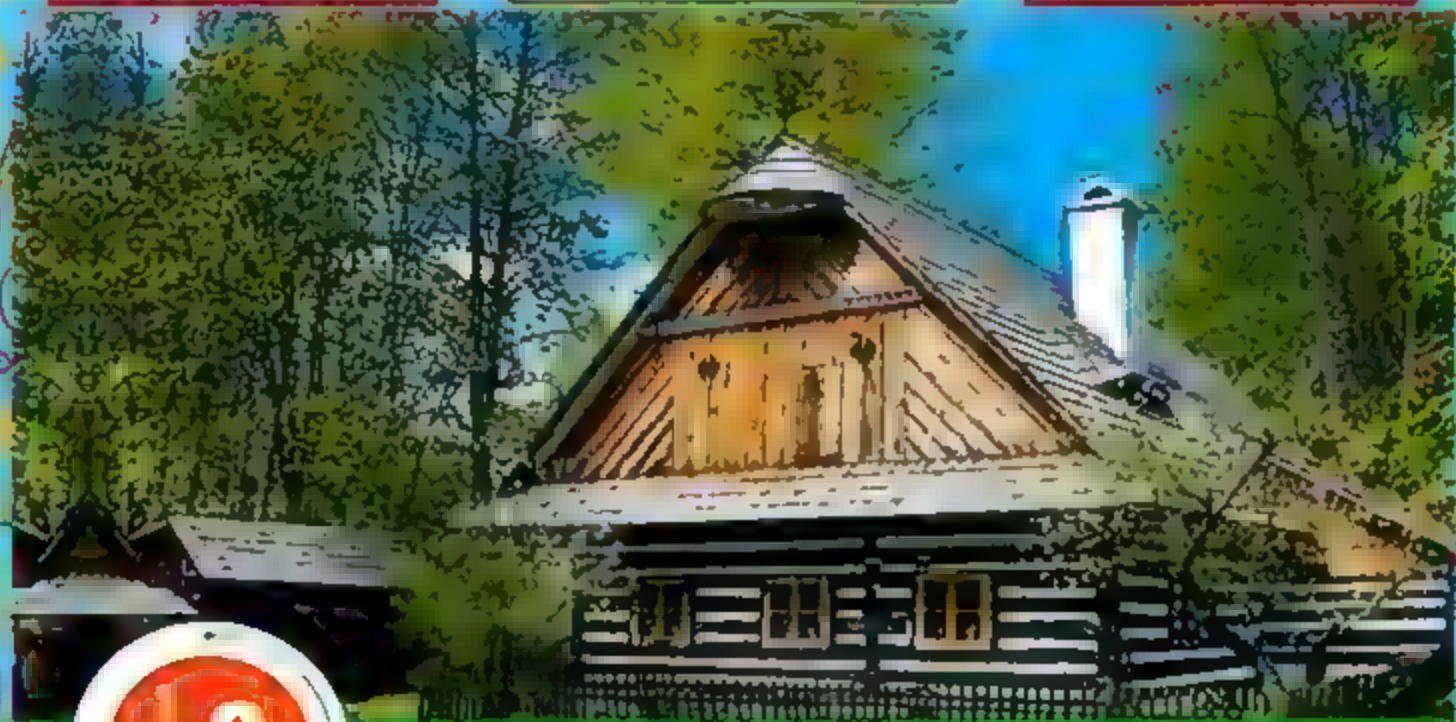
ABC is a triangle in which :  $D \in \overline{AB}$

$m(\angle B) = 90^\circ$ ,  $m(\angle ADE) = 90^\circ$

and  $m(\angle ACB) = m(\angle E)$

Prove that :  $m(\angle BDC) > m(\angle DAE)$





6

## Theorem 2 and its corollary, and theorem 3

From the school book

## 1 Complete the following :

- 1 The ray drawn from the midpoint of a side of a triangle parallel to another side ....
- 2 The line segment joining the midpoints of two sides of a triangle is ..... the third side.
- 3 The length of the line segment joining the midpoints of two sides of a triangle equals .....

## 4 In the opposite figure :

If D is the midpoint of  $\overline{AB}$ ,  $\overline{DE} \parallel \overline{BC}$

, then ..... is the midpoint of .....

## 5 In the opposite figure :

If D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

, then ..... //

## 6 In the opposite figure :

$\because$  D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

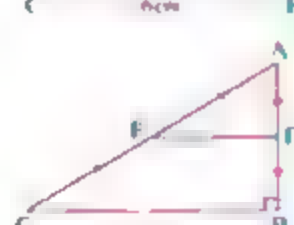
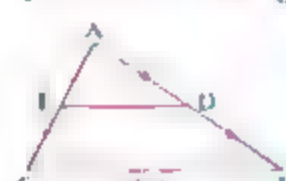
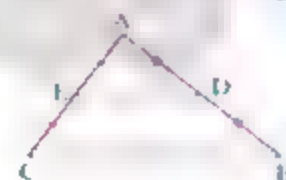
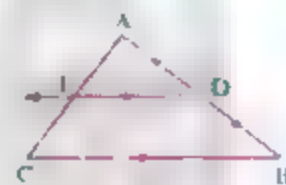
,  $BC = 6$  cm.

$\therefore DE = \dots\dots\dots$  cm.

## 7 In the opposite figure :

If  $m(\angle B) = 90^\circ$ , D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

, then  $m(\angle ADE) = \dots\dots\dots$



13

8 In the opposite figure :

If D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

• and the perimeter of the triangle ABC = 24 cm.

• then the perimeter of the triangle ADE = ..... cm.

9 In the opposite figure :

• D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

• AD = 3 cm, AE = 2 cm, and DE = 4 cm.

• The perimeter of the figure DBCE = ..... cm

10 In the opposite figure :

If the perimeter of the square ABCD = 20 cm.

•  $\overline{MF} \parallel \overline{CB}$  where  $F \in \overline{AB}$

• then AF = ..... cm.

11 In the opposite figure :

• The perimeter of the rhombus ABCD = 24 cm.

E is the midpoint of  $\overline{AB}$

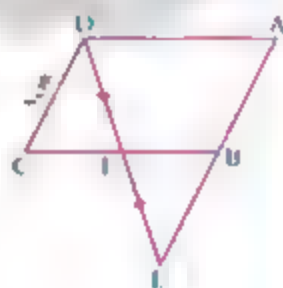
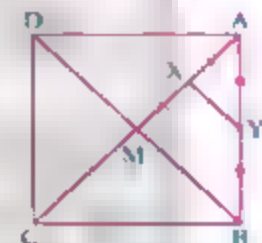
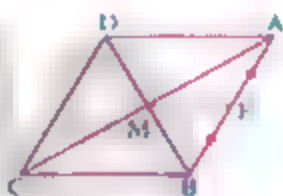
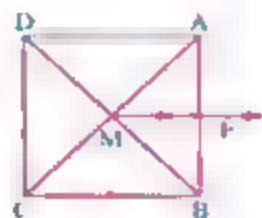
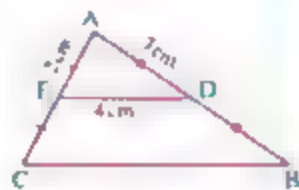
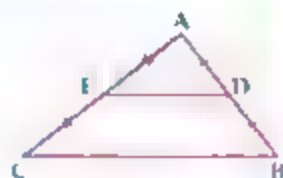
• ME = ..... cm

12 In the opposite figure :

• ABCD is a square, X and Y are the midpoints

of  $\overline{AM}$  and  $\overline{AB}$  respectively and AC = 12 cm

• XY = ..... cm,  $m(\angle AYX) = \dots\dots\dots^\circ$



2 In the opposite figure :

ABCD is a parallelogram, DC = 5 cm.

•  $F \in \overline{BC}$ ,  $\overline{DF} \cap \overline{AB} = \{E\}$ ,

if DF = FE

Complete the following proof to find the length of  $\overline{BE}$

Given

R.T.F.

Proof

• ABCD is a

•  $\overline{AD} \parallel \dots\dots\dots$

•  $\overline{BF} \parallel$

• In  $\triangle AED$  :

• F is the midpoint of ..... ,  $\overline{BF} \parallel$



## Exercise 6

∴ B is the midpoint of \_\_\_\_\_

∴ AB = \_\_\_\_\_

∴ AB = \_\_\_\_\_ = \_\_\_\_\_ cm. (Properties of \_\_\_\_\_)

∴ BE = \_\_\_\_\_ cm.

(The req.)

## 3 In the opposite figure :

ABC is a triangle in which  $CA = CB$ ,

E is the midpoint of  $\overline{AB}$ ,  $\overline{EF} \parallel \overline{AC}$ ,

H and G are the two midpoints of  $\overline{BD}$  and  $\overline{CD}$  respectively

Complete the following proof to prove that :  $EF = GH$

Given

R.T.P.

Proof

In  $\triangle ABC$  :

∴ E is the midpoint of \_\_\_\_\_,  $\overline{EF} \parallel$

∴ F is the midpoint of \_\_\_\_\_

∴  $EF = \frac{1}{2}$  \_\_\_\_\_

∴ In  $\triangle BDC$  :

∴ H is the midpoint of \_\_\_\_\_

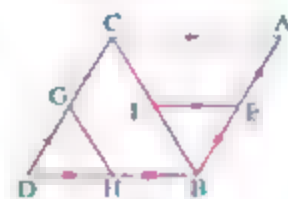
∴ \_\_\_\_\_ is the midpoint of  $\overline{CD}$

∴  $GH = \frac{1}{2}$  \_\_\_\_\_

∴  $CA =$  \_\_\_\_\_

∴  $EF =$  \_\_\_\_\_

(Q.E.D.)



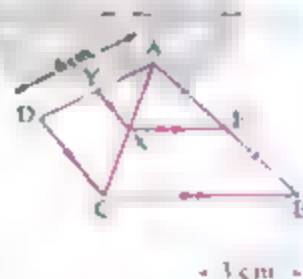
## 4 In the opposite figure :

$AE = EB$ ,  $\overline{EX} \parallel \overline{BC}$ ,

$\overline{XY} \parallel \overline{CD}$

and  $AD = 6$  cm.

Find the length of :  $\overline{AY}$



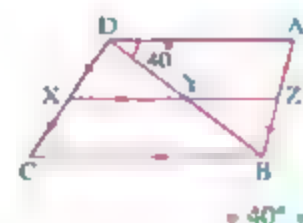
## 5 In the opposite figure :

X is the midpoint of  $\overline{CD}$

∴ Z is the midpoint of  $\overline{AB}$

∴  $\overline{XY} \parallel \overline{CB}$ ,  $m(\angle ADB) = 40^\circ$

Find :  $m(\angle ZYB)$

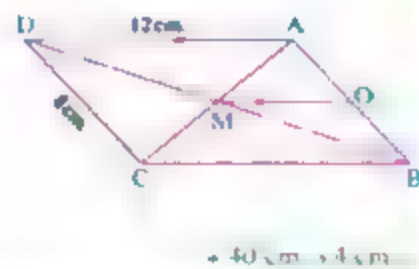


Unit 3

6 In the opposite figure :

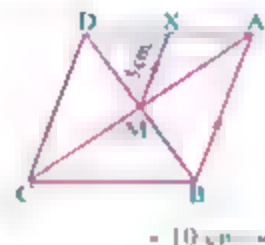
ABCD is a parallelogram ,  $\overline{AC} \cap \overline{BD} = \{M\}$ Draw  $\overline{MO} \parallel \overline{AD}$  to cut  $\overline{AB}$  at OIf  $AD = 12$  cm. and  $DC = 8$  cm.

, then find : 1 The perimeter of ABCD

2 The length of  $\overline{AO}$ 

7 In the opposite figure :

ABCD is a parallelogram , its diagonals intersect at M

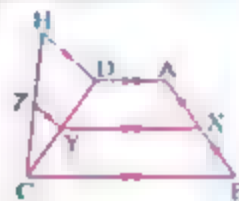
Draw  $\overline{MX} \parallel \overline{BA}$  to intersect  $\overline{AD}$  at X1 Prove that : X is the midpoint of  $\overline{AD}$ 2 If  $MX = 5$  cm. , then find the length of  $\overline{CD}$ 8 ABCD is a parallelogram , its diagonals intersect at M ,  $\overline{ME} \parallel \overline{BA}$  and cuts  $\overline{AD}$  at EProve that :  $ME = \frac{1}{2} DC$ 

9 In the opposite figure :

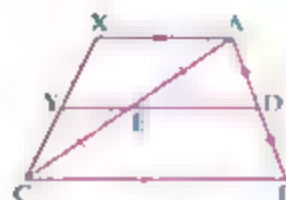
ABCD is a parallelogram ,

 $BC = CH$  ,  $H \in \overline{BC}$ Draw  $\overline{AH}$  to cut  $\overline{DC}$  at OProve that :  $AO = OH$ 

10 In the opposite figure :

ABCD is a trapezium ,  $\overline{AD} \parallel \overline{BC}$  , X is the midpoint of  $\overline{AB}$ If  $\overline{AD} \parallel \overline{XY}$  where  $Y \in \overline{DC}$  ,  $\overline{YZ} \parallel \overline{DH}$ Prove that :  $CZ = ZH$ 

11 In the opposite figure :

 $AD = DB$  ,  $AE = EC$  , $\overline{AX} \parallel \overline{BC}$  ,  $\overline{DE} \cap \overline{XC} = \{Y\}$ Prove that : Y is the midpoint of  $\overline{XC}$ 

## Exercice 6

12 In the opposite figure :

ABCD is a quadrilateral in which :

X and Z are the midpoints of  $\overline{AB}$

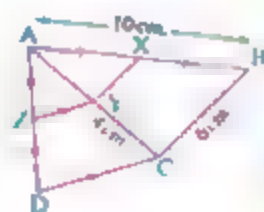
and  $\overline{AD}$  respectively and  $Y \in \overline{AC}$  such that :

$\overline{YZ} \parallel \overline{CD}$  and  $YC = 4$  cm.

If  $BC = 6$  cm. and  $AB = 10$  cm

, then find : 1 The length of  $\overline{AY}$

2 The perimeter of  $\triangle AXY$



$$= 4 \text{ cm} + 12 \text{ cm} =$$

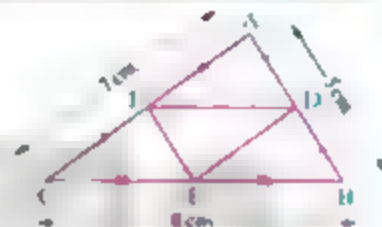
13 In the opposite figure :

$AB = 5$  cm.,  $BC = 8$  cm.,

$AC = 7$  cm., D, E and F are the midpoints of

$\overline{AB}$ ,  $\overline{BC}$  and  $\overline{CA}$  respectively.

Calculate the perimeter of :  $\triangle DEF$



$$= 10 \text{ cm} =$$

14 In the opposite figure :

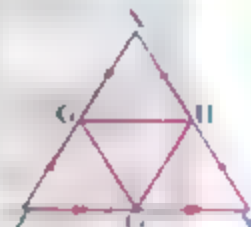
XYZ is a triangle in which :

H, O and G are the midpoints of  $\overline{XY}$ ,  $\overline{YZ}$

and  $\overline{ZX}$  respectively.

If the perimeter of  $\triangle HOG$  is 18 cm.,

then find the perimeter of :  $\triangle XYZ$



$$= 36 \text{ cm} =$$

15 ABC is a triangle, if X, Y and Z are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{AC}$  respectively.

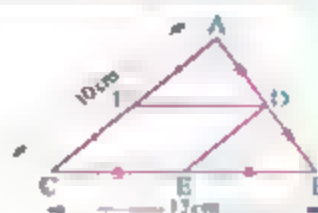
Prove that : The perimeter of  $\triangle XYZ = \frac{1}{2}$  that of  $\triangle ABC$

16 In the opposite figure :

ABC is a triangle in which D, E and F are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{CA}$  respectively,

$BC = 12$  cm.,  $AC = 10$  cm.

Find the perimeter of the figure DECF



$$= 22 \text{ cm} =$$



## Unit 3

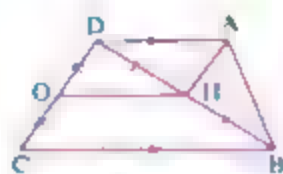
17 In the opposite figure :

$$\overline{AD} \parallel \overline{BC}, AD = \frac{1}{2} BC,$$

H is the midpoint of  $\overline{BD}$ ,

O is the midpoint of  $\overline{CD}$

Prove that : AHOD is a parallelogram.

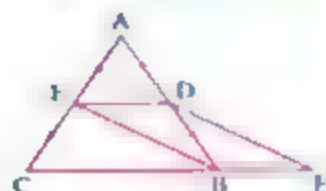


18 In the opposite figure :

D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively,

$F \in \overline{CB}$  where  $BF = \frac{1}{2} BC$

Prove that : BEDF is a parallelogram.



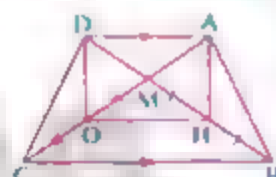
19 In the opposite figure :

ABCD is a trapezium in which  $\overline{AD} \parallel \overline{BC}$

and  $AD = \frac{1}{2} BC$  and  $\overline{AC} \cap \overline{DB} = \{M\}$

Let H and O be the midpoints of  $\overline{MB}$  and  $\overline{MC}$  respectively

Prove that : AHOD is a parallelogram.

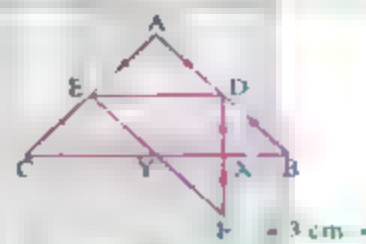


20 In the opposite figure :

D is the midpoint of  $\overline{AB}$ , E is the midpoint of  $\overline{AC}$ ,

$\overline{DE} \cap \overline{BC} = \{X\}$ ,  $DX = XF$ ,  $BC = 12$  cm.

Find the length of :  $\overline{XY}$



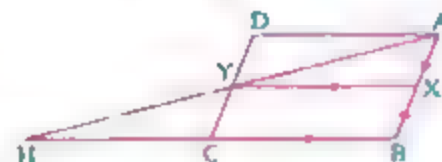
21 In the opposite figure :

ABCD is a parallelogram, X is the midpoint of  $\overline{AB}$

Draw  $\overline{XY} \parallel \overline{BC}$  to cut  $\overline{DC}$  at Y

Draw  $\overline{AY}$  to cut  $\overline{BC}$  at H

Prove that : C is the midpoint of  $\overline{BH}$



22 In the opposite figure :

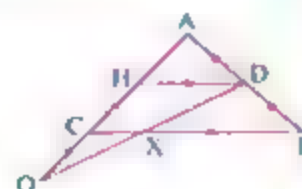
ABC is a triangle, D is the midpoint of  $\overline{AB}$ ,

$\overline{DH} \parallel \overline{BC}$ ,  $O \in \overline{AC}$  such that  $HC = CO$

Prove that :  $CO = \frac{1}{3} AO$

If we draw  $\overline{DO}$  to cut  $\overline{BC}$  at X,

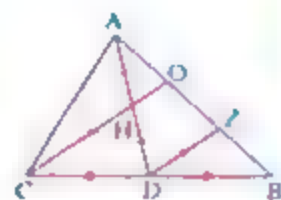
then prove that :  $OX = XD$



## Exercise 6

23 In the opposite figure :

ABC is a triangle , D is the midpoint of  $\overline{BC}$   
and H is the midpoint of  $\overline{AD}$   
Draw  $\overline{CH}$  to cut  $\overline{AB}$  at O,  
then draw  $\overline{DZ} \parallel \overline{CO}$  to cut  $\overline{AB}$  at Z  
Prove that :  $AO = OZ = ZB$



24 ABCD is a parallelogram M is the intersection point of its diagonals ,  
Draw  $\overline{CE} \parallel \overline{BD}$  to cut  $\overline{AB}$  at E and  $\overline{AD}$  at F  
Prove that : 1  $AB = BE$   
2  $AD = DF$

25 In the opposite figure :

ABCD is a trapezium in which  $\overline{AD} \parallel \overline{BC}$   
Let H be the midpoint of  $\overline{AB}$  ,  
 $\overline{HX} \parallel \overline{BC}$  and  $\overline{XY} \parallel \overline{DB}$   
Prove that : Y is the midpoint of  $\overline{BC}$



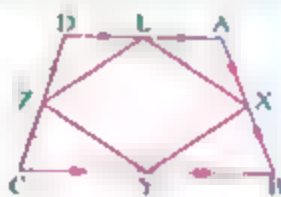
26 ABCD is a trapezium in which  $\overline{AD} \parallel \overline{BC}$  , L is the midpoint of  $\overline{AB}$  , draw  $\overline{EX} \parallel \overline{BC}$   
to cut  $\overline{DB}$  at X ,  $\overline{DC}$  at Y , and draw  $\overline{YZ} \parallel \overline{DB}$  to cut  $\overline{BC}$  at Z  
Prove that :  $XD = YZ$

27 ABC is a triangle in which  $AB = 9 \text{ cm}$  ,  $AC = 8 \text{ cm}$  ,  $D \in \overline{AB}$  ,  
 $E \in \overline{AB}$  such that  $AD = DE = EB$  and  $\overline{DX}$  ,  $\overline{EY}$  are drawn parallel to  $\overline{BC}$  and  
cutting  $\overline{AC}$  at X and Y respectively , where  $DX = 4 \text{ cm}$ .  
Calculate : The perimeter of the shape DEYX

4 cm

28 In the opposite figure :

ABCD is a quadrilateral in which X , Y , Z and L  
are the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  ,  $\overline{CD}$  and  $\overline{DA}$  respectively.  
Prove that : XYZL is a parallelogram.



29 ABC is a triangle in which  $AB = AC$  , X , Y and Z are the midpoints of  $\overline{AB}$   
 ,  $\overline{BC}$  and  $\overline{CA}$  respectively. Prove that : AXYZ is a rhombus.

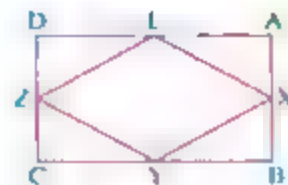
3

30 In the opposite figure :

ABCD is a rectangle and X, Y, Z and L are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$  and  $\overline{DA}$  respectively

Prove that :

- 1 XYZL is a rhombus.
- 2 The perimeter of the rhombus = 2 BD

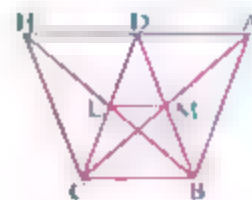


31 In the opposite figure :

ABCD and DBCH are two parallelograms having a common base  $\overline{BC}$  and on one side of  $\overline{BC}$

Let  $\overline{AC} \cap \overline{BD} = \{M\}$  and  $\overline{DC} \cap \overline{BH} = \{L\}$

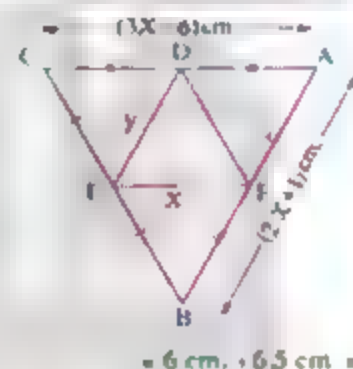
Prove that : 1  $\overline{ML} \parallel \overline{BC}$       2  $ML = \frac{1}{4} AH$



32 Connecting with algebra :

In the opposite figure :

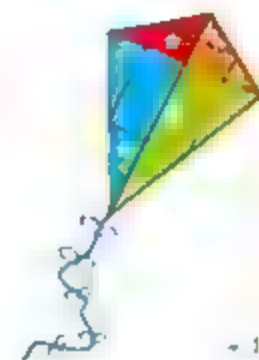
Find the value of each of : X and y



### Life Application

33 Sara wants to design a kite whose two diagonals are of length 64 cm. and 90 cm.

She wants to put a stripe to decorate the kite such that the stripe joins the midpoints of the sides of the kite. How long is the stripe ?





## Exercise 6



For excellent pupils

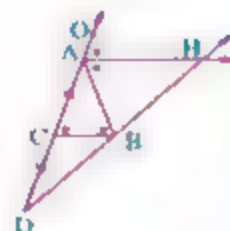
34 In the opposite figure :

ABC is a triangle in which  $m(\angle ABC) = m(\angle ACB)$

$D \in \overline{AC}$  such that  $AC = CD$  and  $O \in \overline{CA}$

Let  $\overline{AH}$  bisect  $\angle BAO$  such that :  $\overline{AH} \cap \overline{DB} = \{H\}$

Prove that :  $DB = BH$



35 ABCD is a quadrilateral in which  $\overline{AC} \perp \overline{BD}$  and X, Y, Z and L are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$  and  $\overline{DA}$  respectively.

Prove that : XYZL is a rectangle whose area equals  $\frac{1}{4} AC \times BD$

For the next year

ask for



EL-MOFASSER

in

Math, Science  
& English



17

# Pythagoras' theorem

From the school book

1 In each of the following figures, find the length of the unknown side :

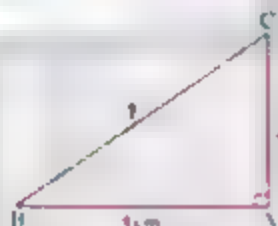


Fig. (1)



Fig. (2)

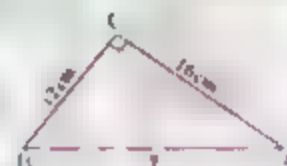


Fig. (3)

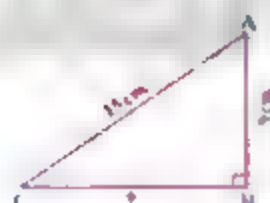


Fig. (4)



Fig. (5)

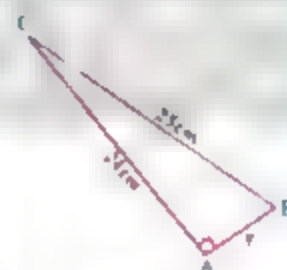


Fig. (6)

2 In the opposite figure :

ABCD is a square whose side length = 4 cm.

and  $E \in \overline{BC}$  where  $CE = 3$  cm.

Complete the following proof to find the length of  $\overline{DE}$

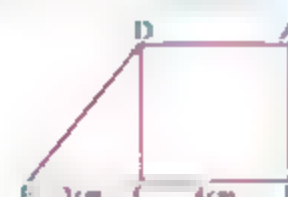
$\therefore$  ABCD is a square.

$\therefore DC =$       cm  $\wedge m(\angle DCB) =$        $^\circ$

$\therefore$  In  $\triangle DCE : (DE)^2 = ($        $)^2 + (CE)^2 = ($        $)^2 + (3)^2 =$       +      =     

$\therefore DE =$       cm.

(The req.)



## Exercise 7

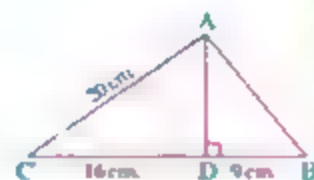
3 In the opposite figure :

$\overline{AD} \perp \overline{BC}$  ,  $BD = 9 \text{ cm}$  ,

$DC = 16 \text{ cm}$  , and  $AC = 20 \text{ cm}$  .

Find :

- 1  $AD$
- 2  $AB$
- 3 The area of  $\triangle ABC$



$$= 12 \text{ cm} \cdot 15 \text{ cm} \cdot 150 \text{ cm}^2$$

4 In the opposite figure :

$ABC$  is a triangle and  $\overline{AD} \perp \overline{BC}$

If  $AD = 24 \text{ cm}$  .

$AB = 26 \text{ cm}$  .

and  $AC = 30 \text{ cm}$  .

Find  $BC$  and calculate the area of  $\triangle ABC$



$$= 28 \text{ cm} \cdot 336 \text{ cm} =$$

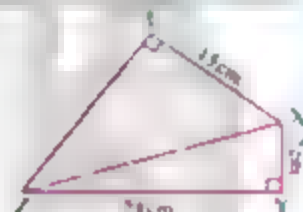
5 In the opposite figure :

$XYZL$  is a quadrilateral in which :

$m(\angle XYZ) = m(\angle XLZ) = 90^\circ$  ,

$XY = 7 \text{ cm}$  ,  $YZ = 24 \text{ cm}$  , and  $XL = 15 \text{ cm}$  .

Find : The length of each of  $\overline{XZ}$  and  $\overline{ZL}$  .



$$= 24 \text{ cm} \cdot 20 \text{ cm} =$$

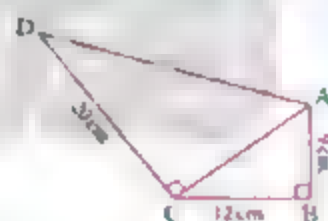
6 In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$  ,  $AB = 9 \text{ cm}$  ,

$BC = 12 \text{ cm}$  , and  $DC = 20 \text{ cm}$  .

Find :

- 1 The length of  $\overline{AC}$
- 2 The length of  $\overline{AD}$
- 3 The perimeter of the figure  $ABCD$
- 4 The area of the figure  $ABCD$



$$= 15 \text{ cm} \cdot 25 \text{ cm} \cdot 66 \text{ cm} \cdot 204 \text{ cm}^2 =$$

7 In the opposite figure :

$\triangle ABD$  is a right-angled triangle at  $B$

$AB = 8 \text{ cm}$  ,  $AD = 17 \text{ cm}$  .

and  $C \in \overline{BD}$  such that  $AC = 10 \text{ cm}$  .

Find : The length of each of  $\overline{CB}$  ,  $\overline{BD}$  and  $\overline{CD}$



$$= 6 \text{ cm} \cdot 15 \text{ cm} \cdot 9 \text{ cm} =$$



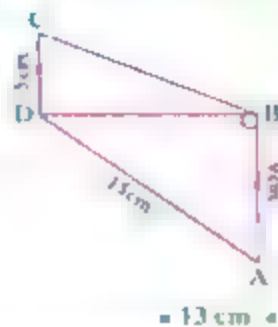
Unit 3

8 In the opposite figure :

$$m(\angle ABD) = 90^\circ, \overline{BA} \parallel \overline{CD}$$

$$AB = 9 \text{ cm.}, AD = 15 \text{ cm.}$$

$$\text{and } DC = 5 \text{ cm.}$$

Calculate the length of :  $\overline{BC}$ 

$$= 13 \text{ cm.}$$

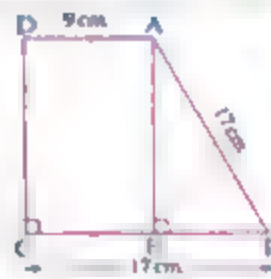
9 In the opposite figure :

ABCD is a trapezium,  $\overline{AD} \parallel \overline{BC}$ 

$$m(\angle DCB) = 90^\circ, \overline{AE} \perp \overline{BC}$$

$$\text{If } AB = BC = 17 \text{ cm. and } AD = 9 \text{ cm.}$$

find the length of  $\overline{DC}$  and calculate the area of the trapezium ABCD

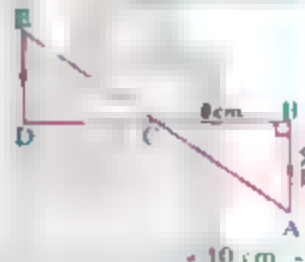


$$= 15 \text{ cm.}, 195 \text{ cm}^2$$

10 In the opposite figure :

$$\overline{BD} \cap \overline{AE} = \{C\}, \overline{AB} \parallel \overline{DE}$$

$$AB = 6 \text{ cm.}, BC = 8 \text{ cm.}$$

and C is the midpoint of  $\overline{BD}$ Calculate the length of :  $\overline{CE}$ 

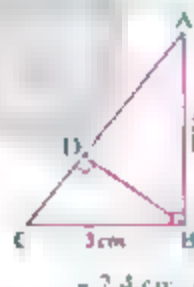
$$= 10 \text{ cm.}$$

11 In the opposite figure :

ABC is a right-angled triangle at B

$$\overline{BD} \perp \overline{AC}, AB = 4 \text{ cm.}$$

$$\text{and } BC = 3 \text{ cm.}$$

Calculate the length of :  $\overline{BD}$ 

$$= 2.4 \text{ cm.}$$

12 Complete the following :

- In the right-angled triangle, the area of the square on the hypotenuse equals
- If XYZ is a right-angled triangle at X,  $XY = 12 \text{ cm.}$  and  $XZ = 9 \text{ cm.}$ , then  $YZ = \dots\dots\dots \text{ cm.}$
- If ABC is a right-angled triangle at B,  $AB = 20 \text{ cm.}$  and  $AC = 25 \text{ cm.}$ , then  $BC = \dots\dots\dots \text{ cm.}$

## Exercise 7

- 4 In the opposite figure :

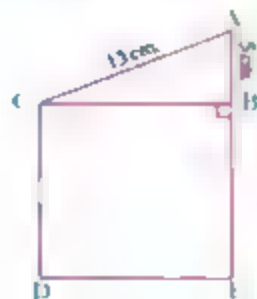
If  $m(\angle ABC) = 90^\circ$

,  $AB = 5$  cm.

and  $AC = 13$  cm.

, then the area of

the square  $BEDC = \dots\dots\dots \text{cm}^2$



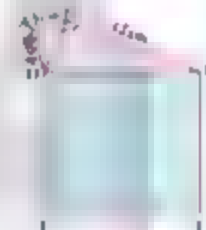
- 5 A rectangle is of length 8 cm and width 6 cm ,  
then the length of its diagonal equals  $\dots\dots\dots$  cm.

- 6 If the area of a rectangle equals  $60 \text{ cm}^2$  and its width is 5 cm ,  
then the length of its diagonal =  $\dots\dots\dots$  cm.

- 7 If  $\triangle ABC$  is right-angled at B ,  
then the side length of the shaded  
square =  $\dots\dots\dots$  cm.



- 8 If  $\triangle ABD$  is right-angled at A  
and  $\triangle BCD$  is right-angled at B ,  
then the area of the shaded  
square =  $\dots\dots\dots \text{cm}^2$



- 9 If the triangles  $ABC$  ,  $ACD$  and  $DCE$   
are right-angled at B , A and D respectively ,  
 $AB = BC = AD = DE = 1$  cm.  
, then the area of the shaded  
square =  $\dots\dots\dots \text{cm}^2$



- 13 Choose the correct answer from those given :

- 1 In the opposite figure :

Which of the following relations is true ?

(a)  $X = 4^2 + 3^2$  (b)  $X^2 = 4^2 - 3^2$

(c)  $X^2 + 9 = 16$  (d)  $X^2 = 25$



- 2 In the opposite figure :

Which of the following relations is true ?

(a)  $X + 3 + X = 15$  (b)  $X^2 + 3X = 108$

(c)  $(X + 3)^2 = 15 - X^2$  (d)  $X^2 + 6X + 9 = 225$

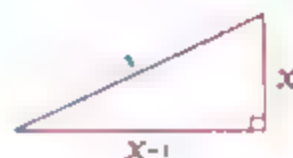


33

3 In the opposite figure :

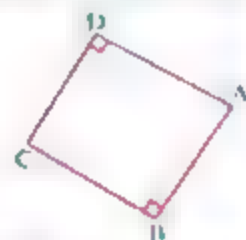
Which of the following relations is true ?

- (a)  $X^2 + (X-1)^2 = 5$       (b)  $X + (X-1) = 25$   
 (c)  $X^2 - X = 12$       (d)  $(X-1)^2 - X^2 = 25$

4 If ABCD is a square , then  $(AC)^2 =$ 

- (a) AB      (b)  $(AB)^2$       (c)  $2(AB)^2$       (d)  $4(AB)^2$

14 In the opposite figure :

If  $m(\angle B) = m(\angle D) = 90^\circ$ Prove that :  $(AB)^2 + (BC)^2 = (AD)^2 + (DC)^2$ 

15 In the opposite figure :

ABC is a triangle ,  $m(\angle B) = 90^\circ$ ,  $\overline{AE} \parallel \overline{BC}$  , If  $BC = 12$  cm ,  $AC = 20$  cm.,  $D \in \overline{AB}$  where  $BD = 9$  cm , and  $AE = 2 BC$ Find : The length of each of  $\overline{AD}$  and  $\overline{ED}$ 

$$= 7 \text{ cm} + 25 \text{ cm} =$$

## Life Applications

10 A window cleaner has a ladder which is 5 metres long.

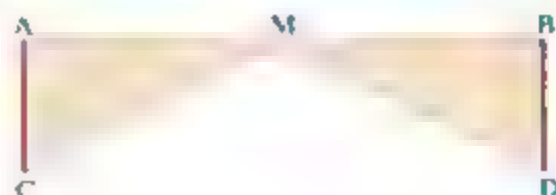
He places it so that it reaches a window till 4 metres from the ground.

How far is the wall from the foot of the ladder ?



$$= 3 \text{ m}$$

17 A wooden bridge  $\overline{AB}$  of length 15 m. is built horizontally on two vertical walls  $\overline{AC}$  and  $\overline{BD}$  resting on two supports  $\overline{MC}$  and  $\overline{MD}$

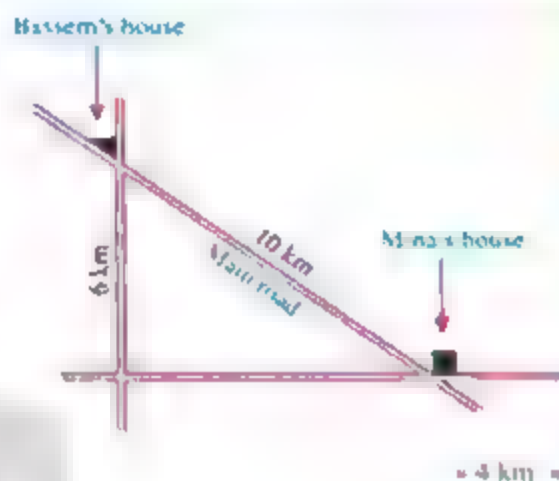
if  $AC = 4$  m. and  $AM = MB$ Calculate the length of the support  $\overline{MC}$ 

$$= 8.5 \text{ m}$$



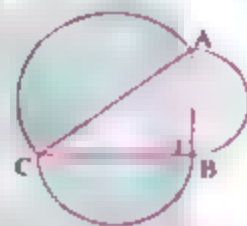
## Exercice 7

- 18 If Mina wants to go to the house of his friend Bassem.
- What is the distance saved if he takes the main road instead of the other two roads ?



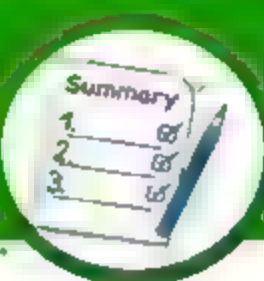
For excellent pupils

- 19 If  $\triangle ABC$  is right-angled at  $B$ ,  $D$  is the midpoint of  $\overline{BC}$ ,  
prove that :  $(AC)^2 - (AD)^2 = 3(BD)^2$
- 20 In the opposite figure :  
Prove that the sum of areas of the two semicircles  
drawn on the two sides of the right angle in a right-angled triangle  
equals the area of the semicircle drawn on the hypotenuse.  
[Given the area of the circle =  $\pi r^2$ ]



# Summary of the second part of unit 3

## From lesson 5 to lesson 7



- The sum of the measures of the interior angles of a triangle is  $180^\circ$
- The measure of an exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.
- If two angles of one triangle equal two angles of another triangle in measure , then the third angle of the first triangle is equal in measure to the third angle of the other triangle.
- If the sum of measures of two angles in a triangle equals  $90^\circ$  , then the third angle is right.
- If the sum of measures of two angles in a triangle is less than  $90^\circ$  , then the third angle is obtuse.
- If the sum of measures of two angles in a triangle is more than  $90^\circ$  , then the third angle is acute.
- If the measure of an angle in a triangle equals the sum of measures of the other two angles , then the triangle is right-angled.
- The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side
- The line segment joining the midpoints of two sides of a triangle is parallel to the third side.
- The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side.

### Pythagoras' theorem :

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.

### In the opposite figure :

If  $\Delta ABC$  is right-angled at B, then

- $(AC)^2 = (AB)^2 + (BC)^2$
- $(AB)^2 = (AC)^2 - (BC)^2$
- $(BC)^2 = (AC)^2 - (AB)^2$



## Exams on the second part of unit three from lesson (5) to lesson (7)



### Model 1

Answer the following questions :

1 Choose the correct answer from those given :

- 1 The measure of the exterior angle at any vertex of the equilateral triangle =  
 (a)  $60^\circ$  (b)  $120^\circ$  (c)  $150^\circ$  (d)  $30^\circ$
- 2 A rectangle of length 4 cm and width 3 cm , then the length of its diagonal =  
 (a) 25 cm. (b) 3 cm. (c) 4 cm. (d) 5 cm.
- 3 Any triangle has at least two ..... angles.  
 (a) acute (b) obtuse (c) right (d) reflex
- 4 If the measures of two angles in a triangle are  $30^\circ$  and  $50^\circ$  , then the triangle is  
 (a) acute-angled. (b) right-angled.  
 (c) obtuse-angled. (d) equilateral.
- 5 If  $\triangle ABC$  is right at B ,  $AB = 20$  cm. and  $AC = 25$  cm , then  $BC =$  ..... cm.  
 (a) 225 (b) 400 (c) 15 (d) 10
- 6 In  $\triangle ABC$  if  $m(\angle B) = m(\angle A) + m(\angle C)$  , then  $\angle B$  is  
 (a) acute. (b) right. (c) obtuse. (d) reflex.

2 Complete the following :

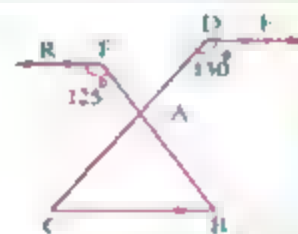
- 1 The measure of the exterior angle of a triangle equals .....
- 2 In  $\triangle ABC$  : if  $m(\angle B) > m(\angle A) + m(\angle C)$  , then  $\angle A$  is .....
- 3 The line segment joining the midpoints of two sides of a triangle is ..... to the third side.
- 4 In the right-angled triangle , the area of the square on the hypotenuse equals .....
- 5 The ray drawn from the midpoint of a side in a triangle parallel to another side .....

3 [a] In the opposite figure :

$$\overline{DE} \parallel \overline{FR} \parallel \overline{BC}$$

$$, m(\angle CDE) = 130^\circ \text{ and } m(\angle RFB) = 125^\circ$$

Calculate the measures of the angles of :  $\triangle ABC$

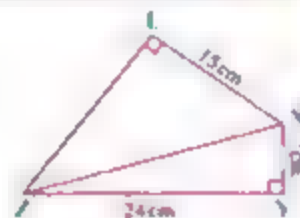
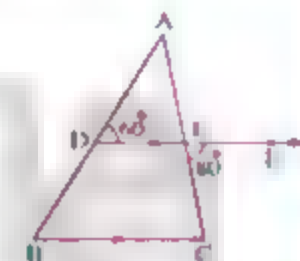
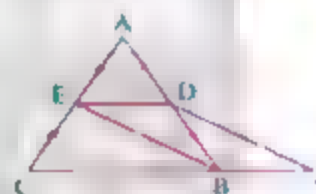




3

**(b) In the opposite figure :** $AB = 6 \text{ cm.}, BC = 9 \text{ cm.}$ 

$, AC = 8 \text{ cm.}, D, E \text{ and } F \text{ are the midpoints of } \overline{AB}, \overline{BC} \text{ and } \overline{CA} \text{ respectively.}$

Calculate the perimeter of :  $\triangle DEF$ **4 (a) In the opposite figure :** $XYZI$  is quadrilateral in which : $m(\angle XYZ) = m(\angle XLZ) = 90^\circ, XY = 7 \text{ cm.}$  $, YZ = 24 \text{ cm. and } XL = 15 \text{ cm.}$ Find the length of each of :  $\overline{XZ}$  and  $\overline{LZ}$ **(b) In the opposite figure :** $ABC$  is a triangle ,  $m(\angle ADE) = 60^\circ$  $, m(\angle FEC) = 80^\circ$  $, D \in \overline{AB}, \overline{DF} \parallel \overline{BC}$ and  $\overline{AC} \cap \overline{DF} = \{E\}$ Find : The measures of the interior angles of  $\triangle ABC$ **5 (a) In the opposite figure :** $D$  and  $E$  are midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively $, F \in \overline{CB}$  where  $BF = \frac{1}{2} BC$ Prove that :  $BEDF$  is a parallelogram.**(b) In the opposite figure :** $ABC$  is a triangle in which :  $\overline{AD} \perp \overline{BC}$ If  $AD = 24 \text{ cm.}, AB = 26 \text{ cm. and } AC = 30 \text{ cm.}$ 1 Find :  $BC$ 2 Calculate the area of :  $\triangle ABC$ **Model 2**

Answer the following questions :

**1 Choose the correct answer from the given ones :**1 In  $\triangle ABC$  ,  $m(\angle A) + m(\angle B) = 110^\circ$  $, \text{ then } m(\angle C) =$ (a)  $110^\circ$ (b)  $90^\circ$ (c)  $70^\circ$ (d)  $55^\circ$

- 2  $\triangle XYZ$  is right-angled at  $Y$ ,  $XY = 12$  cm,  $YZ = 5$  cm  
 then  $XZ = \dots\dots\dots$  cm.

(a) 169 (b) 25 (c) 17 (d) 13

- 3  $\triangle ABC$  is right-angled at  $B$ , if the measure of the exterior angle at  $A$  is  $120^\circ$   
 then  $m(\angle C) = \dots\dots\dots$

(a)  $60^\circ$  (b)  $90^\circ$  (c)  $120^\circ$  (d)  $30^\circ$

- 4 The length of the line segment joining the midpoints of two sides of a triangle is equal to  $\dots\dots\dots$  the length of the third side.

(a) twice (b) half (c) third (d) quarter

- 5 In  $\triangle ABC$ ,  $m(\angle A) = m(\angle C) - m(\angle B)$ ,  $m(\angle B) = 50^\circ$   
 then  $m(\angle A) = \dots\dots\dots$

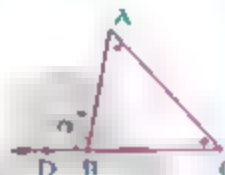
(a)  $40^\circ$  (b)  $90^\circ$  (c)  $50^\circ$  (d)  $45^\circ$

- 6 In the opposite figure :

$D \in \overline{CB}$ ,  $m(\angle ABD) = 100^\circ$

,  $m(\angle A) = m(\angle C)$ , then  $m(\angle C) = \dots\dots\dots$

(a)  $40^\circ$  (b)  $80^\circ$  (c)  $50^\circ$  (d)  $100^\circ$



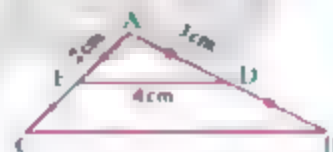
- 2 Complete the following :

- 1 If the measure of an angle of a triangle is greater than the sum of measures of the other two angles, then the triangle is  $\dots\dots\dots$

- 2 In  $\triangle ABC$ ,  $m(\angle B) = 90^\circ$ , then  $(AB)^2 = \dots\dots\dots$

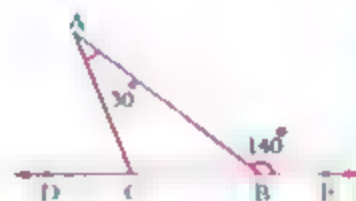
- 3 In the opposite figure :

The perimeter of  
 the figure DBCE =  $\dots\dots\dots$  cm.



- 4 In the opposite figure :

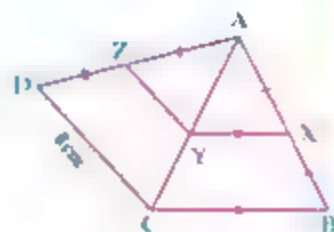
$m(\angle ACD) = \dots\dots\dots^\circ$



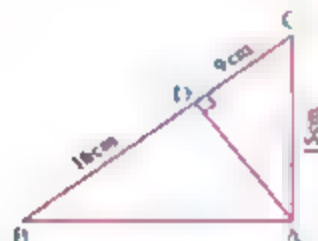
- 5 If the area of a rectangle is  $12$  cm<sup>2</sup> and its length is 4 cm, then its diagonal length =  $\dots\dots\dots$  cm.

Unit 3

3 [a] In the opposite figure :

X is the midpoint of  $\overline{AB}$ ,  $\overline{XY} \parallel \overline{BC}$  , Z is the midpoint of  $\overline{AD}$ ,  $CD = 8 \text{ cm}$ Find : The length of  $\overline{YZ}$ 

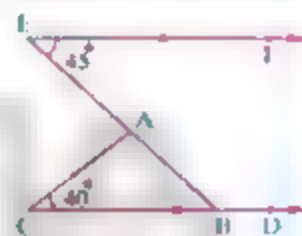
[b] In the opposite figure :

 $AC = 15 \text{ cm}$  ,  $DC = 9 \text{ cm}$  ,,  $DB = 16 \text{ cm}$  ,  $\overline{AD} \perp \overline{BC}$ Find with proof the length of each of :  $\overline{AB}$  ,  $\overline{AD}$ 

4 [a] In the opposite figure :

 $\overline{EF} \parallel \overline{CD}$  ,  $m(\angle E) = 45^\circ$ ,  $m(\angle C) = 40^\circ$ 

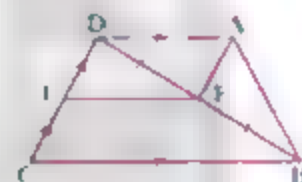
Find :

The measures of angles of  $\triangle ABC$  ,  $m(\angle ABD)$ 

[b] In the opposite figure :

 $\overline{AD} \parallel \overline{BC}$  ,  $AD = \frac{1}{2} BC$ , E is the midpoint of  $\overline{BD}$ , F is the midpoint of  $\overline{CD}$ 

Prove that : AEFD is a parallelogram.



5 [a] In the opposite figure :

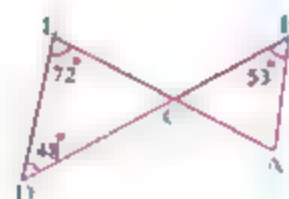
 $m(\angle B) = m(\angle ACD) = 90^\circ$ ,  $AB = 4 \text{ cm}$  ,  $BC = 3 \text{ cm}$  ,  $CD = 12 \text{ cm}$ 

Find : 1 The perimeter of the figure ABCD

2 The area of the figure ABCD



[b] In the opposite figure :

 $m(\angle B) = 53^\circ$  ,  $m(\angle E) = 72^\circ$ ,  $m(\angle D) = 48^\circ$ Find : 1  $m(\angle A)$ 2  $m(\angle BCE)$ 



8

## Geometric transformations

From the school book

- 1 Describe the type of the geometric transformation (reflection, translation or rotation) in each of the following :



- 2 Write below each shape the type of the geometric transformation (reflection, translation or rotation) :

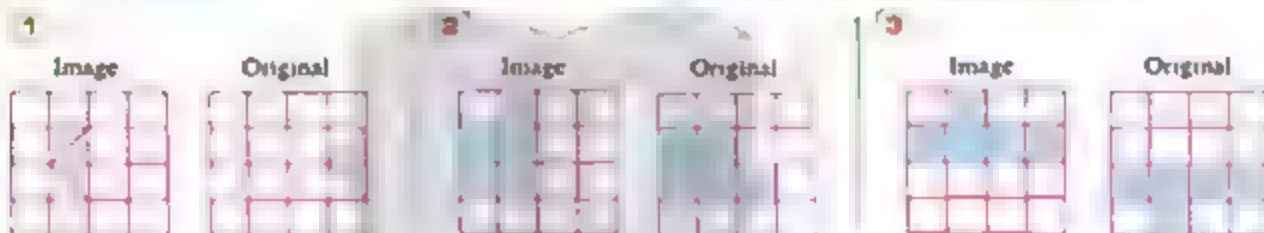


Unit 3

- 3 Write below each shape the type of the geometric transformation (reflection , translation or rotation) :



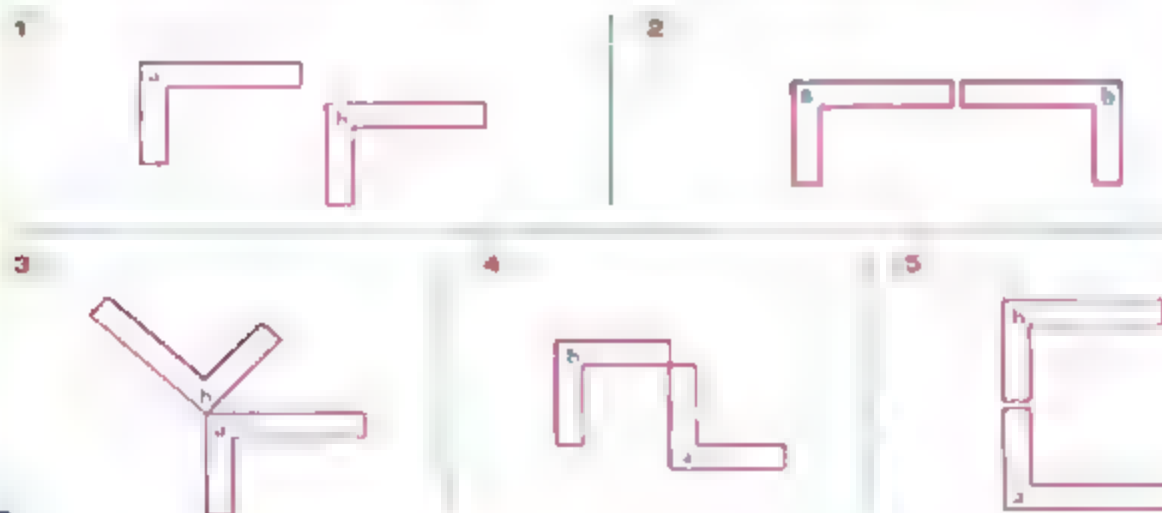
- 4 Write the type of the geometric transformation in each of the following shapes :



- 5 Describe the type of transformation in each of the following figures (reflection , translation , rotation) :



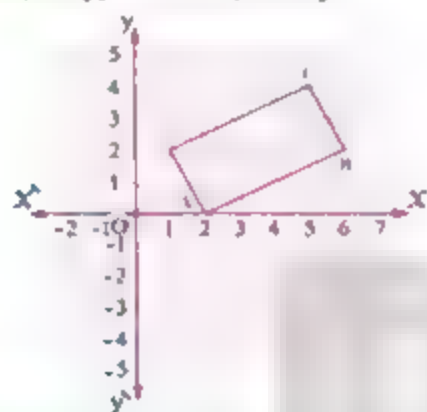
- 6 Figure b is the image of figure a by a geometric transformation. Identify each transformation as (a translation , a reflection or a rotation) :



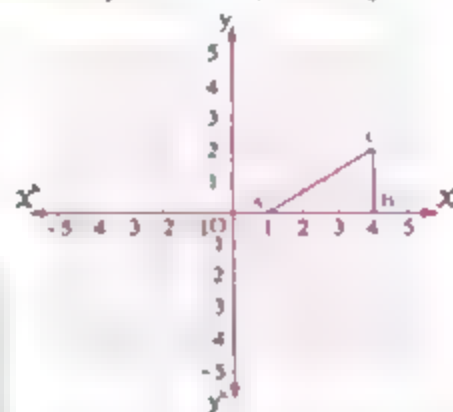
## Exercise

7 Draw the image of each figure according to the shown transformation, then describe each type :

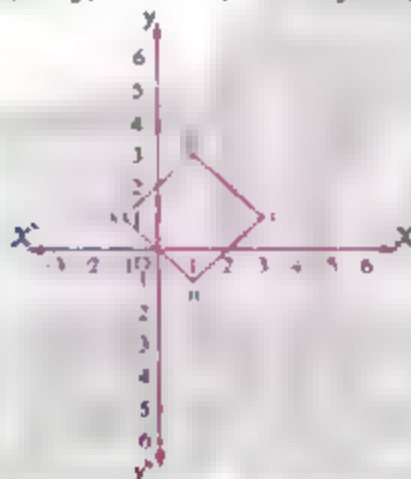
1.  $(X, y) \rightarrow (-X, y)$



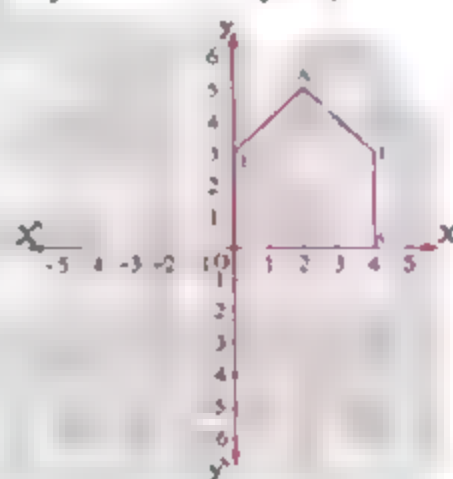
2.  $(X, y) \rightarrow (-X, -y)$



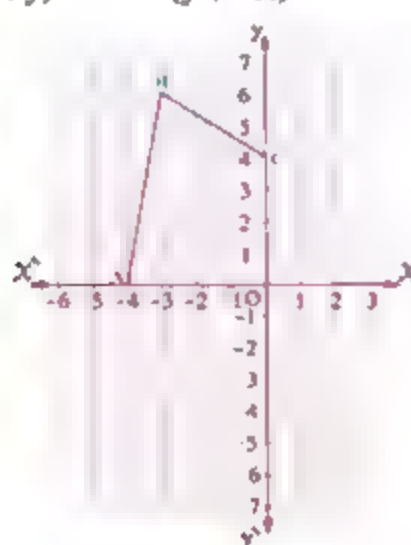
3.  $(X, y) \rightarrow (X + 2, y + 3)$



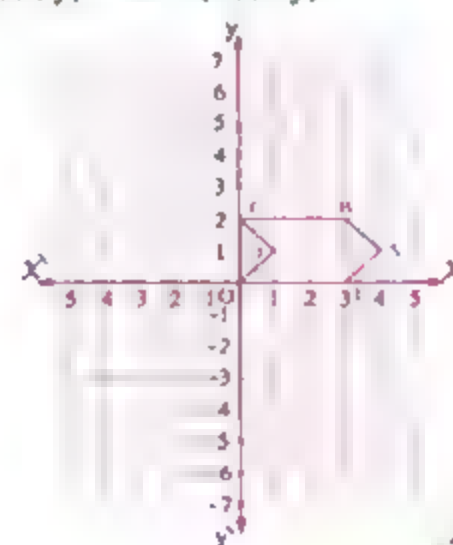
4.  $(X, y) \rightarrow (X, y - 3)$



5.  $(X, y) \rightarrow (y, -X)$



6.  $(X, y) \rightarrow (-X, y)$

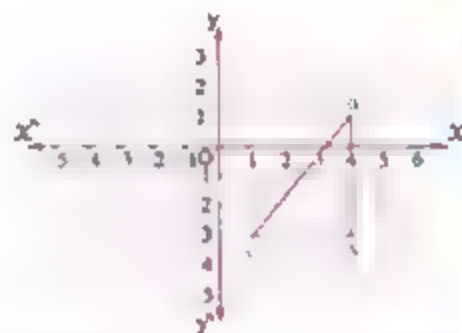




## Unit 3

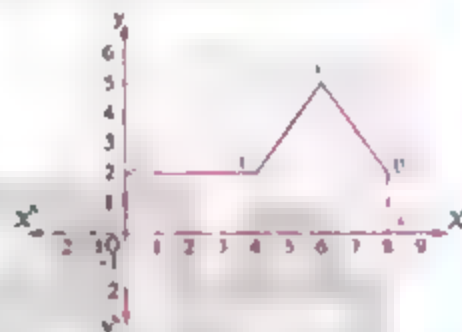
- 8 Map the image of  $\triangle ABC$  where  $A(4, -3)$ ,  $B(4, 1)$ ,  $C(1, -3)$  according to the following transformations then describe its type :

- 1  $(X, y) \longrightarrow (-X, y)$
- 2  $(X, y) \longrightarrow (-X, -y)$
- 3  $(X, y) \longrightarrow (X, y - 2)$
- 4  $(X, y) \longrightarrow (-y, X)$



- 9 Draw the image of the polygon ABCDEO according to each transformation and describe the type :

- 1  $(X, y) \longrightarrow (-X, y)$
- 2  $(X, y) \longrightarrow (X, y + 5)$
- 3  $(X, y) \longrightarrow (-X, -y)$
- 4  $(X, y) \longrightarrow (X - 5, y)$
- 5  $(X, y) \longrightarrow (X, -y)$



- 10 Draw the image of  $\triangle ABC$  where  $A(1, 2)$ ,  $B(3, 2)$  and  $C(3, 5)$  by the following transformations :

- 1  $(X, y) \longrightarrow (X, -y)$
- 2  $(X, y) \longrightarrow (X + 1, y - 3)$
- 3  $(X, y) \longrightarrow (-y, X)$

- 11 On a square lattice, draw  $\triangle ABO$  where  $A(3, 1)$ ,  $B(1, 3)$  and  $O$  is the origin point, and draw its images by the following transformations then describe its type :

- 1  $(X, y) \longrightarrow (X + 1, y - 2)$
- 2  $(X, y) \longrightarrow (X, -y)$
- 3  $(X, y) \longrightarrow (-y, X)$
- 4  $(X, y) \longrightarrow (-X, -y)$

- 12 On a square lattice, draw the quadrilateral ABCD where  $A(1, 1)$ ,  $B(4, 2)$ ,  $C(3, 4)$  and  $D(1, 4)$ , and draw its image by the following transformation, then describe its type :

- 1  $(X, y) \longrightarrow (y, -X)$
- 2  $(X, y) \longrightarrow (-X, y)$
- 3  $(X, y) \longrightarrow (X - 1, y + 1)$



For excellent pupils

- 13 Draw  $\triangle ABC$  whose image  $\triangle A'B'C'$  by the transformation  $(X, y) \longrightarrow (-y, X)$  where  $A'(1, -1)$ ,  $B'(3, 1)$  and  $C'(4, -1)$ , then describe the transformation type.



9

## Reflection in a straight line

From the school book

### First Problem on reflection in the plane:

- 1 Find the image of each of  $A$ ,  $\overline{AB}$  and  $\triangle ABC$  by reflection in the straight line  $L$ :

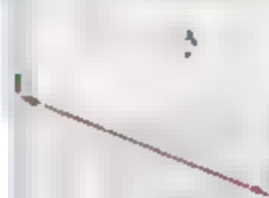


Fig. (1)

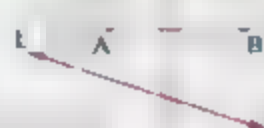


Fig. (2)

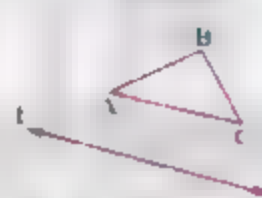


Fig. (3)

- 2 Copy the figures below in your notebook, then draw the images of  $\triangle DEF$  and the circle  $M$  by reflection in  $L$ :



33

3 Draw the axes of symmetry of each of the following figures if they are existed :

1



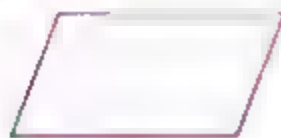
2



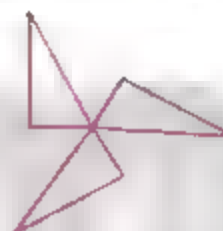
3



4



5



6



4 Draw the triangle ABC in which :  $AB = 6 \text{ cm}$ ,  $m(\angle A) = 90^\circ$  and  $m(\angle B) = 30^\circ$ , then draw its image by reflection in  $\overline{AB}$

5 Draw the image of  $\triangle ABC$  in which :  $AB = 3 \text{ cm}$ ,  $BC = 4 \text{ cm}$ , and  $AC = 5 \text{ cm}$ , by reflection in the straight line containing the shortest side.

6 Draw the image of  $\triangle XYZ$  in which :  $XY = 3 \text{ cm}$ ,  $YZ = 5 \text{ cm}$  and  $ZX = 7 \text{ cm}$ , by reflection in the straight line containing the longest side.

7 Draw  $\triangle ABC$  in which :  $AB = 3 \text{ cm}$ ,  $BC = 6 \text{ cm}$ , and  $m(\angle ABC) = 90^\circ$ , then find its image by reflection in the straight line  $L$  which is perpendicular to  $\overline{BC}$  at  $C$

8 Draw the rectangle ABCD in which :  $AB = 6 \text{ cm}$ , and  $CB = 4 \text{ cm}$ , then draw its image by reflection in  $\overline{AD}$ . Say the name of the resulting figure which consists of the rectangle and its image, then find its perimeter. = 32 cm.

9 Draw the image of the circle M whose radius length is 2 cm, by reflection in the straight line which is far from the centre by 1 cm.

10 Draw the circle N with radius length 2.5 cm, then draw its image by reflection in the straight line which is far from its centre by 2.5 cm.

11 Draw  $\triangle ABC$  where  $BC = 3 \text{ cm}$ ,  $AB = 4 \text{ cm}$ , and  $AC = 5 \text{ cm}$ . If the point D is the image of the point C by reflection in  $\overline{AB}$  Find :

1 The perimeter of  $\triangle ACD$

2 The area of  $\triangle ACD$

= 16 cm, = 12 cm<sup>2</sup>



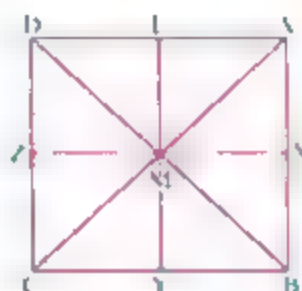
## Exercise 9

## 12 In the opposite figure :

ABCD is a square. M is the point of intersection of its diagonals X, Y, Z and L are the midpoints of its sides  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CD}$  and  $\overline{DA}$  respectively.

Complete the following :

- 1 The image of the point A by reflection in  $\overline{LY}$  is
- 2 The image of the  $\overline{AM}$  by reflection in  $\overline{XM}$  is
- 3 The image of the  $\Delta ALM$  by reflection in  $\overline{LY}$  is
- 4 The image of the  $\Delta ALM$  by reflection in  $\overline{XZ}$  is
- 5 The image of the  $\Delta ALM$  by reflection in  $\overline{AM}$  is
- 6 The image of the  $\Delta AMB$  by reflection in  $\overline{LY}$  is
- 7 The image of the  $\Delta AMB$  by reflection in  $\overline{XZ}$  is
- 8 The image of the square AXML by reflection in  $\overline{LY}$  is  
and by reflection in  $\overline{AM}$  is
- 9 The image of the square ABCD by reflection in  $\overline{LY}$  is
- 10  $\Delta MZD$  is the image of  $\Delta MZC$  by reflection in
- 11  $\Delta AXM$  is the image of  $\Delta CYM$  by reflection in

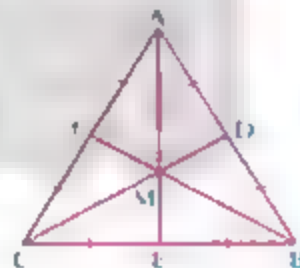


## 13 In the opposite figure :

$\Delta ABC$  is an equilateral triangle, where D, E and F are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{AC}$  respectively, and  $\overline{AE} \cap \overline{BF} \cap \overline{CD} = \{M\}$ .

Complete :

- 1 Axes of symmetry of  $\Delta ABC$  are
- 2  $\overline{AB}$  is the reflected image of  $\overline{AC}$  by reflection in
- 3 The reflected image of  $\overline{AF}$  by reflection in  $\overline{BF}$  is  
and the reflected image of  $\overline{CF}$  in  $\overline{AE}$  is
- 4 The reflected image of  $\Delta AMD$  by reflection in  $\overline{AE}$  is  
 $\therefore m(\angle AMD) = m(\angle \quad)$ , because reflection in a line reserves
- 5 The reflected image of  $\Delta AMB$  by reflection in  $\overline{AE}$  is
- 6  $\Delta BMC$  is the reflected image of by reflection in  $\overline{CD}$ , and the reflected image of by reflection in  $\overline{BF}$   
 $\therefore BM = AM$ , and  $CM = AM$ , because the reflection reserves



## 14 Complete the following :

- 1 The reflection in a plane reserves :

(a)

(b)

(c)

(d)

3

2 If the reflection in a straight line transforms the figure to itself then this straight line is called \_\_\_\_\_

3 The number of axes of symmetry of :

(a) The equilateral triangle is

(c) The scalene triangle is

(e) The rectangle is .....

(g) The square is

(i) The isosceles trapezium is

(b) The isosceles triangle is

(d) The parallelogram is

(f) The rhombus is

(h) The trapezium which is not isosceles is

(j) The circle is

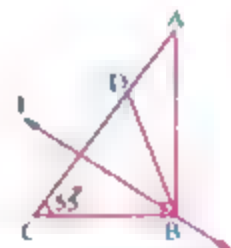
4 In the opposite figure :

If  $m(\angle ABC) = 90^\circ$

and  $m(\angle C) = 55^\circ$ , the straight line L

is the axis of symmetry of  $\triangle DBC$

, then  $m(\angle ABD) = \dots\dots\dots$



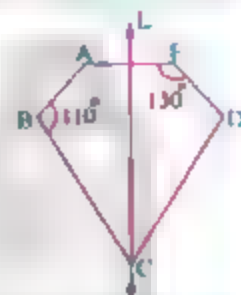
15 In the opposite figure :

If the straight line L

is the axis of symmetry

of the figure ABCDE,

calculate :  $m(\angle BCD)$



16 By using geometric instruments, draw the rectangle ABCD, where  $AB = 3$  cm, and  $BC = 4$  cm. locate  $\bar{A}$  as the reflected image of A by reflection in  $\bar{CD}$  and locate  $\bar{C}$  as the reflected image of C by reflection in  $\bar{AB}$

Prove that :

1  $m(\angle \bar{C}AC) = 2m(\angle CAB)$

2  $\bar{AC} \parallel \bar{A}\bar{C}$

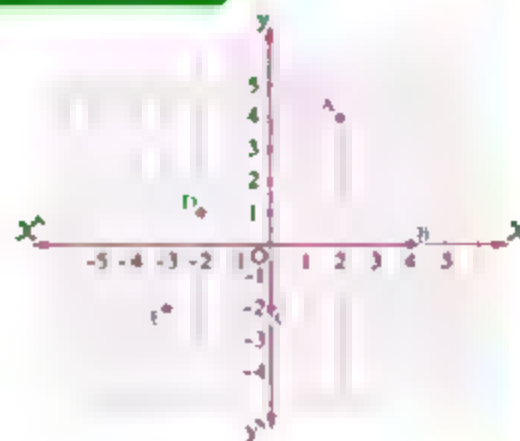
### Second Problems on reflection in the Cartesian plane:

1 In the opposite figure :

Write the coordinates of the image of each point by reflection in :

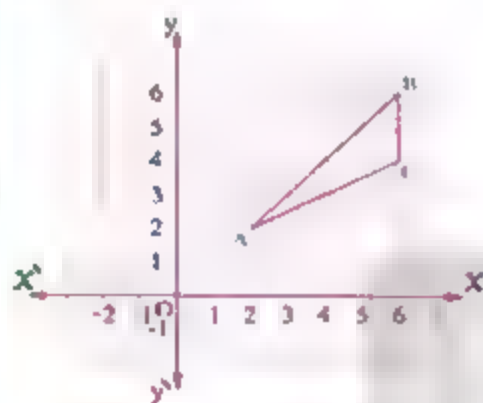
1 The X-axis

2 The y-axis



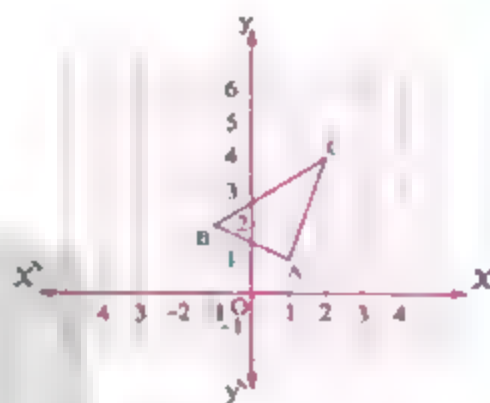
## Exercise 9

- 2 Copy each of the following figures on a lattice and draw the image of the figure by a geometric transformation as shown below each figure, then write the coordinates of each vertex of the figure.



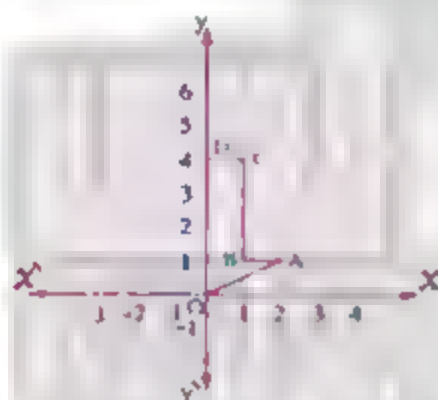
reflection in the  $x$ -axis

Fig. (1)



reflection in the  $y$ -axis

Fig. (2)



reflection in the  $y$ -axis

Fig. (3)



reflection in  $\overline{FG}$

Fig. (4)

- 3 Draw  $\overline{AB}$  where  $A(4, 3)$  and  $B(1, -2)$ , then draw its image by reflection in :
- The  $x$ -axis.
  - The  $y$ -axis.
- 4 If  $A(3, 1)$  and  $B(3, -2)$ , find  $\overline{DC}$  which is the image of  $\overline{AB}$  by reflection in the  $y$ -axis and name the figure  $ABCD$  and calculate its perimeter. → 8 length units
- 5 Find the image of  $\triangle ABC$  where  $A(-6, -1)$ ,  $B(-2, -1)$  and  $C(-5, -6)$  by reflection in the  $x$ -axis.



## Unit 3

- 6 Draw the image of  $\triangle OBC$  where  $O(0, 0)$ ,  $B(3, 0)$  and  $C(-1, 2)$  by reflection in the  $y$ -axis.
- 7 On a square lattice, draw  $\triangle ABC$  where  $A(2, 2)$ ,  $B(3, 4)$  and  $C(-3, 2)$ , then draw  $\triangle \hat{A}\hat{B}\hat{C}$  which is the image of  $\triangle ABC$  by reflection in the  $y$ -axis, then draw  $\triangle \hat{\hat{A}}\hat{\hat{B}}\hat{\hat{C}}$  which is the image of  $\triangle \hat{A}\hat{B}\hat{C}$  by reflection in the  $x$ -axis.
- 8 On a square lattice, draw the rectangle whose vertices are  $A(3, 2)$ ,  $B(8, 2)$ ,  $C(8, 6)$  and  $D(3, 6)$ , then draw its image by reflection in the  $y$ -axis.
- 9 Graph the square  $ABCD$  and its image by reflection in the  $x$ -axis, then compare the lengths of the sides and the area where  $A(0, 2)$ ,  $B(-5, 0)$ ,  $C(-3, -5)$  and  $D(2, -3)$ .
- 10  $ABCD$  is a rectangle in which:  $A(1, 1)$ ,  $B(1, 3)$  and  $C(-3, 3)$ . Determine the coordinates of  $D$  from the graph, then find the image of the rectangle  $ABCD$  by reflection in the  $x$ -axis.
- 11 Draw the image of the square  $ABCD$  where  $A(2, 3)$  and  $B(2, -1)$  by reflection in the  $y$ -axis. What do you notice?
- 12 Draw the image of the rectangle  $XYZL$  where  $X(2, 2)$  and  $Y(-3, 2)$  with width 3 units by reflection in the  $x$ -axis.
- 13 Complete the following table:

No.	The point	Its image by reflection in the $x$ -axis	Its image by reflection in the $y$ -axis
1	$(3, -2)$		
2		$(1, 2)$	
3			$(-2, 4)$
4	$(0, 5)$		
5		$(3, 0)$	
6			$(0, 0)$

14 Complete the following :

- 1 The image of the point  $(1, 3)$  by reflection in the  $X$ -axis is
- 2 The image of the point  $(-2, 5)$  by reflection in the  $y$ -axis
- 3 The image of the point  $(2, -3)$  by reflection in the \_\_\_\_\_ is  $(2, 3)$
- 4 The image of the point  $(-1, -4)$  by reflection in the \_\_\_\_\_ is  $(1, -4)$
- 5 The image of the point  $(0, 3)$  by reflection in the \_\_\_\_\_ is itself.
- 6 The image of the point  $(-5, 0)$  by reflection in the \_\_\_\_\_ is itself.
- 7 The image of the point  $(2, 1)$  by reflection in the  $X$ -axis followed by reflection in the  $y$ -axis is .....
- 8 The image of the point  $(2, -3)$  by reflection in the  $y$ -axis followed by reflection in the  $X$ -axis is ....
- 9 If  $\hat{A}(-2, 3)$  is the image of the point  $A(2, 3)$  by reflection in  $y$ -axis, then the image of the point  $\hat{A}$  by reflection in the  $y$ -axis is



15 Determine on a square lattice the points  $A(5, 4)$ ,  $B(5, 1)$ ,  $C(2, 1)$ ,  $\hat{A}(4, 5)$ ,  $\hat{B}(1, 5)$  and  $\hat{C}(1, 2)$

- 1 If  $\Delta \hat{A}\hat{B}\hat{C}$  is the image of  $\Delta ABC$  by reflection in the straight line  $L$ , draw this straight line.
- 2 If the figure  $ABB\hat{A}$  is the image of the figure  $CBB\hat{C}$  by reflection in the straight line  $M$ , draw this straight line.

16 If the geometric transformation  $(X, y) \rightarrow (y, X)$  is a reflection in a straight line  $L$ , draw on a square lattice the straight line  $L$



10

## Reflection in a point

From the school book

## First Problems on reflection in the plane:

1 Choose the correct answer from the given ones :

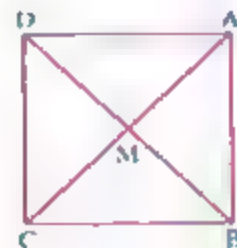
1 If  $\overline{A'B'}$  is the image of  $\overline{AB}$  by reflection in  $M$ , then  $\overline{A'B'}$  $\overline{AB}$ (a)  $>$ (b)  $<$ (c)  $=$ (d)  $\neq$ 

2 In the opposite figure :

The image of  $\overline{AB}$ by reflection in the point  $M$  is ..(a)  $\overline{AM}$ (b)  $\overline{BA}$ (c)  $\overline{BM}$ 

3 In the opposite figure :

ABCD is a square whose diagonals intersect at  $M$ . The image of  $\triangle ABM$  by reflection in  $M$  is  $\triangle$

(a)  $\triangle ADM$ (b)  $\triangle BCM$ (c)  $\triangle DCM$ (d)  $\triangle CDM$ 4 If  $\hat{A}$  is the image of  $A$  by reflection in  $M$  and if  $MA = 5$  cm, then  $A\hat{A} =$ 

(a) 5 cm.

(b) 7 cm.

(c) 10 cm.

(d) 15 cm



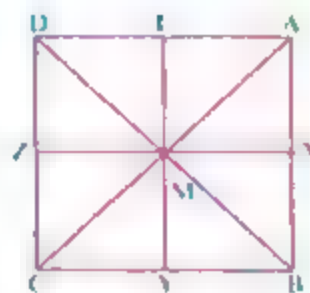
## Exercise 10

2 In the opposite figure :

ABCD is a square whose diagonals intersect at M  
X , Y , Z and L are the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  ,  $\overline{CD}$   
and  $\overline{DA}$  respectively

Complete the following :

- 1 The image of the point A by reflection in M is
- 2 The image of the point X by reflection in M is
- 3 The image of  $\overline{AL}$  by reflection in M is
- 4 The image of  $\overline{MZ}$  by reflection in M is
- 5 The image of  $\overline{BM}$  by reflection in M is
- 6 The image of  $\overline{AX}$  by reflection in X is
- 7 The image of  $\triangle ALM$  by reflection in M is
- 8 The image of  $\triangle BXM$  by reflection in M is
- 9 The image of  $\triangle AMB$  by reflection in M is
- 10 The image of the square AXML by reflection in M is



3 Using the geometric tools , draw the image of each of the following by reflection in A  
(Answer in the same page of the book) :

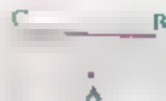


Fig. (1)



Fig. (2)



Fig. (3)

4 Draw  $\triangle ABC$  in which  $AB = BC = 4$  cm and  $AC = 5$  cm , then find its image by reflection in the point B

5 In each of the following figures , draw  $\triangle \hat{A}BC$  as the image of  $\triangle ABC$  by reflection in the point B and mention the name of the figure  $A\hat{C}AC$  giving reason

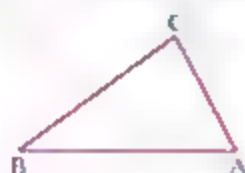


Fig. (1)

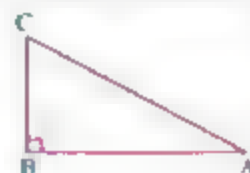


Fig. (2)

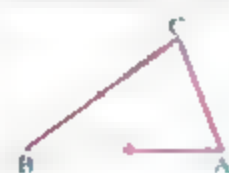


Fig. (3)



Fig. (4)

## Unit 3

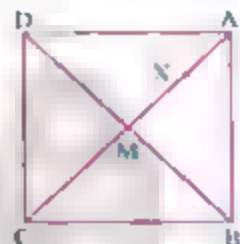
- 6 Draw  $\triangle ABC$  in which  $BC = 3$  cm,  $AB = 4$  cm, and  $m(\angle B) = 90^\circ$ , then draw  $\triangle \hat{A}BC$  as the image of  $\triangle ABC$  by reflection in  $C$ . Prove that the quadrilateral  $ABA\hat{B}$  is a parallelogram.
- 7 Draw the square  $ABCD$  whose side length is 5 cm, then draw its image by reflection in the point  $M$  where  $M$  is the point of intersection of its diagonals. What do you observe?
- 8  $ABC$  is a triangle,  $F$  is the midpoint of  $\overline{AC}$ . Draw  $D$  as the image of  $B$  by reflection in  $F$ . What is the type of the figure  $ABCD$  and what is the type of the triangle  $ABC$  required to transfer the figure  $ABCD$  to :
- 1 Rectangle.
  - 2 Rhombus.

## 9 In the opposite figure :

$ABCD$  is a square,  $M$  is the point of intersection of its diagonals and  $X \in \overline{AM}$ . Find  $Y$  as the image of  $X$  by reflection in  $M$  then,

Prove that : 1  $\triangle DAX \cong \triangle BCY$

2 The figure  $DXBY$  is a parallelogram.



## 10 In the opposite figure :

$ABCD$  is a rectangle,  $M$  is the point of intersection of its diagonals,  $X \in \overline{AD}$  and  $\overline{XM} \cap \overline{BC} = \{Y\}$

Prove that :

1  $Y$  is the reflected image of  $X$  in  $M$

2 The figure  $AXCY$  is a parallelogram.



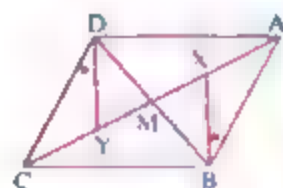
## 11 In the opposite figure :

$ABCD$  is a parallelogram,  $M$  is the point of intersection of its diagonals and  $X \in \overline{AC}$ ,  $Y \in \overline{AC}$  such that  $m(\angle ABX) = m(\angle CDY)$

Prove that :

1  $\triangle ABX$  is the image of  $\triangle CDY$  by reflection in  $M$

2 The figure  $XBYD$  is a parallelogram.



### Second Problems on reflection in the Cartesian plane:

1 Choose the correct answer from those given :

- 1 The image of the point  $(-3, 2)$  by reflection in the origin point is  
 (a)  $(3, 2)$  (b)  $(-3, -2)$  (c)  $(3, -2)$  (d)  $(-3, 2)$
- 2 The point  $(5, -2)$  is the image of the point \_\_\_\_\_ by reflection in the origin point.  
 (a)  $(5, -2)$  (b)  $(-5, -2)$  (c)  $(-5, 2)$  (d)  $(5, 2)$
- 3 The point whose image by reflection in the origin point is itself is  
 (a)  $(0, 1)$  (b)  $(1, 0)$  (c)  $(0, 0)$  (d)  $(-1, 0)$
- 4 The image of the point  $(3, -2)$  by reflection in the origin point followed by reflection in X-axis is  
 (a)  $(3, -2)$  (b)  $(-3, -2)$  (c)  $(-3, 2)$  (d)  $(3, 2)$

2 On a square lattice, draw  $\triangle ABC$  where  $A(3, 1)$ ,  $B(1, 4)$  and  $C(0, 0)$ , then find its image by reflection in the point C

3 In  $XY$ -coordinate plane, draw  $\triangle ABC$ , where  $A(-2, 4)$ ,  $B(5, 0)$  and  $C(3, -3)$ , then find the reflected image of  $\triangle ABC$  in the origin point.

4 On a square lattice, draw  $\triangle ABC$  where  $A(2, -2)$ ,  $B(3, 4)$  and  $C(-3, 2)$ , then map  $\triangle \hat{A}\hat{B}\hat{C}$  as the image of  $\triangle ABC$  by reflection in  $y$ -axis then map  $\triangle \hat{A}\hat{B}\hat{C}$  as the image of  $\triangle \hat{A}\hat{B}\hat{C}$  by reflection in  $X$ -axis. What is the image of  $\triangle ABC$  by reflection in the origin point? What do you deduce?

5 ABCD is a rectangle where  $A(2, 5)$ ,  $B(6, 5)$ ,  $C(6, 8)$  and  $D(2, 8)$ , then find the image of the rectangle ABCD by reflection in the origin point.



### For excellent pupils

6 In the opposite figure :

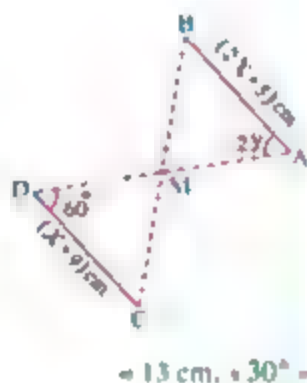
If  $\overline{CD}$  is the image of  $\overline{BA}$

by reflection in the point M and  $BA = (2x + 5)$  cm.

$CD = (x + 9)$  cm. ,  $m(\angle A) = 2y$  and  $m(\angle D) = 60^\circ$

Find :

- 1 The length of  $\overline{CD}$
- 2 The value of  $y$



$= 13 \text{ cm. } + 30^\circ =$





11

## Translation

From the school book

## First Problems on translation in the plane:

- 1 Using the geometric tools, draw the image of each of the following :  
By translation  $MN$  in the direction of  $\overrightarrow{MN}$  as shown in each case.

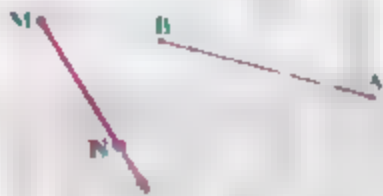


Fig. (1)

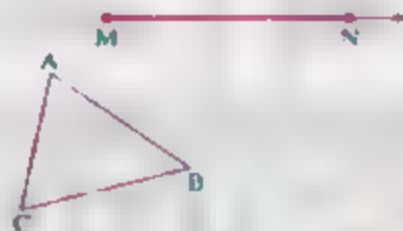
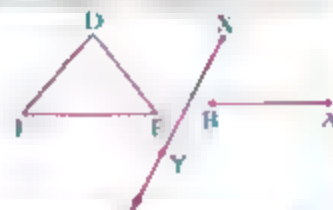


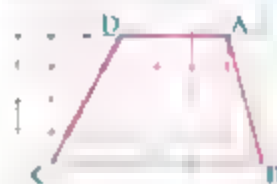
Fig. (2)

- 2 In the opposite figure :

Using the geometric tools, find the image of each of the opposite figures by the translation of displacement  $XY$  in the direction of  $\overrightarrow{XY}$




- 3 Using the grid, draw the image of the figure ABCD by the translation of 4 units in the direction of  $\overrightarrow{BC}$



- 4 [.] Draw a line segment  $\overline{AB}$  where  $AB = 5$  cm, then draw the image of  $\overline{AB}$  by a translation of magnitude of 8 cm. in the direction of  $\overrightarrow{AB}$

## EXERCISE 11

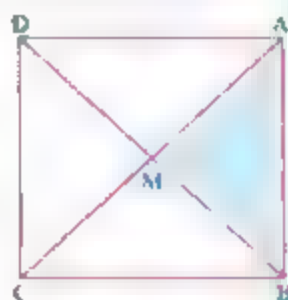
- 5 Using the geometric instruments, draw the square ABCD whose side length is 4 cm, then draw its image by translation of magnitude of 4 cm, in the direction of  $\overrightarrow{AB}$
- 6  Draw  $\triangle ABC$  in which  $AB = 4$  cm,  $BC = 6$  cm, and  $CA = 5$  cm, then draw the image of  $\triangle ABC$  by a translation of magnitude of 3 cm in the direction of  $\overrightarrow{CB}$

- 7  In the opposite figure :

ABCD is a square whose side length is 4 cm

M is the point of intersection of its diagonals. Draw :

- 1 The image of  $\triangle MAB$  by the translation of distance 2 cm, in the direction of  $\overrightarrow{AD}$
- 2 The image of  $\triangle AMB$  by the translation AM in the direction of  $\overrightarrow{AM}$

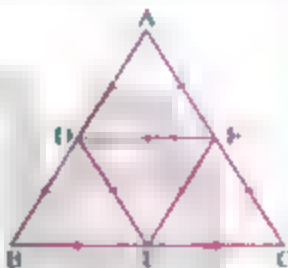


- 8  In the opposite figure :

The triangles ADF, BDE, DEF and EFC are congruent.

Complete :

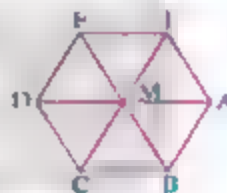
- 1 The image of  $\triangle ADF$  by a translation of magnitude of AD in the direction of  $\overrightarrow{AD}$  is
- 2  $\triangle FEC$  is the image of  $\triangle DBE$  by a translation of magnitude ..... in the direction of .....



- 9 In the opposite figure :

ABCDEF is a regular hexagon. Complete the following :

- 1 The image of the point D by translation DM in the direction of  $\overrightarrow{DM}$  is ...
- 2 The image of  $\overline{AF}$  by translation ED in the direction of  $\overrightarrow{ED}$  is
- 3 The image of  $\triangle MCD$  by translation EF in the direction of  $\overrightarrow{EF}$  is
- 4 The translation which makes  $\triangle DME$  the image of  $\triangle MAF$  is

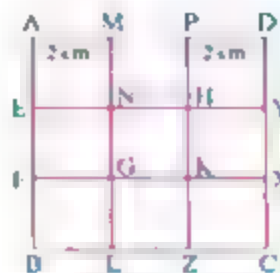


- 10  In the opposite figure :

ABCD is a square and all the interior squares are congruent.

Complete :

- 1 The image of  $\overline{AE}$  by a translation of magnitude of 2 cm, in the direction of  $\overrightarrow{GK}$  is
- 2 The image of the square AENM by a translation of magnitude of 4 cm, in the direction of  $\overrightarrow{PK}$  is
- 3 The square MNHP is the image of the square GLZK by a translation of magnitude ..... in the direction of



## Unit 3

- 11  $\Delta ABC$  is right-angled at B where  $AB = 3$  cm. and  $BC = 4$  cm. If  $\Delta \hat{A}\hat{B}\hat{C}$  is the image of  $\Delta ABC$  by translation of a distance 3 cm in the direction of  $\overline{CB}$   
Prove that : The figure  $A\hat{A}\hat{C}\hat{C}$  is a parallelogram.
- 12 Draw  $\Delta ABC$  which is right-angled at B , in which  $AB = BC = 3$  cm. , then find the image of  $\Delta ABC$  by translation of a distance 3 cm in the direction of  $\overline{AB}$  , then  
Prove that : The figure  $B\hat{B}\hat{C}\hat{C}$  is a square.
- 13 ABCD is a rectangle , where  $E \in \overline{AD}$  Find the translated image of  $\Delta ABE$  by translation of a magnitude DA in the direction of  $\overline{AD}$  If  $\hat{E}$  is the image of E by the same translation.  
Prove that : The figure  $BC\hat{E}\hat{E}$  is a parallelogram.
- 14 ABCD is a parallelogram ,  $\overline{BE} \perp \overline{AD}$  cutting it at E Find  $\Delta \hat{A}\hat{B}\hat{D}$  as the image of  $\Delta ABE$  by translation of a distance ED in the direction of  $\overline{AD}$  , then  
Prove that : The figure  $EB\hat{B}\hat{D}$  is a rectangle.

## Second &gt; Problems on translation in the Cartesian plane:

## 1 Complete the following :

- The image of the point  $(2, 5)$  by translation  $(X, y) \rightarrow (X + 2, y + 1)$  is
- The image of the point  $(3, 2)$  by translation  $(X, y) \rightarrow (X + 3, y - 2)$  is
- The image of the point  $(-5, 4)$  by translation  $(X, y) \rightarrow (X + 4, y - 5)$  is
- The image of the point  $(-2, -5)$  by translation  $(X, y) \rightarrow (X - 2, y)$  is
- The image of the point  $(3, -2)$  by translation  $(X, y) \rightarrow (X, y + 3)$  is

## 2 Choose the correct answer from those given :

- The image of the point  $(-1, 2)$  by translation of magnitude of 3 units in the positive direction of the X-axis is  
(a)  $(-1, 5)$  (b)  $(2, 2)$  (c)  $(-2, 2)$  (d)  $(-1, 3)$
- The image of the point  $(-3, 4)$  by translation of magnitude of 4 units in the negative direction of the y-axis is  
(a)  $(-3, 0)$  (b)  $(-7, 4)$  (c)  $(-3, 8)$  (d)  $(-1, 4)$
- If  $\hat{A}(3, -3)$  is the image of A by translation  $(X, y) \rightarrow (X - 1, y - 4)$  , then the point A is  
(a)  $(2, -7)$  (b)  $(4, 1)$  (c)  $(-4, -1)$  (d)  $(2, 1)$
- The image of the point  $(-1, 4)$  by the translation  $(3, -2)$  followed by reflection in the X-axis is  
(a)  $(2, 2)$  (b)  $(-2, 2)$  (c)  $(-2, -2)$  (d)  $(2, -2)$



## Exercise 11

- 5 If the point  $(a, -1)$  is the image of  $(2, 4)$  by the translation

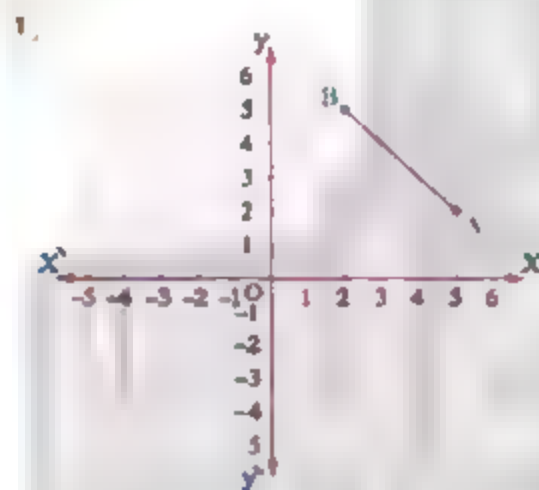
$(X, y) \longrightarrow (X + 1, y - b)$ , then  $(a, b)$  is

- (a)  $(3, 3)$  (b)  $(1, 3)$  (c)  $(3, 5)$  (d)  $(1, -5)$

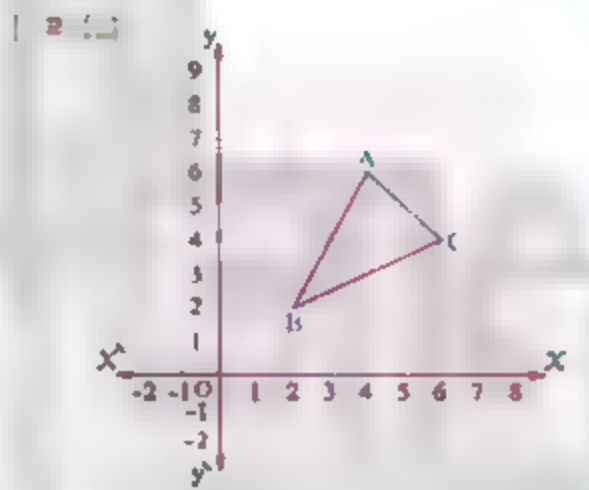
- 6 If  $\hat{A}$  is the image of the point  $A(2, 3)$  by reflection in the  $y$ -axis, then  $A$  is the image of  $\hat{A}$  by the translation

- (a)  $(X, y) \longrightarrow (X + 4, y)$  (b)  $(X, y) \longrightarrow (X, y + 6)$   
(c)  $(X, y) \longrightarrow (X - 4, y)$  (d)  $(X, y) \longrightarrow (X, y - 6)$

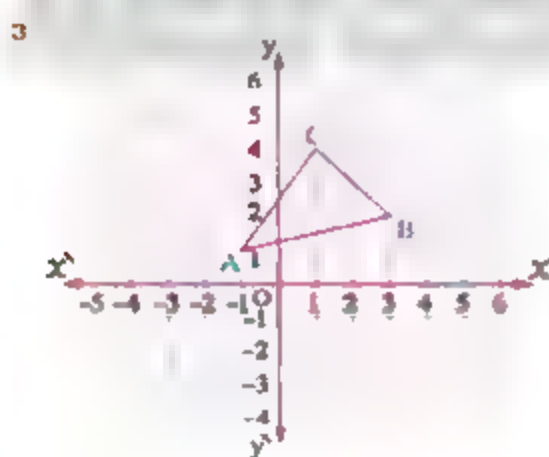
- 3 Find the image of each of the following figures by the translation shown under each figure :



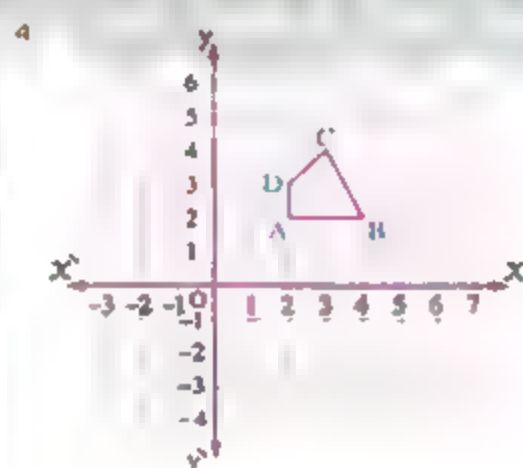
$$(X, y) \longrightarrow (X - 3, y - 4)$$



$$(X, y) \longrightarrow (X + 2, y + 3)$$



$$(X, y) \longrightarrow (X + 2, y)$$



$$(X, y) \longrightarrow (X + 3, y - 2)$$

## Unit 3

- 4 Copy the graph, then draw the image of the parallelogram ABCD under each of the following translations :

1  $(X, y) \longrightarrow (X + 5, y + 2)$

2  $(X, y) \longrightarrow (X - 8, y - 1)$

3  $(X, y) \longrightarrow (X + 2, y - 4)$

4  $(X, y) \longrightarrow (X - 4, y + 2)$



- 5 Find the image of each of the following points by the translation

$(X, y) \longrightarrow (X + 2, y - 3)$  followed by the translation  $(X, y) \longrightarrow (X - 3, y + 1)$

1 A (4, -2)

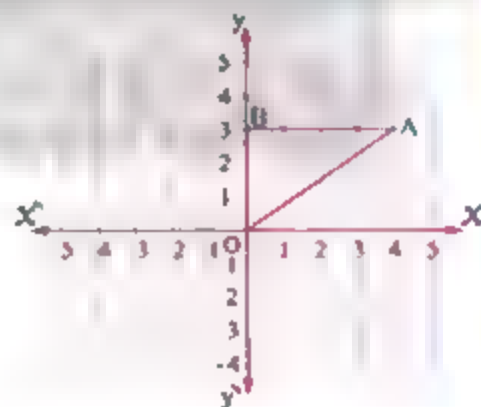
2 B (-1, 3)

3 C (0, 2)

- 6 On a square lattice, draw  $\overline{AB}$  where A (2, 3) and B (4, 1), then draw the image of  $\overline{AB}$  by the translation  $(X, y) \longrightarrow (X + 3, y + 2)$

- 7 Using the square lattice, draw  $\triangle OBC$  where O is the origin point, B (3, 0), C (0, 2), then draw its image by the translation  $(X, y) \longrightarrow (X - 4, y + 1)$

- 8 Find the image of  $\triangle AOB$  by the translation of magnitude = AO and in the direction of  $\overline{AO}$



- 9 Using the lattice, find the image of each of the following points by the translation of LM in the direction of  $\overline{LM}$  where L (1, 3) and M (4, 5)

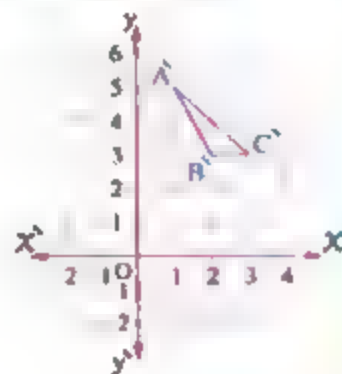
1 B (-2, 3)

2 C (5, 4)

3 D (3, 0)

## Exercise 11

- 10 On a square lattice, draw  $\triangle ABC$  where  $A(2, 1)$ ,  $B(1, -1)$  and  $C(0, 1)$ , then draw its image by a translation of  $2\overline{AB}$  in the direction of  $\overline{AB}$ .
- 11 A square has vertices  $A(1, 1)$ ,  $B(4, 2)$ ,  $C(3, 5)$  and  $D(0, 4)$ .
- Graph the square and its image under the translation which maps vertex  $A$  onto vertex  $B$ .
  - Write the mapping rule for the translation.
- 12 Use the translation:  $(x, y) \longrightarrow (x + 2, y + 3)$  to locate the point whose image is  $(2, 3)$ .
- 13 If the image of the point  $A(1, 1)$  by translation in the Cartesian plane is  $\hat{A}(2, 2)$ , find the images of the points  $O(0, 0)$ ,  $B(-1, 3)$  and  $C(-3, 5)$  by the same translation.
- 14 If  $A(-3, 1)$  and  $B(1, -2)$ , write the mapping rule of the translation that makes  $B$  the image of  $A$ .
- 15 If  $A(3, 2)$ ,  $B(5, 1)$ , find:
- $\hat{C}$  which is the image of  $C(1, -1)$  under translation of  $AB$  in the direction of  $\overline{AB}$ .
  - $\hat{D}$  whose image is  $\hat{D}(2, 1)$  under translation of  $AB$  in the direction of  $\overline{AB}$ .
- 16 The point  $\hat{A}(3, -3)$  is the image of the point  $A$  by the translation  $(x, y) \longrightarrow (x - 1, y - 4)$ . Locate  $A$  then by the same translation, draw the image of  $\triangle ABC$  where  $B(5, 0)$  and  $C(-1, -2)$ .
- 17 In the opposite figure:
- Copy the graph, then draw the triangle  $ABC$  whose image is  $\triangle \hat{A}\hat{B}\hat{C}$  by the translation  $(x, y) \longrightarrow (x + 2, y + 3)$ .



- 18 If  $A(-2, 1)$  and  $B(1, 3)$ , find on a square lattice the image of  $\overline{AB}$  by reflection in  $X$ -axis followed by the translation  $(x, y) \longrightarrow (x + 4, y)$ .



## Unit 3

18 State whether the graph shows a reflection or a translation :

- 1 Name the line of reflection
- 2 Describe the translation.

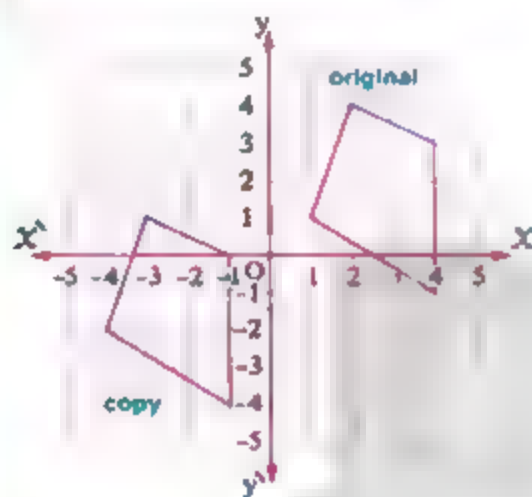


Fig. (1)

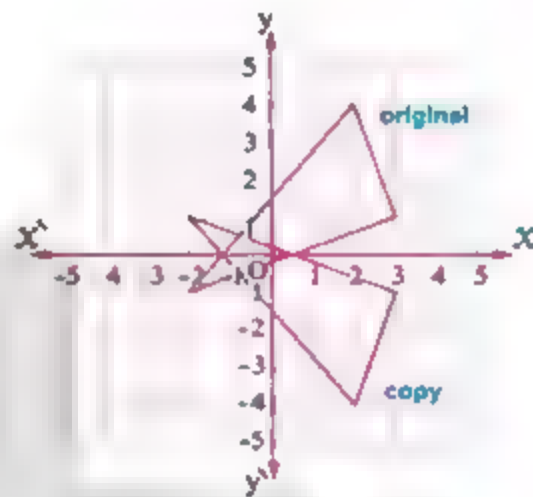


Fig. (2)

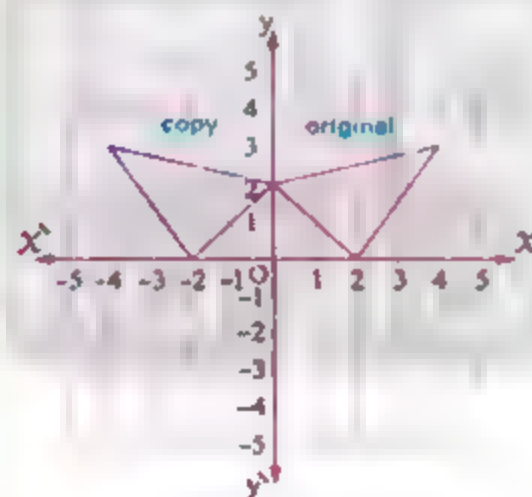


Fig. (3)

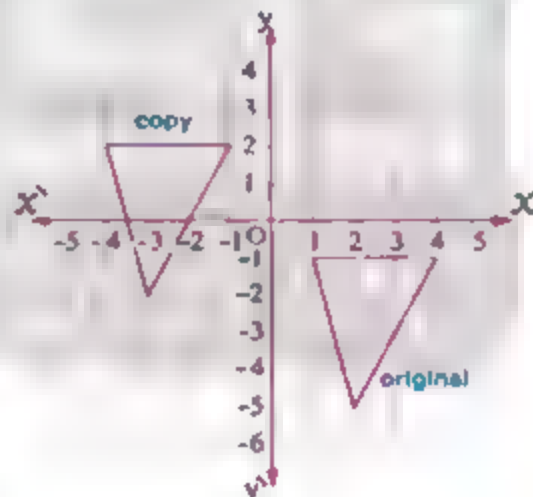


Fig. (4)



For excellent pupils

- 20 Draw  $\triangle ABC$  on a square lattice where  $A(4, 4)$ ,  $B(4, 2)$  and  $C(1, 2)$ , then draw its image by the translation of magnitude 3 AB in the direction of  $\overrightarrow{AB}$
- 21 If  $A(2, 1)$  is the image of  $B$  by reflection in  $X$ -axis followed by reflection in  $y$ -axis, state the translation which makes  $A$  the image of the point  $B$



12

## Rotation

From the school book

## First Problems orientation in the plane :

- 1 Draw the image of the point A ,  $\overline{AB}$  and  $\triangle ABC$  by the required rotation :

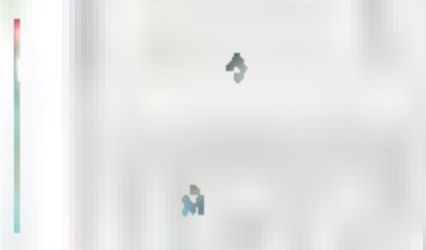


Fig. (1)  
 $R(M, 30^\circ)$



Fig. (2)  
 $R(M, -60^\circ)$



Fig. (3)  
 $R(M, 60^\circ)$

- 2 Use the geometric tools to draw  $\overline{AB}$  with length 3 cm , then draw its image by rotation  $R(B, 135^\circ)$
- 3 Draw the equilateral triangle ABC with side length 6 cm. Draw the image of the triangle ABC by rotation  $R(A, 60^\circ)$
- 4 Draw the triangle ABC in which  $AB = 5$  cm ,  $BC = 6$  cm and  $CA = 7$  cm , then draw the image of  $\triangle ABC$  by rotation :
- $R(A, 180^\circ)$        $R(A, 360^\circ)$

## Unit 3

5 Draw the triangle XYZ in which  $XY = XZ = 3$  cm. and  $YZ = 4$  cm. , then draw the image of  $\Delta XYZ$  in each of the two cases :

- 1 By rotation about X with an angle of measure  $90^\circ$
- 2 By rotation about X with an angle of measure  $270^\circ$

6 Draw  $\Delta ABC$  in which  $AB = 5$  cm. ,  $AC = 3$  cm. ,  $m(\angle A) = 40^\circ$  , then draw  $\hat{C}$  the image of C by rotation  $R(A, +40^\circ)$  ,  $\hat{B}$  the image of B by rotation  $R(A, -40^\circ)$

7 Draw the square ABCD with side length 5 cm Draw the image of the square ABCD :

- 1 By rotation  $R(B, 90^\circ)$
- 2 By rotation  $R(A, 180^\circ)$

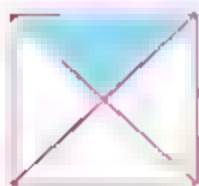
8 Using the geometric tools , draw the square ABCD with side length 4 cm. , then draw its image by rotation about its centre (The point of diagonals intersection) with an angle of measure  $90^\circ$

9 Draw the rectangle ABCD in which  $BC = 6$  cm. ,  $AB = 4$  cm. Draw the image of the rectangle ABCD :

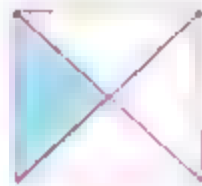
- 1 By rotation  $R(A, 90^\circ)$
- 2 By rotation  $R(M, 180^\circ)$  where M is the point of intersection of its diagonals

10 Choose the correct answer from those given :

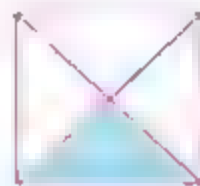
- 1 Which of these figures represents the rotation of the opposite square about its centre with an angle of measure  $270^\circ$  ?



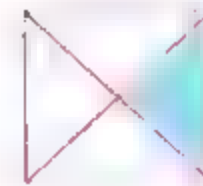
(a)



(b)



(c)

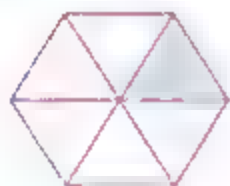


(d)

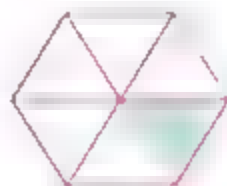


## Exercise 12

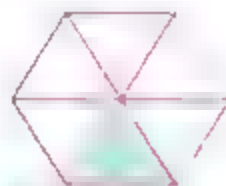
- 2 Which of these figures represents the rotation of the opposite regular hexagon about its centre with an angle of measure  $(-120^\circ)$  ?



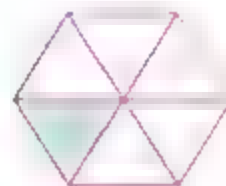
(a)



(b)



(c)



(d)

- 3 In the opposite figure :

If B is the midpoint of  $\overline{AC}$ , then the image of  $\overline{AC}$  by rotation about B with an angle of measure  $180^\circ$  is

(a)  $\overline{AC}$ (b)  $\overline{AB}$ (c)  $\overline{CA}$ (d)  $\overline{CB}$ 

- 4 In the opposite figure :

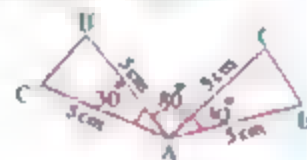
$\overline{CD}$  is the image of  $\overline{AB}$  under a rotation about M

and the measure of its angle is .....

(a)  $75^\circ$ (b)  $30^\circ$ (c)  $-30^\circ$ (d)  $150^\circ$ 

- 5 In the opposite figure :

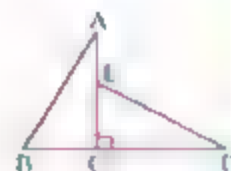
$\triangle A'B'C'$  is the image of  $\triangle ABC$  by a rotation about A with an angle of measure

(a)  $-110^\circ$ (b)  $80^\circ$ (c)  $110^\circ$ (d)  $140^\circ$ 

- 6 In the opposite figure :

$\triangle ABC$  is the image of  $\triangle DEC$  which is right-angled

at C by rotation about C with an angle of measure

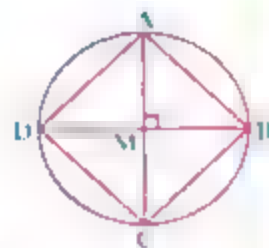
(a)  $90^\circ$ (b)  $-90^\circ$ (c)  $180^\circ$ (d)  $360^\circ$

## Unit 3

## 11 In the opposite figure :

The radius length of circle M is 3 cm.

$\overline{AC}$  and  $\overline{BD}$  are two perpendicular diameters in it.



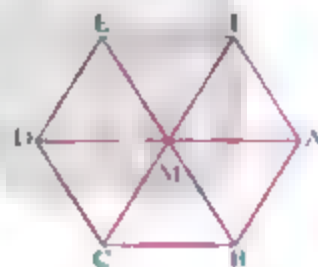
Complete :

- By the rotation  $R(M, 90^\circ)$ , then the image of the point A is \_\_\_\_\_ and the image of the point B is \_\_\_\_\_.  
 $\therefore$  The image of  $\overline{AB}$  is \_\_\_\_\_ and the image of  $\overline{AB}$  is \_\_\_\_\_.
- By rotation  $R(M, -90^\circ)$ , the image of  $\overline{AB}$  is \_\_\_\_\_ and the image of  $\overline{AB}$  is \_\_\_\_\_ and the image of  $\overline{AB}$  is \_\_\_\_\_.
- By rotation  $R(M, 180^\circ)$ , the image of the point A is \_\_\_\_\_, the image of the point B is \_\_\_\_\_.  
 $\therefore$  The image of  $\overline{AB}$  is \_\_\_\_\_.
- By rotation  $R(M, 180^\circ)$ , the image of  $\overline{AB}$  is \_\_\_\_\_.

## 12 In the opposite figure :

ABCDEF is a regular hexagon whose centre is M

Complete the following :



- The image of the point E by rotation about M with an angle of measure  $120^\circ$  is \_\_\_\_\_.
- The image of  $\overline{AF}$  by rotation about M with an angle of measure  $180^\circ$  is \_\_\_\_\_.
- The image of  $\overline{DE}$  by rotation about M with an angle of measure  $-60^\circ$  is \_\_\_\_\_.
- The image of  $\triangle MCD$  by rotation about M with an angle of measure  $300^\circ$  is \_\_\_\_\_.
- $\triangle ABM$  is the image of  $\triangle CDM$  by rotation about \_\_\_\_\_ with an angle of measure \_\_\_\_\_.
- $\triangle BMC$  is the image of \_\_\_\_\_ by rotation about M with an angle of measure  $(-120^\circ)$ .

## Exercise 12

- 13 Referring to the opposite figure ,  
choose the correct answer from those given :



Fig. (1)



Fig. (2)



Fig. (3)

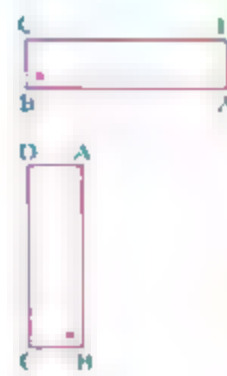


Fig. (4)

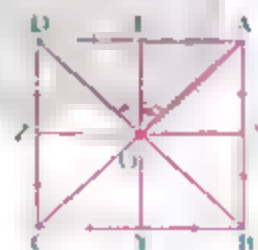
- 1 The image of the figure by reflection in  $\overline{AD}$  is  
(a) fig. (1)      (b) fig. (2)      (c) fig. (3)      (d) fig. (4)
- 2 The image of the figure by rotation about A with an angle of measure  $90^\circ$  is  
(a) fig. (1)      (b) fig. (2)      (c) fig. (3)      (d) fig. (4)
- 3 The image of the figure by translation to the right is  
(a) fig. (1)      (b) fig. (2)      (c) fig. (3)      (d) fig. (4)
- 4 The image of the figure by rotation about A with an angle of measure  $180^\circ$  is  
(a) fig. (1)      (b) fig. (2)      (c) fig. (3)      (d) fig. (4)

- 14 In the opposite figure :

ABCD is a square , O is the point of intersection of its diagonals , X , Y , Z and L are the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  ,  $\overline{CD}$  and  $\overline{DA}$  respectively.

Find :

- 1 The image of  $\triangle AXO$  by reflection in  $\overline{AO}$  followed by another reflection in  $\overline{LO}$
- 2 The image of  $\triangle AXO$  by rotation  $R(O, 90^\circ)$



- 15 ABC is a right-angled triangle at B with  $AB = 5$  cm. and  $BC = 12$  cm. Find :

- 1 X as the image of B by translation 9 cm. in the direction of  $\overline{BA}$
- 2 Y as the image of B by rotation  $R(A, -90^\circ)$
- 3 The length of  $\overline{XY}$

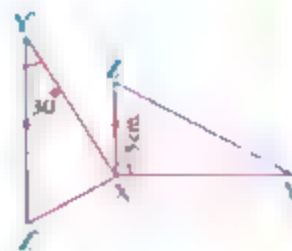


53

16 In the opposite figure :

If the point  $X$  is the centre of rotation such that  $\hat{Y}$  is the image of  $Y$  and  $\hat{Z}$  is the image of  $Z$  if  $XZ \parallel \hat{Y}\hat{Z}$  Find :

- 1 The measure of the angle of rotation.
- 2 The length of  $X\hat{Z}$



$\angle YXZ = 120^\circ$  ,  $XZ = 5 \text{ cm}$  .

### Second Problems on rotation in the Cartesian plane:

1 Complete the following :

- 1 The image of the point  $(2, -3)$  by rotation about the origin point with an angle of measure  $90^\circ$  is ..... and with an angle of measure  $180^\circ$  is .....
- 2 The image of the point  $(-1, 0)$  by rotation about the origin point with an angle of measure  $90^\circ$  is ..... and with an angle of measure  $360^\circ$  is .....
- 3 The point  $(3, -2)$  is the image of the point  $(2, 3)$  by rotation about the origin point with an angle of measure .....
- 4 The image of the point ..... by rotation about the origin point with an angle of measure  $90^\circ$  is  $(-1, 4)$
- 5 The image of the point ..... by rotation about the origin point with an angle of measure  $(-180^\circ)$  is  $(5, -2)$
- 6 The image of the point  $(-3, 7)$  by rotation with an angle of measure  $90^\circ$  about the origin point followed by reflection in y-axis is .....
- 7 The image of the point  $(-2, 0)$  by translation  $(X, y) \rightarrow (X + 3, y - 1)$  followed by rotation about the origin point with an angle of measure  $90^\circ$  is .....
- 8 The rotation with an angle of measure  $90^\circ$  about the origin point maps the point  $(X, -y)$  onto the point .....
- 9 The image of  $(a, b)$  is the same point by rotation about the origin point with an angle of measure .....
- 10 If the image of the point  $(X, y)$  by rotation about the origin point with an angle of measure  $90^\circ$  is  $(a, b)$  , then  $a + y =$  .....

Exercise 12

2 Complete the following diagram :



by reflection in the X-axis

→ The point ( , )

by reflection in the y-axis

→ The point ( , )

by reflection in the origin point

→ The point ( , )

by translation  $(X, y) \rightarrow (X - 3, y + 4)$

→ The point ( , )

by rotation  $R(O, 90^\circ)$

→ The point ( , )

by rotation  $R(O, -90^\circ)$

→ The point ( , )

by rotation  $R(O, \pm 180^\circ)$

→ The point ( , )

by rotation  $R(O, \pm 360^\circ)$

→ The point ( , )

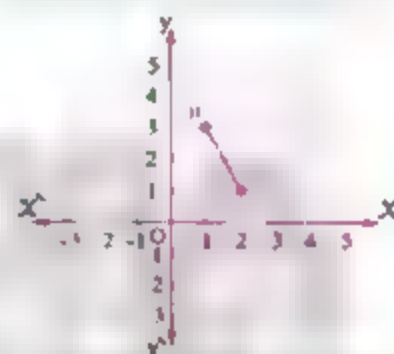
3 In the opposite figure :

The point A (2, 1) and B (1, 3)

Find the image of  $\overline{AB}$

by rotation about the origin point

with an angle of measure  $90^\circ$



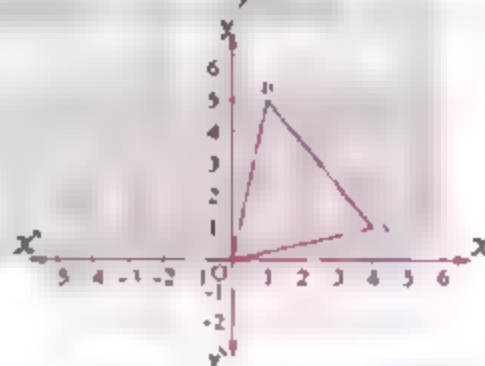
4 On the lattice, draw the image of  $\triangle OAB$

by rotation about the origin with an angle

of measure .

1  $90^\circ$

2  $180^\circ$



5 In the opposite figure :

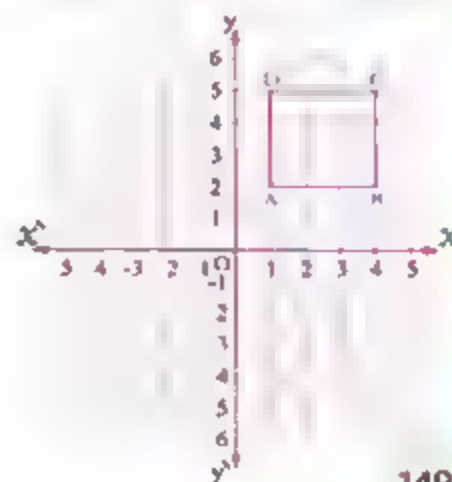
Draw the image of the square ABCD

by rotation about the origin with an angle

of measure :

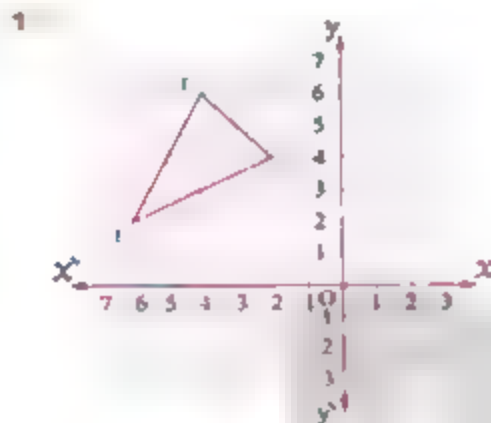
1  $90^\circ$

2  $180^\circ$

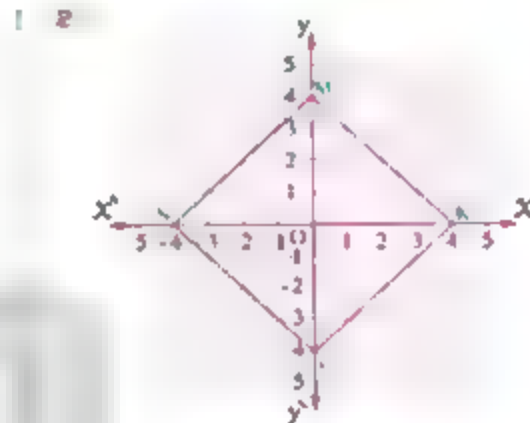


3

6. Copy each figure on a graph paper. Draw their images under the transformation indicated. Give the coordinates of the images vertices in each case.

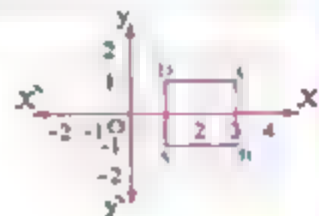


Rotation of  $90^\circ$   
Clockwise about O



Rotation of  $90^\circ$   
anticlockwise about O

7. Draw on graph paper  $\triangle ABC$  where  $A(3, -1)$ ,  $B(5, 2)$  and  $C(-2, 4)$ , then draw its image by rotation about the origin point with an angle of measure  $180^\circ$ .
8. In an orthogonal Cartesian coordinate system, determine the two points  $A(3, 0)$  and  $B(0, 2)$ , then draw the image of  $\triangle AOB$  by rotation about «O» with an angle of measure  $90^\circ$  where O is the origin point.
9. A rectangle has vertices  $A(-1, -2)$ ,  $B(7, 2)$ ,  $C(5, 6)$  and  $D(-3, 2)$ .  
Graph the rectangle and its image under rotation about the origin where  $(x, y) \rightarrow (-y, x)$ .
10. On a lattice, draw the quadrilateral ABCD where  $A(0, 4)$ ,  $B(4, 4)$ ,  $C(7, 0)$  and  $D(0, 0)$ , then draw its image by rotation :  
 1.  $R(O, 90^\circ)$                       2.  $R(O, -180^\circ)$
11. If the image of the point C by rotation with an angle of measure  $90^\circ$  about the origin is  $\hat{C}(-4, 5)$ , locate the point C, then draw its image  $\hat{C}$  by rotation about the origin with an angle of measure  $180^\circ$ .
12. Draw the image of the square ABCD by rotation about A with an angle of measure  $90^\circ$ .





## Exercise 12

- 13 On a lattice, draw  $\triangle ABC$  where  $A(4, 4)$ ,  $B(4, 2)$  and  $C(1, 2)$ , then draw its image by rotation about the point  $B$  with an angle of measure  $180^\circ$ .
- 14 On graph paper, draw the rectangle  $ABCD$  with vertices  $A(0, 0)$ ,  $B(0, 2)$ ,  $C(4, 2)$  and  $D(4, 0)$ .
- (a) Draw three images formed by rotating the rectangle about the origin through an angle of measure
- 1  $90^\circ$
  - 2  $180^\circ$
  - 3  $270^\circ$
- (b) What are the coordinates of the centre of the rectangle?
- (c) Draw three images formed by rotating the rectangle about its centre through an angle of measure
- 1  $90^\circ$
  - 2  $180^\circ$
  - 3  $270^\circ$



For excellent pupils

- 15 In the opposite figure :

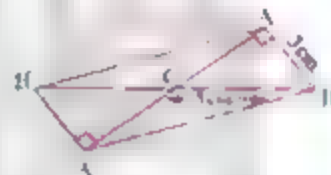
$ABC$  is a right-angled triangle at  $A$

$AB = 3$  cm. and  $BC = 5$  cm.

If  $\triangle CA'B'$  is the image of  $\triangle CAB$  by rotation about  $C$  with an angle of measure  $180^\circ$

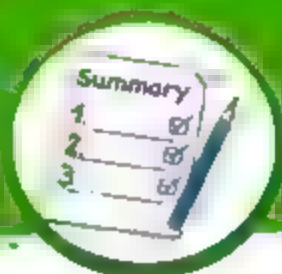
Find the area of  $\triangle AA'B'$

$= 12 \text{ cm}^2$



# Summary of the third part of unit 3

From lesson 8 to lesson 12



## • Properties of reflection in a straight line :

- 1 Reserves the lengths of the line segments.
- 2 Reserves the measures of angles.
- 3 Reserves parallelism.
- 4 Doesn't reserve the orientation of the vertices of the figure.
- 5 Reserves the betweenness.

## • Properties of reflection in a point :

- 1 Reserves the lengths of the line segments.
- 2 Reserves the measures of angles.
- 3 Reserves parallelism.
- 4 Reserves the orientation of vertices of the figure
- 5 Reserves the betweenness.

## • Properties of translation :

- 1 Reserves the lengths of the line segments.
- 2 Reserves the measures of angles.
- 3 Reserves the parallelism.
- 4 Reserves the orientation of vertices of the figure.
- 5 Reserves the betweenness.

## • To determine the translation, we should know two important elements which are :

- 1 The magnitude of the translation.
- 2 The direction of the translation.

## • Properties of rotation :

- 1 Reserves the lengths of the line segments.
- 2 Reserves the measures of angles.
- 3 Reserves the parallelism.
- 4 Reserves the orientation of vertices of the figure.
- 5 Reserves the betweenness.

• The rotation is determined completely by the following elements :

- 1 The centre of rotation.
- 2 The measure of the angle of rotation.
- 3 The direction of rotation.

• Summary for geometrical transformations (reflection • translation • rotation) in the Cartesian plane :

by reflection in the  $x$ -axis

→ The point  $(X, -y)$

by reflection in the  $y$ -axis

→ The point  $(-X, y)$

by reflection in the origin point

→ The point  $(-X, -y)$

by translation  $(X, y) \Rightarrow (X + k, y + l)$

→ The point  $(X + k, y + l)$

by rotation  $R(O, 90^\circ)$  ( $\frac{1}{4}$  turn)

→ The point  $(-y, X)$

by rotation about  $O$  with an angle of measure  $(-90^\circ)$  or  $(270^\circ)$

→ The point  $(y, -X)$

by rotation  $R(O, \pm 180^\circ)$  ( $\frac{1}{2}$  turn)

→ The point  $(-X, -y)$

by rotation  $R(O, \pm 360^\circ)$   
(identity rotation)

→ The point  $(X, y)$



## Exercises on the third part of unit three from lesson (18) to lesson (21)



### Model 1

Answer the following questions :

1 Choose the correct answer from the given ones :

- 1 The image of the point  $(-3, -5)$  by reflection in  $X$ -axis is  
 (a)  $(3, -5)$       (b)  $(3, 5)$       (c)  $(-3, 5)$       (d)  $(-3, -5)$
- 2 If  $\hat{A}$  is the image of  $A$  by reflection in  $M$  and  $MA = 6$  cm, then  $A\hat{A} =$   
 (a) 6 cm.      (b) 3 cm.      (c) 12 cm.      (d) 9 cm.
- 3 The image of the point  $(-2, 3)$  by translation of magnitude of 4 units in the negative direction of  $y$ -axis is  
 (a)  $(2, 3)$       (b)  $(-2, 7)$       (c)  $(-6, 3)$       (d)  $(-2, -1)$
- 4 The image of the point  $(-4, 2)$  by rotation around the origin point with an angle of measure  $90^\circ$  is  
 (a)  $(-2, 4)$       (b)  $(-2, -4)$       (c)  $(4, -2)$       (d)  $(2, -4)$
- 5 The image of the point  $(2, -7)$  by reflection in the origin point is  
 (a)  $(2, 7)$       (b)  $(-2, 7)$       (c)  $(-2, -7)$       (d)  $(2, -7)$
- 6 If the image of the point  $(5, -2)$  by rotation around the origin point is itself, then the measure of angle of rotation is  
 (a)  $90^\circ$       (b)  $180^\circ$       (c)  $270^\circ$       (d)  $360^\circ$

2 Complete the following :

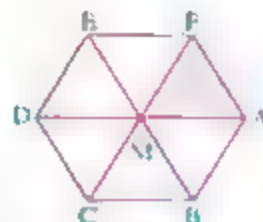
- 1 The number of axes of symmetry of the rhombus = ..... and the number of axes of symmetry of the parallelogram = .....
- 2 The reflection in a straight line reserves ..... , ..... , .....
- 3 The image of the point  $(-3, -4)$  by translation  $(-4, 1)$  is .....
- 4 The image of the point  $(-2, 5)$  by rotation  $R(O, -90^\circ)$  is .....
- 5 The point  $(-2, 4)$  is the image of the point ..... by reflection in the origin point.

- 3 [a] Draw  $\triangle ABC$  in which :  $A(3, 2)$  ,  $B(-1, 5)$  and  $C(0, -3)$  , then draw its image by reflection in the  $y$ -axis.

[b] In the opposite figure :

If  $ABCDEF$  is a regular hexagon whose centre is  $M$  , find :

- 1 The image of  $\triangle AMF$  by reflection in  $\overline{AM}$
- 2 The image of  $\triangle AMF$  by reflection in  $M$
- 3 The image of  $\triangle AMF$  by translation of distance  $AM$  in the direction of  $\overline{AM}$
- 4 The image of  $\triangle AMF$  by rotation  $R(M, 120^\circ)$



- 4 [a]  $ABCD$  is a rectangle where :  $A(4, 3)$  ,  $B(4, 1)$  ,  $C(-1, 1)$  and  $D(-1, 3)$  , draw the rectangle and its image by rotation  $R(O, 180^\circ)$

[b] Draw the image of  $\triangle ABC$  where :  $AB = 3$  cm. ,  $BC = 4$  cm. and  $AC = 5$  cm. by reflection in the straight line which contains the longest side.

- 5 [a] Draw  $\triangle ABC$  in which :  $AB = 4$  cm. ,  $BC = 6$  cm. and  $AC = 5$  cm. , then draw its image by translation of magnitude 3 cm. in the direction of  $\overline{CB}$

[b] Draw  $\triangle ABC$  in which :  $AB = BC = 4$  cm. and  $AC = 5$  cm. , then find its image by reflection in point  $B$

### Model 2

Answer the following questions :

- 1 Choose the correct answer from the given ones :

- 1 The image of the point  $(-7, 4)$  by rotation around the origin point with an angle of measure  $180^\circ$  is  
 (a)  $(-4, -7)$  (b)  $(4, 7)$  (c)  $(7, -4)$  (d)  $(-7, -4)$
- 2 The point whose image is itself by reflection in the origin point is  
 (a)  $(0, 1)$  (b)  $(1, 0)$  (c)  $(0, -1)$  (d)  $(0, 0)$
- 3 The image of the point  $(-3, 2)$  by translation of magnitude 4 units in the positive direction of the  $X$ -axis is  
 (a)  $(1, 2)$  (b)  $(-3, 6)$  (c)  $(-7, 2)$  (d)  $(-3, -2)$
- 4 The image of the point  $(-10, 0)$  by reflection in the  $X$ -axis is  
 (a)  $(10, 0)$  (b)  $(-10, 0)$  (c)  $(0, -10)$  (d)  $(0, 10)$
- 5 The image of the point  $(5, -7)$  by rotation  $R(O, 90^\circ)$  is  
 (a)  $(-7, -5)$  (b)  $(7, 5)$  (c)  $(-7, 5)$  (d)  $(-5, 7)$



- 6 If  $\hat{A}(-4, 5)$  is the image of A by translation  $(-2, 3)$ , then the point A is  
 (a)  $(-6, 8)$  (b)  $(-2, 8)$  (c)  $(-2, 2)$  (d)  $(-6, 2)$

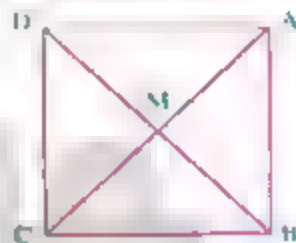
**2 Complete the following :**

- 1 The rotation in the plane reserves .....
- 2 The number of axes of symmetry of the isosceles trapezium = .....
- 3 The measure of the angle of the identity rotation equals .....
- 4 The translation in the plane is determined by .....
- 5 The point  $(-3, 4)$  is the image of the point  $(3, 4)$  by reflection in .....

- 3 [a]** Draw on a graph paper  $\triangle ABC$  where  $A(2, 0)$ ,  $B(3, 2)$ ,  $C(4, 1)$ , then draw its image by reflection in the X-axis.

**[b] In the opposite figure :**

ABCD is a square, M is the point of intersection of its diagonals, find :



- 1 The image of  $\triangle AMB$  by reflection in M
- 2 The image of  $\overline{AD}$  by translation of magnitude DC in direction of  $\overline{DC}$

- 4 [a]** If the image of the point  $(1, 2)$  by a translation is  $(1, 4)$

Find :

- 1 The translation.
- 2 The image of the point  $(3, 1)$  by the same translation.

- [b]** Draw the equilateral triangle ABC of side length 5 cm, then draw its image by rotation  $R(B, 60^\circ)$

- 5 [a]** Find the image of  $A(-3, 2)$  by translation BC in the direction of  $\overline{BC}$  where  $B(1, 3)$ ,  $C(4, 5)$

- [b]** On the square lattice, draw  $\triangle ABC$  where  $A(1, 1)$ ,  $B(1, 4)$ ,  $C(5, 1)$ , then draw its image by rotation  $R(O, -90^\circ)$



# SKILLS

TIMSS Problems



## Accumulative basic skills

1 Choose the correct answer from the given ones :

- 1 A square of area  $144 \text{ cm}^2$ , then its perimeter =  $\text{cm}$   
 (a) 12 (b) 48 (c) 288 (d) 576
- 2 A rectangle, its length is 6 cm, and its perimeter is 16 cm, then its area =  $\text{cm}^2$   
 (a) 10 (b) 8 (c) 12 (d) 16
- 3 The supplementary of the angle whose measure is  $30^\circ$ , is an angle of measure  
 (a)  $30^\circ$  (b)  $60^\circ$  (c)  $120^\circ$  (d)  $150^\circ$
- 4 Which of the following figures is suitable to form a circle ?



(a)



(b)



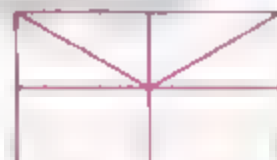
(c)



(d)

- 5 The area of the shaded part from the area of the figure =

(a)  $\frac{1}{8}$  (b)  $\frac{1}{2}$  (c)  $\frac{3}{8}$  (d)  $\frac{3}{4}$



- 6 The great number of triangles in the opposite figure =

(a) 4 (b) 6 (c) 8 (d) 10



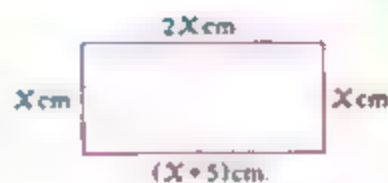
- 7 If X is an angle, then  $m(\angle X) + m(\text{reflex } \angle X) =$

(a) two right angles. (b) three right angles.  
 (c) five right angles. (d) four right angles.

## Basic skills

- 8 The area of the rectangle in the opposite figure = .....  $\text{cm}^2$

(a) 50 (b) 30 (c) 20 (d) 15



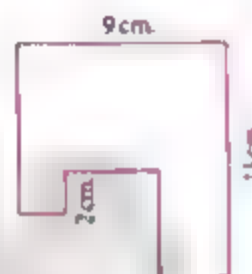
- 9 The perimeter of the opposite figure = ..... cm.

(a) 22 (b) 24 (c) 44 (d) 120



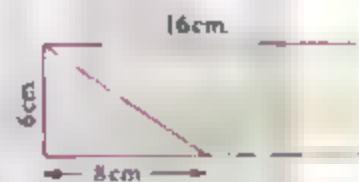
- 10 The perimeter of the opposite figure = ..... cm.

(a) 99 (b) 44 (c) 22 (d) 20



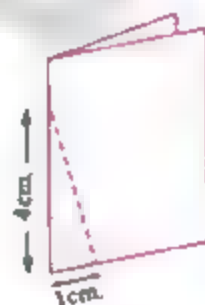
- 11 The area of the shaded part in the opposite figure = ...  $\text{cm}^2$

(a) 24 (b) 44 (c) 48 (d) 72



- 12 A rectangular piece of paper is folded as in the opposite figure, and a part of it is cut off with aligning the dashed line, then at opening the cut small part, it will be in the form of

(a) an equilateral triangle. (b) an isosceles triangle.  
(c) a right-angled triangle (d) two isosceles triangles.



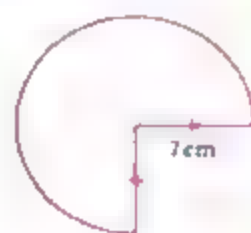
2 Complete the following :

- 1 A cube, area of one of its faces is  $25 \text{ cm}^2$ , then its volume = ...  $\text{cm}^3$
- 2 A cuboid, its volume =  $48 \text{ cm}^3$ , if the length of its base equals 6 cm. and its width equals 4 cm., then its height = ... cm
- 3 The angle whose measure is  $89^\circ$ , is ... angle.

## Accumulative basic skills &gt;

4. If  $m(\angle A) = 2m(\angle B)$ ,  $\angle A$  complements  $\angle B$ , then  $m(\angle A) =$  .

5. The area of the opposite figure = .....  $\text{cm}^2$  ( $\pi = \frac{22}{7}$ )



6. The perimeter of the opposite figure = ..... cm. ( $\pi = \frac{22}{7}$ )



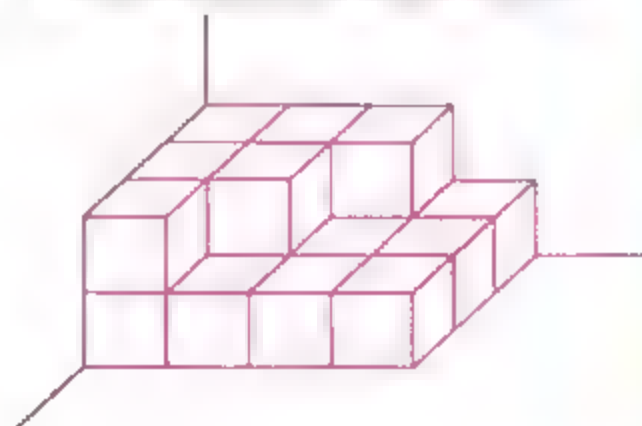
7. The number of acute angles in the opposite figure is



8. In the opposite figure :  
 $m(\angle B) + m(\angle C) + m(\angle D) =$  .



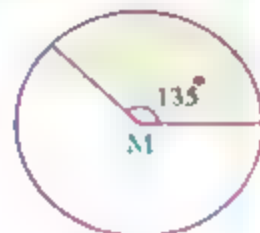
9. The volume of the opposite figure = ..... volume units.





## Basic skills

- 10 The percentage of the area of the shaded part to the area of the circle is ...



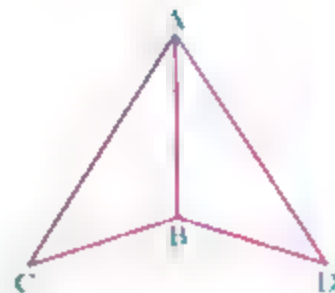
- 11 In the opposite figure :

If  $\triangle ABC \cong \triangle ABD$ ,

the perimeter of the figure ABCD = 20 cm.

and  $AB = 6$  cm.

, then the perimeter of  $\triangle ABC =$  ..... cm.



- 12 In the opposite figure :

ABCD is a square of area  $49 \text{ cm}^2$

, if  $EC = 15$  cm., then the area of  $\triangle ABE =$  .....  $\text{cm}^2$





EL-MONASSER

# In Mathematics

## Notebook

For **1<sup>st</sup>** Prep.  
Second Term

- Quizzes
- Final Revision
- Final Examinations



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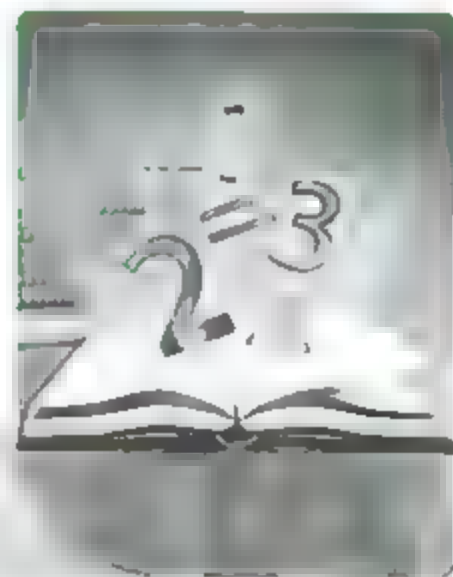
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By  
A group of supervisors

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(2 models + model for the merge students)
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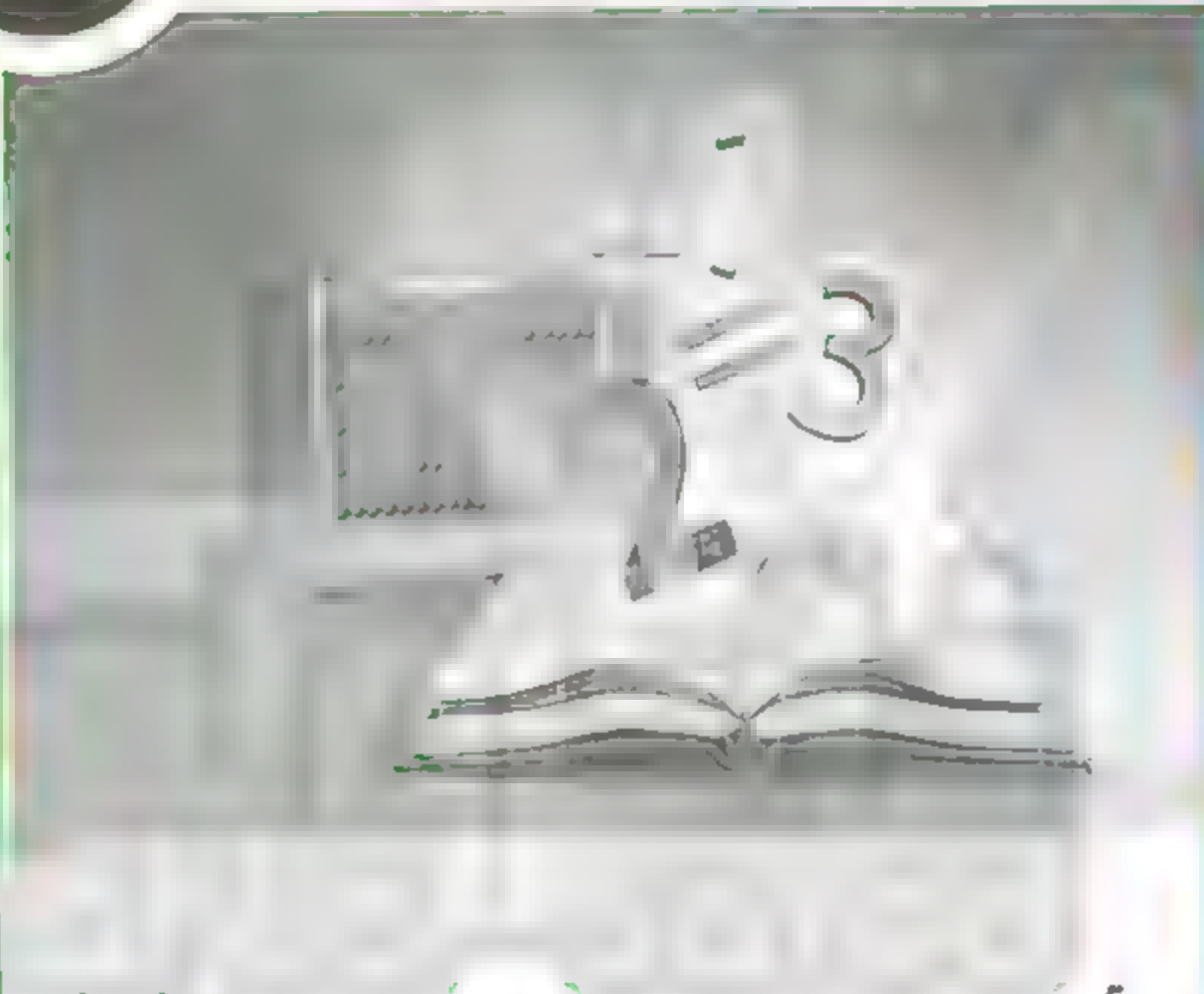
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First

## Algebra and Statistics



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(2 models + model for the merge students)
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# Quizzes

on Algebra and Statistics



## Algebra and Statistics

## Quiz 1

on lesson 1 – unit 1



1 Choose the correct answer from those given :

1 The multiplicative inverse of the number  $\left(\frac{3}{4}\right)^{\text{zero}}$  is

(a)  $-1$

(b)  $-\frac{4}{3}$

(c)  $\frac{4}{3}$

(d)  $1$

2 If  $a = b$ , then  $\left(\frac{5}{7}\right)^{a-b} =$ 

(a)  $\frac{5}{7}$

(b)  $\frac{7}{5}$

(c)  $1$

(d) zero

3 If  $x = -\frac{2}{3}$ ,  $y = 2$ , then  $x^y =$ 

(a)  $\frac{4}{9}$

(b)  $-\frac{4}{9}$

(c)  $\frac{8}{27}$

(d)  $-\frac{8}{27}$

2 [a] Simplify to the simplest form :

$$\left(-\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 + \left(-\frac{2}{9}\right)^2$$

[b] If  $x = -\frac{3}{2}$ ,  $y = \frac{1}{2}$ ,  $z = -\frac{3}{4}$ ,find in the simplest form the numerical value of :  $\left(\frac{xz}{y}\right)^2$ 

## Quiz 2

till lesson 2 – unit 1



1 Choose the correct answer from those given :

$(a^2)^4 =$

(a)  $a^6$

(b)  $a^8$

(c)  $a^2$

(d)  $a^4$

2 The quarter of the number  $4^{20} =$ 

(a)  $4^5$

(b)  $4^{10}$

(c)  $4^{19}$

(d)  $1^{20}$

3  $2^5 + 2^5 =$ 

(a)  $4^5$

(b)  $2^{10}$

(c)  $2^6$

(d)  $4^{10}$

2 [a] Find each of the following in the simplest form :

$$\left(\frac{3}{5}\right)^7 + \left(\frac{3}{5}\right)^5 \times \frac{3}{5}$$

$$\frac{x^5 \times x^4}{x^3 \times x^2 \times x^4} \text{ where } x \neq 0$$

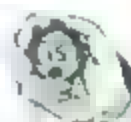
$$\left(-\frac{2}{3}\right)^3$$

[b] If  $a = -\frac{1}{2}$ ,  $b = 2$ ,  $c = \frac{3}{4}$ ,find the numerical value of the expression :  $a^3 b^2 + b^2 c - 8 abc$



## Quiz 3

till lesson 3 - unit 1

time  
15 min

1 Choose the correct answer from those given :

1) If  $X^{-1} = \frac{1}{2}$ , then  $X =$ 

(a)  $\frac{1}{2}$

(b)  $-\frac{1}{2}$

(c) 2

(d) -2

2  $\frac{6a^2 X^4}{2a^3 X^3} =$  where  $a \neq 0$ 

(a)  $3aX$

(b)  $3a^5 X^7$

(c)  $\frac{3X}{a}$

(d)  $\frac{3}{aX}$

3  $5^6 \times 5^{-6} =$ 

(a) 5

(b) 1

(c) 12

(d) zero

2 [a] Find each of the following in the simplest form :

1  $5^2 \times 5^3$

2  $\left(\frac{3^4 \times 7^2}{7^3 \times 3^2}\right)^{-1}$

3  $\left(-\frac{X^3}{y^2}\right)^{-2}$  such that  $X \neq 0, y \neq 0$

[b] If  $X = \frac{1}{2}$  and  $y = \frac{1}{8}$ , find the value of the expression :  $(2X - y)^{-2} \times y$ 

## Quiz 4

till lesson 4 - unit 1

time  
15 min

1 Choose the correct answer from those given :

1  $2.37 \times 10^{-4} =$

(a) 0.00237

(b) 0.000237

(c) 23700

(d) 0.0000237

2  $3y^{-1} = \dots\dots\dots$ ,  $y \neq 0$ 

(a)  $\frac{1}{3}y$

(b)  $\frac{3}{y}$

(c)  $y^{-3}$

(d)  $\frac{1}{3y}$

3 Which of the following =  $\frac{1}{4}$  million ?

(a)  $25 \times 10^5$

(b)  $0.25 \times 10^5$

(c)  $0.25 \times 10^6$

(d)  $0.25 \times 10^4$

2 [a] Write the result of each of the following on the standard form :

1  $(5.8 \times 10^7) + (3.2 \times 10^5)$

2  $(65.5 \times 10^{-2}) + (5 \times 10^2)$

3  $60000 \times 5000$

[b] Calculate the value of each of the following :

1  $\frac{(3^{-2})^4}{3^{-5} \times 3^{-2}}$

2  $\frac{(-3)^7 \times (-3)^{-2}}{(-3)^5}$

## Algebra and Statistics

## Quiz 5

till lesson 5 - unit 1



1 Choose the correct answer from those given :

1 If  $0.000237 = 2.37 \times 10^n$ , then  $n =$

(a) 4

(b) 3

(c) -4

(d) -3

2  $-11 + 3 \times 7 =$

(a) 11

(b) 10

(c) -56

(d) 1

3  $3^2 + 3^2 + 3^2 =$

(a)  $3^3$

(b)  $9^6$

(c)  $9^2$

(d)  $3^6$

2 [a] Calculate the value of each of the following :

1  $8 \times 2^2 - 7 \times (4 + 1)$

2  $2((5^2 + 1) - (4^2 - 1))$

3  $\frac{5 + 2 \times 5}{2^2 + 1}$

[b] Find the value of :  $\frac{7^{-3} \times 7^4}{7 \times 7^{-2}}$

## Quiz 6

till lesson 6 - unit 1



1 Choose the correct answer from those given :

1  $\sqrt{(-7)^2} =$

(a) 49

(b) 7

(c) -7

(d)  $\pm 7$

2  $\sqrt{10^2 - 8^2} =$

(a) 2

(b) 6

(c)  $\pm 2$

(d)  $\pm 6$

3  $\sqrt{9 + 16} = 3 +$

(a) 4

(b) 5

(c) 2

(d) 22

2 [a] 1 If  $(AB)^2 = 144 \text{ cm}^2$ ,  $(BC)^2 = 625 \text{ cm}^2$  and  $B \in \overline{AC}$ , then find the length of :  $\overline{AC}$ 

2 Reduce to the simplest form :  $(-\frac{1}{2})^3 \times \sqrt{\frac{25}{9}} \times \sqrt{(\frac{8}{5})^2} \times 3^{-1}$

[b] If  $a = \frac{1}{3}$ ,  $b = -\frac{2}{3}$ , find the value of :  $|(a^3 + b^3)^{-1}|$

## Quiz 7

till lesson 7 - unit 1



1 Choose the correct answer from those given :

1 The S.S. of the equation :  $2x = -6$  in  $\mathbb{Z}$  is

- (a)  $\{-3\}$  (b)  $\emptyset$  (c)  $\{-6\}$  (d)  $\{3\}$

2 If  $3y = 6$ , then  $6y =$ 

- (a) 2 (b) 3 (c) 6 (d) 12

3 The number 0.000014 =

- (a)  $1.4 \times 10^{-5}$  (b)  $1.4 \times 10^5$  (c)  $1.4 \times 10^{-6}$  (d)  $1.4 \times 10^6$

2 [a] 1 Find the S.S. of the following equation in  $\mathbb{Q}$  :  $3(2x - 1) = 15$ 

2 Two natural numbers , the difference between them is 5 and their sum is 15 find the two numbers.

[b] Find the value of :  $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^{\text{zero}}$ 

## Quiz 8

till lesson 8 - unit 1



1 Choose the correct answer from those given :

1 The S.S. of the inequality :  $-5x < \text{zero}$  in  $\mathbb{Q}$  is

- (a)  $\emptyset$  (b)  $\mathbb{Q}_+$  (c)  $\mathbb{Q}_-$  (d)  $\mathbb{Z}_+$

2 The quarter of the number  $2^{10}$  is

- (a)  $2^5$  (b)  $2^9$  (c)  $2^8$  (d)  $1^{10}$

3 If  $x < y$  ,  $y < z$  , then

- (a)  $x > z$  (b)  $x < z$  (c)  $x = z$  (d)  $x \geq z$

2 [a] 1 Find the S.S. of the following inequality in  $\mathbb{Q}$  :  $2 - 3x < -4$ 2 Find the S.S. of the following equation in  $\mathbb{Q}$  :  $5x + 4 = 6$ [b] If  $a = -\frac{1}{2}$  ,  $b = 2$  ,  $c = \frac{3}{2}$  , find the numerical value of the expression :  $a^2 b^3 + (a + c)^5$



## Quiz 9

till lesson 2 - unit 2



1 Choose the correct answer from those given :

- 1 As flipping a fair coin once , the probability of appearing a head is  
 (a) 5 (b) 5 % (c) 50 % (d)  $\frac{1}{5}$
- 2 If the probability of occurring an event is  $\frac{5}{8}$  , then the probability of not occurring of the same event =  
 (a)  $\frac{5}{8}$  (b)  $\frac{3}{8}$  (c) 1 (d)  $\frac{1}{2}$
- 3  $x^4 + x^{-2} =$   
 (a)  $x^2$  (b)  $x^{-6}$  (c)  $x^{-8}$  (d)  $x^6$

2 [a] Find the S.S. of the following equation in Q :  $3(x + 1) - x = 8$ 

[b] A card is drawn randomly from 8 cards numbered from 1 to 8 Write the sample space , then find the probability of each of the following

- 1 Appearing an even number      2 Appearing a number divisible by 3

# Final Revision

of Algebra and Statistics



## Revision for the important rules of algebra and statistic

**Remember** The repeated multiplication and the laws of powers

• If  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers,  $m$  and  $n$  are two integers, then :

①  $\left(\frac{a}{b}\right)^n \times \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n+m}$  when multiplying the like base, add their powers (indices)

②  $\left(\frac{a}{b}\right)^n \div \left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\right)^{n-m}$  where  $\frac{a}{b} \neq 0$

when dividing like bases, we subtract their powers (indices)

③  $\left(\left(\frac{a}{b}\right)^n\right)^m = \left(\frac{a}{b}\right)^{n \times m}$

④  $\left(\frac{a}{b} \times \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \times \left(\frac{c}{d}\right)^n$

⑤  $\left(\frac{a}{b} \div \frac{c}{d}\right)^n = \left(\frac{a}{b}\right)^n \div \left(\frac{c}{d}\right)^n$  where  $\frac{c}{d} \neq 0$

• If  $\frac{a}{b}$  is a rational number, then  $\left(\frac{a}{b}\right)^0 = 1$  where  $a \neq 0$

• If  $a$  is a rational number,  $a \neq 0$ ,  $n$  is a positive integer, then  $a^{-n} = \frac{1}{a^n}$ ,  $a^n = \frac{1}{a^{-n}}$

• If  $\frac{a}{b}$  is a rational number, where  $\frac{a}{b} \neq 0$ ,  $n$  is a positive integer, then  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

**Example**

Simplify each of the following to the simplest form :

1  $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3$

2  $\left(\frac{2}{9}\right)^7 \div \left(\frac{2}{9}\right)^5$

3  $\left(-\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

4  $\frac{6^{-3} \times 6^5}{6^2}$

5  $\left(\frac{5^3 \times 5^4}{5^5}\right)^{-2}$

6  $\frac{(-4x^3y^4)^2}{(-2xy^2)^4}$

**Solution**

①  $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)^5 = \frac{2^5}{3^5} = \frac{32}{243}$

②  $\left(\frac{2}{9}\right)^7 \div \left(\frac{2}{9}\right)^5 = \left(\frac{2}{9}\right)^2 = \frac{2^2}{9^2} = \frac{4}{81}$

③  $\left(-\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = -\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2 = -\left(\frac{1}{2}\right)^5 = -\frac{1}{32}$

④  $\frac{6^{-3} \times 6^5}{6^2} = 6^{-3+5-2} = 6^0 = 1$

⑤  $\left(\frac{5^3 \times 5^4}{5^5}\right)^{-2} = (5^{3+4-5})^{-2} = (5^2)^{-2} = 5^{-4} = \frac{1}{5^4} = \frac{1}{625}$

⑥  $\frac{(-4x^3y^4)^2}{(-2xy^2)^4} = \frac{(-4)^2 \times x^6 \times y^8}{(-2)^4 \times x^4 \times y^8} = \frac{16x^6y^8}{16x^4y^8} = x^2$  where  $x, y \neq 0$



**Remember** The standard scientific notation of the rational number

The number is written in the standard form as  $a \times 10^n$  where  $1 \leq |a| < 10$  and  $n \in \mathbb{Z}$

**For example :** Each of the following two numbers is written in its standard form :

$$\bullet 4.6 \times 10^8 \quad \bullet 5.27 \times 10^{-6} \quad \bullet -9.6 \times 10^{10} \quad \bullet 1 \times 10^{-13}$$

Each of the previous numbers is the product of two numbers

- The first number could be positive or negative and its absolute value must be greater than or equal 1 and less than 10
- The second number expresses the powers of the number 10 (These powers could be positive or negative)

### Example 1

Put each of the following numbers in the standard form :

1 8 200 000 000

2 0 000 000 135

3  $45 \times 10^8$

4  $706.4 \times 10^5$

5  $-0.0015 \times 10^{-9}$

### Solution

(1)  $8\,200\,000\,000 = 8.2 \times 10^9$

(2)  $0\,000\,000\,135 = 1.35 \times 10^{-7}$

(3)  $45 \times 10^8 = 4.5 \times 10 \times 10^8 = 4.5 \times 10^9$

(4)  $706.4 \times 10^5 = 7.064 \times 10^2 \times 10^5 = 7.064 \times 10^7$

(5)  $-0.0015 \times 10^{-9} = -1.5 \times 10^{-3} \times 10^{-9} = -1.5 \times 10^{-12}$

### Noticethat

To move the decimal point 9 places towards left , we multiplied by  $10^9$

### Noticethat

To move the decimal point 7 places towards right , we multiplied by  $10^{-7}$

### Example 2

Find the value of n in each of the following :

1  $500\,000 = 5 \times 10^n$

2  $0.00\,052 = 5.2 \times 10^n$

3  $7\,293 = n \times 10^3$

### Solution

1)  $\because 500\,000 = 5 \times 10^5$

$\therefore n = 5$

2)  $\because 0.00\,052 = 5.2 \times 10^{-4}$

$\therefore n = -4$

3)  $\because 7\,293 = 7\,293 \times 10^3$

$\therefore n = 7\,293$

## Algebra and Statistics

**Remember** The order of mathematical operations

The order of performing the mathematical operations is as the following :

- 1 Perform the operations within parentheses (interior parentheses then exterior ones)
- 2 Evaluate the powers.
- 3 Perform multiplications and divisions in order from left to right
- 4 Perform additions and subtractions in order from left to right.

**Remember that**

In the problems containing fractions , we should perform the operations in the numerator and denominator before divisions

**Example**

Calculate the value of each of the following :

$$1 \quad 4 - 3 [4 - 2 (6 - 3)] + 2$$

$$(2) \quad 8 \times 2^2 - 7 \times (4 + 1)$$

$$3 \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2}$$

**Solution**

$$\begin{aligned} (1) \quad 4 - 3 [4 - 2 (6 - 3)] + 2 &= 4 - 3 [4 - 2 \times 3] + 2 && \text{(the interior parentheses)} \\ &= 4 - 3 [4 - 6] + 2 && \text{(multiplication inside parentheses)} \\ &= 4 - 3 [-2] + 2 && \text{(subtraction inside parentheses)} \\ &= 4 + 6 + 2 && \text{(multiplication by parentheses)} \\ &= 4 + 3 && \text{(division)} \\ &= 7 && \text{(addition)} \end{aligned}$$

$$\begin{aligned} (2) \quad 8 \times 2^2 - 7 \times (4 + 1) &= 8 \times 2^2 - 7 \times 5 && \text{(addition inside parentheses)} \\ &= 8 \times 4 - 7 \times 5 && \text{(powers)} \\ &= 32 - 35 && \text{(multiplication)} \\ &= -3 && \text{(subtraction)} \end{aligned}$$

$$(3) \quad \frac{11 - (5 - 4)}{5^2 - 10 \times 2} - \frac{11}{25} - \frac{1}{20} = \frac{10}{5} = 2$$

**Remember** The square root of a perfect square rational number

- The square root of the perfect square rational number 'a' is the number whose square equals 'a'

For example :  $\sqrt{16} = 4$  ,  $-\sqrt{16} = -4$  ,  $\pm \sqrt{16} = \pm 4$

- It is meaningless to find  $\sqrt{a}$  if a is a negative rational number.

•  $\sqrt{a^2} = |a|$

For example :  $\sqrt{(-3)^2} = 3$  ,  $-3$

- When there is an addition or a subtraction operation under the square root , it must be performed first before finding the square root.

**Example 1**

Find each of the following in the simplest form :

1  $\sqrt{36}$

2  $-\sqrt{\frac{16}{25}}$

3  $\pm \sqrt{2\frac{1}{4}}$

4  $\sqrt{\left(\frac{2}{7}\right)^2}$

5  $-\sqrt{0.25}$

6  $\sqrt{16+9}$

7  $\sqrt{100-36}$

8  $\sqrt{\frac{16a^8}{49d^4}}$

**Solution**

1  $\sqrt{36} = 6$  because  $6^2 = 36$

2  $-\sqrt{\frac{16}{25}} = -\frac{4}{5}$  because  $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

3  $\pm \sqrt{2\frac{1}{4}} = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2}$

4  $\sqrt{\left(\frac{2}{7}\right)^2} = \frac{2}{7}$

5  $-\sqrt{0.25} = -\sqrt{\frac{25}{100}} = -\frac{5}{10} = -\frac{1}{2}$

6  $\sqrt{16+9} = \sqrt{25} = 5$

7  $\sqrt{100-36} = \sqrt{64} = 8$

8  $\sqrt{\frac{16a^8}{49d^4}} = \frac{4a^4}{7d^2}$

**Example 2**

Simplify each of the following to the simplest form :

1  $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2$

2  $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$

**Solution**

1  $-\frac{2}{7} \times \sqrt{\frac{49}{4}} \times \left(\frac{2}{7}\right)^2 = -\frac{2}{7} \times \frac{7}{2} \times \frac{4}{49} = -\frac{4}{49}$

2  $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$



## Algebra and Statistics

**Remember** The properties of the equality relation for solving the equation

- We can add any rational number to both sides of the equation

*For example :* If  $X - 1 = 5$ , then  $X - 1 + 1 = 5 + 1$  i.e.  $X = 6$

- We can subtract any rational number from both sides of the equation

*For example :* If  $X + 3 = 2$ , then  $X + 3 - 3 = 2 - 3$  i.e.  $X = -1$

- We can multiply both sides of the equation by the same rational number.

*For example :* If  $\frac{1}{3} X = 2$ , then  $\frac{1}{3} X \times 3 = 2 \times 3$  i.e.  $X = 6$

- We can divide both sides of the equation by the same rational number not equal to zero

*For example :* If  $7 X = 14$ , then  $\frac{7 X}{7} = \frac{14}{7}$  i.e.  $X = 2$

**Exercise 1**

Find in Q the solution set of each of the following equations :

①  $2 X - 5 = 13$

②  $3 X + 4 = 2 (X + 1)$

**Solution**

①  $\because 2 X - 5 = 13$  "Adding 5 to both sides"

$\therefore 2 X - 5 + 5 = 13 + 5$

$\therefore 2 X = 18$  "Dividing both sides by 2"

$\therefore \frac{2 X}{2} = \frac{18}{2}$

$\therefore X = 9$

$\therefore$  The S.S. =  $\{9\}$

② Notice that the variable (X) exists in the two sides, then we try to collect it in one side (say the left side)

$\therefore 3 X + 4 = 2 (X + 1)$

Using the distribution property

$\therefore 3 X + 4 = 2 X + 2$

Subtracting  $2 X$  from both sides

$\therefore 3 X - 2 X + 4 = 2 X - 2 X + 2$

$\therefore X + 4 = 2$

Subtracting 4 from both sides

$\therefore X + 4 - 4 = 2 - 4$

$\therefore X = -2$

$\therefore$  The S.S. =  $\{-2\}$

## Remarks for solving word problems

- If a number =  $X$ , then its twice =  $2X$  and its three times =  $3X$ .
- If a number =  $X$  and another number exceeds it by 5, then the other number =  $X + 5$ .
- If a number =  $X$  and another number decreases than it by 5, then the other number =  $X - 5$ .
- If the age of a man now =  $X$  years, then :
  - His age after 3 years =  $(X + 3)$  years.
  - His age 3 years ago =  $(X - 3)$  years.
- Three consecutive integers are  $X, X + 1$  and  $X + 2$ .
- Three consecutive natural (even or odd) numbers are  $X, X + 2$  and  $X + 4$ .
- The perimeter of a rectangle =  $2(\text{length} + \text{width})$ .
- The perimeter of a square =  $\text{side length} \times 4$ .
- The perimeter of the triangle = the sum of its sides lengths.
- The area of the triangle =  $\frac{1}{2}$  the base length  $\times$  the height.
- The sum of measures of the interior angles of the triangle =  $180^\circ$ .

## Example 1

Two natural numbers, one of them is thrice of the other, if the sum of them is 16. Find the two numbers.

## Solution

Let one of the two numbers be  $X$ .

- ∴ The other number is thrice of this number ∴ The other number =  $3X$
- ∴ The sum of the two numbers = 16 ∴  $X + 3X = 16$
- ∴  $4X = 16$  "Dividing by 4" ∴  $X = 4$
- ∴ One of the two numbers = 4, the other number =  $3 \times 4 = 12$

## Example 2

A rectangle with length equals twice its width and its perimeter = 18 cm. Find the dimensions of the rectangle.

## Solution

- Let the width of the rectangle be  $X$  cm. ∴ Its length =  $2X$  cm.
- ∴ The perimeter of the rectangle =  $2(\text{length} + \text{width})$
- ∴  $18 = 2(2X + X)$  ∴  $18 = 2 \times 3X$
- ∴  $18 = 6X$  ∴  $X = 3$
- ∴ The width of the rectangle = 3 cm, and its length = 6 cm.

## Algebra and Statistics

**Remember** The properties of Inequality for solving the Inequalities

- We can add any rational number to both sides of the inequality without change in the inequality relation.

*For example :* If  $X - 1 > 5$  , then  $X - 1 + 1 > 5 + 1$  *i.e.*  $X > 6$

- We can subtract any rational number from the two sides of the inequality without change in the inequality relation

*For example :* If  $X + 3 < 2$  , then  $X + 3 - 3 < 2 - 3$  *i.e.*  $X < -1$

- We can multiply any positive rational number by both sides of the inequality without change in the inequality relation.

*For example :* If  $\frac{1}{3} X < 2$  , then  $\frac{1}{3} X \times 3 < 2 \times 3$  *i.e.*  $X < 6$

- We can multiply any negative rational number by both sides of the inequality with change in the inequality relation.

*For example :* If  $\frac{1}{3} X < 2$  , then  $-\frac{1}{3} X \times -3 > 2 \times -3$  *i.e.*  $X > -6$

- We can divide any positive rational number by both sides of the inequality without change in the inequality.

*For example :* If  $7 X > 14$  , then  $\frac{7 X}{7} > \frac{14}{7}$  *i.e.*  $X > 2$

- We can divide any negative rational number by both sides of the inequality with change in the inequality

*For example :* If  $-7 X > 14$  , then  $\frac{-7 X}{7} < \frac{14}{7}$  *i.e.*  $X < -2$

**Example**

Find in  $\mathbb{Q}$  the solution set of each of the following inequalities :

①  $2X - 5 > 5$

②  $4 - 2X \leq 2$

**Solution**

①  $2X - 5 > 5$  "Adding 5 to both sides"

$\therefore 2X > 10$  "Multiplying both sides by  $\frac{1}{2}$ "

$\therefore X > 5$

②  $4 - 2X \leq 2$  "Adding 4 to both sides"

$\therefore -2X \leq -2$  "Dividing both sides by  $(-2)$ "

Notice that : the change of inequality sign

$\therefore X \geq 1$

$\therefore 2X - 5 + 5 > 5 + 5$

$\therefore \frac{1}{2} \times 2X > 10 \times \frac{1}{2}$

$\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X > 5\}$

$\therefore 4 + 4 - 2X \leq 4 + 2$

$\therefore \frac{-2X}{2} \geq \frac{2}{2}$

$\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X \geq 1\}$



**Remember The probability**

- The probability of any event occurrence  $A \subset S$  is denoted by  $P(A)$  and it is given by using the relation :  $P(A) = \frac{\text{The number of elements of the event} = A}{\text{The number of elements of sample space} = S} = \frac{n(A)}{n(S)}$
- The probability of the impossible event = 0      • The probability of the certain event = 1
- The value of probability of any event is not less than zero and not more than one i.e.  $0 \leq \text{The probability of an event occurrence} \leq 1$
- It is meaningless that the probability of the occurrence of an event is 140% or - 0.2

**Example 1**

A fair die is rolled once and we observe the apparent number on the upper face , what is the probability of getting :

- the number 4
- an even number
- a number greater than or equal to 5
- a prime number
- a number greater than 6
- a number smaller than 10

**Solution**

- The probability of getting the number 4 =  $\frac{1}{6}$
- The probability of getting an even number =  $\frac{3}{6} = \frac{1}{2}$
- The probability of getting a number greater than or equal 5 =  $\frac{2}{6} = \frac{1}{3}$
- The probability of getting a prime number =  $\frac{3}{6} = \frac{1}{2}$
- The probability of getting a number greater than 6 =  $\frac{0}{6} = 0$
- The probability of getting a number smaller than 10 =  $\frac{6}{6} = 1$

**Example 2**

A bag contains 4 red balls , 6 green balls and 5 black balls , if a ball is drawn randomly from it , calculate :

- The probability that the drawn ball is green
- The probability that the drawn ball is black.
- The probability that the drawn ball is not red

**Solution**

- The probability that the drawn ball is green =  $\frac{\text{The number of green balls}}{\text{The total number of balls}} = \frac{6}{15} = \frac{2}{5}$
- The probability that the drawn ball is black =  $\frac{\text{The number of black balls}}{\text{The total number of balls}} = \frac{5}{15} = \frac{1}{3}$
- The probability that the drawn ball is not red =  $\frac{6+5}{15} = \frac{11}{15}$

# Final Examinations

on Algebra and Statistics



## Model Examination of the School Book



on Algebra and Statistics

## Model

1

Answer the following questions :

## 1 Complete :

1)  $\frac{81}{625} = \left(\frac{25}{9}\right)$

2) If  $7 - 2X = 3$ , then  $X = \dots\dots\dots$  where  $X \in \mathbb{R}$

3)  $3^{-1} + 4^{-1} = \dots\dots\dots$

4) The standard form of the number  $0.7 \times 0.005 = \dots\dots\dots$

5) The probability of the certain event = ..

## 2 Choose the correct answer :

1 The sum of the probabilities for all possible outcomes of a randomly experiment is

(a) zero

(b) 1

(c)  $> 1$

(d)  $< 1$

2 If  $3a = \sqrt{4}b$ , then  $\frac{a}{b} = \dots\dots\dots$

(a)  $2:3$

(b)  $3:2$

(c)  $3:4$

(d)  $4:3$

3  $\left(\frac{2}{3}\right)^{-3}$  equals

(a)  $\frac{-27}{8}$

(b)  $\frac{8}{27}$

(c)  $\frac{8}{27}$

(d)  $\frac{27}{8}$

4 There are 21 boys and 15 girls in a classroom, one pupil is chosen randomly, the probability that the chosen pupil is a girl =

(a)  $\frac{5}{12}$

(b)  $\frac{7}{12}$

(c)  $\frac{4}{7}$

(d)  $\frac{5}{8}$

5  $\sqrt{(-8)^2 + (-6)^2} = \dots\dots\dots$

(a)  $|-10|$

(b)  $\pm 10$

(c) 14

(d) -14

6 10 % of L.E.  $2\frac{1}{2} =$  L.E

(a)  $\frac{1}{4}$

(b)  $\frac{1}{2}$

(c) 1

(d) 25

## 3

[a] Simplify to the simplest form :  $\left(\frac{3}{7}\right)^0 \times \left(\frac{5}{2}\right)^2 \times \sqrt{6\frac{1}{4}}$

[b] Find the numerical value of the expression :

$3a + 8a + (4b)$  when  $a = 4$  ,  $b = -2$



## Algebra and Statistics

4 [a] Find in  $\mathbb{Q}$  the S.S. of :  $3x + 1 = 25$

[b] Find the value of :  $\frac{8 \times 8}{8^4}$

5 [a] A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.

The distance in thousand (km.)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought a tyre of this type , what is the probability of change it :

- 1 Before traveled 50 thousand km.
- 2 After traveled more than 100 thousand km.

[b] Find in  $\mathbb{Q}$  the S.S. of :  $2x + 5 < 16$

## Model

Answer the following questions :

1 Complete :

1)  $\left(\frac{-2}{3}\right)^0 = \dots\dots\dots$

2)  $\sqrt{\frac{16}{49}} = \dots\dots\dots$

3) The probability of the impossible event =

4 1 , 2 , 3 , 5 , 8 , ..... , ..... (In the same pattern)

5 If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present students that day is

2 Choose the correct answer :

1)  $2^3 \times 2^3 =$

(a)  $2^6$

(b)  $2^8$

(c)  $2^{15}$

(d)  $2^{53}$

2 Which of the following is the greatest ?

(a)  $2.3 \times 10^4$

(b)  $2.3 \times 10^5$

(c)  $3.2 \times 10^4$

(d)  $3.2 \times 10^5$

3  $(X^2)^3 \times X^6 =$

(a)  $X^{12}$

(b)  $X^{-12}$

(c)  $X$

(d) 1

4 Which of the following may be probability of an event ?

(a) -0.35

(b) 87 %

(c) 1.05

(d) 130 %

5 If  $-X > 4$ , then

(a)  $X > -4$

(b)  $X > 4$

(c)  $X < -4$

(d)  $X < 4$

6 Area of a rectangle of length 120 cm and width 80 cm equals  $m^2$ 

(a) 9600

(b) 400

(c) 9.6

(d) 0.96

3 [a] Two integers numbers, the smaller one is  $2X$  and the greater is  $5X$ , if the difference between them is 30 Find the two numbers.[b] Find the value of :  $5^4 \times 5^3$  in the simplest form.4 [a] Find in  $\mathbb{Q}$  the S.S. of each of the following :

1  $(3X + 2) + 5 = 13$

2  $2X + 15 < 19$

[b] Find the value of the expression in the simplest form :

$\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

5 [a] If a regular die is thrown once and observed the number on upper face,

find the probability of each of the following :

1, Getting a prime even number

2, Getting an odd number less than 4

[b] If  $X = -\frac{1}{2}$ ,  $y = -\frac{3}{4}$ , find in the simplest form :  $\left(\frac{y}{X^2}\right)^2$

## Algebra and Statistics

## Model examination for the merge students

Answer the following questions :

## 1 Choose the correct answer :

1)  $\left(\frac{-2}{3}\right)^2 =$

(a)  $\frac{4}{9}$

(b)  $\frac{4}{9}$

(c)  $\frac{4}{6}$

(d)  $\frac{4}{6}$

2)  $\left(\frac{4}{7}\right)^0 =$

(a) 0

(b) 1

(c)  $\frac{4}{7}$

(d) 1

3)  $2 \times 6 - 4 \times 2 =$

(a) 4

(b) 8

(c) 10

(d) 2

4)  $(7)^{-2} =$

(a) 49

(b)  $\frac{1}{49}$

(c) 14

(d) -14

5)  $\sqrt{9+16} =$

(a) 7

(b) 5

(c) 25

(d) -7

## 2 Complete each of the following :

1) If  $X + 2 = 6$ , then  $X =$

2) When tossing a coin once, then the probability of the appearance of a tail =

3) The probability of the impossible event =

4)  $\sqrt{\left(\frac{2}{3}\right)^2} =$

5)  $7(6^2 - 5 \times 6) =$

## 3 Complete the solution to find the result :

1)  $12 \times 2^2 + 24 + 3^2 = 12 \times \dots + 24 +$

$= \dots + 24 + \dots = \dots + \dots = \dots$

2)  $\frac{8+20-4}{8-4} = \frac{4}{\dots} = \dots$



4 Put (✓) or (X) :

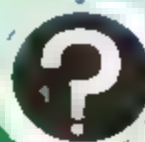
- 1 If  $2X + 3 = 7$ , then  $X = 2$  ( )
- 2  $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^6$  ( )
- 3  $(X^2)^3 = X^6$  ( )
- 4  $\left(\frac{3}{2}\right)^2 = -\frac{9}{4}$  ( )
- 5  $\sqrt{100 - 64} = 2$  ( )

5 A card is drawn randomly from 8 cards numbered from 1 to 8

Join from column (A) to column (B) :

Column (A)	Column (B)
1 The event of getting an even number equals .....	• $\frac{1}{2}$
2 The probability of getting an even number equals	• $\{8, 6, 4, 2\}$
3 The event of getting a number $> 6$ equals	• 1
4 The probability of getting a number $< 9$ equals .....	• $\frac{1}{8}$
5 The probability of getting a number 8 equals	• $\{8, 7\}$

## Schools Examinations



on Algebra and Statistics

1

Cairo Governorate

AL Morhe Directorate of Education  
Modern Language Schools

Answer the following questions :

1 Choose the correct answer :

1)  $7x^{-1} =$

(a)  $7x$

(b)  $\frac{7}{x}$

(c)  $7x^2$

(d) 0

2 If  $-x < 2$ , then

(a)  $x < 2$

(b)  $x \leq 2$

(c)  $x > 2$

(d)  $x > -2$

3  $\frac{6a^2x^4}{2a^1x^1} =$  where  $a \neq 0, x \neq 0$

(a)  $3a^2x$

(b)  $3a^2x^7$

(c)  $\frac{3x}{a}$

(d)  $\frac{3}{a^2x}$

4  $\sqrt{(-6)^2 + (-8)^2} = 25$

(a) 15

(b) 10

(c) 6 (d) 8

5 If  $5x = 35$ , then  $2x + 1 =$

(a) 7

(b) 8 (c) 15

(d) 7

6  $2^3 \cdot 2^5 =$

(a)  $2^2$

(b)  $2^8$

(c)  $2^{13}$

(d)  $2^{53}$

2 Complete :

1  $\sqrt{\frac{25x^2y^2}{36}} =$

2 The S.S. of :  $x + 21 = 8$  in  $\mathbb{Q}$  is

3 The additive inverse of  $\left(-\frac{2}{5}\right)^2 =$

4 The probability of certain event =

5  $5 + 8 + 2 - 3 \times 4 =$

3 [a] If  $x = \frac{3}{2}, y = \frac{1}{2}, z = \frac{4}{3}$ , find the numerical value of :  $x^2y^2z^2$

[b] Find the S.S. of :  $5x + 8 = 13 - 2x, x \in \mathbb{Q}$

4 [a] Find the S.S. in  $\mathbb{Q}$  :  $3x - 1 \leq 5$

[b] Find the value of :  $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

Some school exams  
are modified to include  
what was canceled  
last year

- 5 [a] A box contain 4 white balls , 5 red balls , 6 blue balls . One ball is drawn randomly , Find the probability of the drawn ball is :

1 red                      2 black                      3 white or blue

- [b] Two natural numbers , one of them twice the other and their sum is 24  
Find the two numbers

2

Cairo Governorate

El Mansh Directorate  
El Dragan Elwan School



Answer the following questions :

- 1 Choose the correct answer :

1 If  $0 \in \{5, x-3\}$  , then  $x =$

(a) 0                      (b) -5                      (c) 3                      (d) -3

2  $3^4 \times 3^3 =$

(a)  $3^{12}$                       (b)  $3^7$                       (c) 3                      (d)  $3^{-1}$

3 The S.S. of the inequality :  $x < 3$  in  $\mathbb{Z}$  is

(a)  $\{0\}$                       (b)  $\{0, 1, 2\}$                       (c)  $\{1, 2\}$                       (d)  $\emptyset$

4  $\frac{6a^3x^4}{3a^2x^5} =$  where  $a \neq 0, x \neq 0$

(a)  $2a^3x$                       (b)  $2a^2x^7$                       (c)  $\frac{3x}{a}$                       (d)  $\frac{3}{a^2x}$

5  $\left(\frac{2}{3}\right)^3 =$

(a)  $\frac{27}{8}$                       (b)  $\frac{-8}{27}$                       (c)  $\frac{8}{27}$                       (d)  $\frac{-27}{8}$

6 If the probability of success of a student is 0.6 , then the probability of his failure is

(a) 1                      (b)  $\frac{1}{10}$                       (c)  $\frac{4}{10}$                       (d)  $\frac{6}{10}$

- 2 Complete the following :

1 The multiplicative inverse of 7 is

2 0.00025 in scientific notation =

3  $\left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 =$

4  $\sqrt{16+9} = 4 +$

5 A class has 36 pupils , 25 of them are boys , if a pupil is chosen randomly , then the probability that the pupil is a girl =



## Algebra and Statistics

- 3 [a] Find the simplest form of :  $\left(\frac{7^{-2} \times 7^3}{7^1}\right)^2$   
 [b] Simplify and find the value of :  $\left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{2}{3}\right)^0$   
 [c] \* Find the value of :  $12 \times 2^2 + 24 \div 3^2$

- 4 [a] 1 Calculate :  $\sqrt{100 - (-8)^2}$   
 2 If  $x = \frac{1}{2}$  ,  $y = \frac{2}{3}$  , then find the value of :  $(x^2 y^2)^{-3}$   
 [b] Find in  $\mathbb{Q}$  the S.S. of the inequality :  $3x + 6 > 3$

- 5 [a] Find the solution set in  $\mathbb{Q}$  :  $4x - 5 = 27$   
 [b] A fair die is rolled once. Calculate the probability of appearance :  
 1 an even number. 2 a number greater than 4

3

Cairo Governorate

El-Zaiton Zone  
Taher Gaber El Anany Language School

Answer the following questions :

- 1 Choose the correct answer :

1 If  $4x = 20$  , then  $3x - 1 = \dots$

(a) 13 (b) 14 (c) 15 (d) 16

2  $3^{10} + 3^{10} + 3^{10} = \dots$

(a)  $9^{30}$  (b)  $3^{30}$  (c)  $3^{10}$  (d)  $3^{11}$

3 If  $\frac{4x-1}{2x+3} = 0$  where  $x \in \mathbb{Q}$  , then  $x =$

(a)  $-\frac{3}{2}$  (b)  $\frac{2}{3}$  (c)  $\frac{1}{4}$  (d) 0

4 A class contains 40 students , 36 of them are succeed in a test , then the probability of failed is

(a) 0.1 (b) 0.9 (c) 0.3 (d)  $\frac{2}{3}$

5  $\sqrt{(8)^2 + (6)^2} = 8 + \dots$

(a) 14 (b) 10 (c) 2 (d) 6

6  $\frac{1-5|+1}{3} \dots \mathbb{Z}$

(a)  $\in$  (b)  $\notin$  (c)  $\subset$  (d)  $\not\subset$

- 2 Complete the following :

1 If  $0.0000016 = 1.6 \times 10^n$  , then  $n =$

- 2 The multiplicative invers of  $\sqrt{5\frac{4}{9}}$  =
- 3 When a fair die is tossed once , then the probability of getting an even prime number = .....
- 4 If  $\frac{3}{4} X = 75$  , then  $\sqrt{X}$  =
- 5 25 % from ..... = 8

3 [a] Find in the simplest form :  $\left(\frac{5^5 \times 5^{-2}}{5^4}\right)^2$

[b] Find the S.S. in Q :  $-2 < 4X + 1 < 6$

4 [a] Simplify :  $\left(\frac{-2}{7}\right)^{-2} \times \sqrt{\frac{16}{49}} \times \left(\frac{-1}{7}\right)^0$

- [b] If the length of a rectangle is 5 cm. more than its width and its perimeter is 42 cm. Find the area of this rectangle.

5 [a] Simplify :  $\frac{(4X^3Y^2)^2}{(2XY^3)^3}$  , then find the value when  $X = \frac{1}{2}$  ,  $Y = 3$

- [b] A team plays 30 matches in national league its drawn probability is 0.2 and its win probability is 0.7 Calculate the number of loss matches

4

Giza Governorate

El-Dokki Directorate  
Modern Mother language School

Answer the following questions :

- 1 Choose the correct answer :

[a] If  $\left(\frac{1}{2}\right)^X = 8$  , then  $X$  =

- (a) 4 (b) -4 (c) 3 (d) -3

[a]  $\sqrt{\left(\frac{-2}{9}\right)^2}$  =

- (a)  $\pm \frac{2}{9}$  (b)  $\frac{2}{9}$  (c)  $\frac{-2}{9}$  (d)  $\frac{2}{3}$

3 If  $X + 4 = 10$  , then  $5X$  = .....

- (a) 30 (b) 20 (c) 12.5 (d) 25

4 If  $(2^X)^Y = 8$  , then

- (a)  $X + Y = 3$  (b)  $XY = 3$  (c)  $X - Y = 3$  (d)  $\frac{X}{Y} = 3$

5 If  $X$  is a rational number where  $-X > 4$  , then

- (a)  $X > -4$  (b)  $X > 4$  (c)  $X < -4$  (d)  $X < 4$

## Algebra and Statistics

8 If a letter is selected randomly from the word "SCHOOL", then the probability that the letter is O equals

- (a) 2 (b)  $\frac{1}{2}$  (c)  $\frac{1}{3}$  (d)  $\frac{1}{6}$

2 Complete :

1 If  $\sqrt{x+3} = 3$ , then  $x =$

2  $\left(1\frac{1}{2}\right)^{-2} =$

3  $2^5 \times 5^5 = 10$

4  $4 \times 3^2 - 20 =$

5 The probability of the certain event is

3 (a) Find in the simplest form :  $\left(\frac{5}{7}\right)^0 \times \left(-\frac{3}{2}\right)^2 \times \sqrt{\frac{16}{9}}$

(b) Write the result of the following in the standard form of  $a \times 10^n$ ,  $n \in \mathbb{Z}$  :

$(4.4 \times 10^3) \times (3 \times 10^5)$

4 (a) Find in  $\mathbb{Q}$  the solution set of the equation :  $3x + 1 = 25$

(b) Find the value of :  $\frac{3^{-4} \times 3^1}{3^1}$  in the simplest form

5 (a) Find in  $\mathbb{Q}$  the solution set of the inequality :  $2x + 5 < 9$

(b) If a die is rolled once and the number of dots on the upper face is observed, write down the sample space, then find the probability of the following events :

1 getting a number greater than 6

2 getting a number satisfies the inequality  $1 < x < 6$

3 getting a number divisible by 3

5

Giza Governorate

Oronate Directorate  
El Sadat E.L.S

Answer the following questions :

1 Choose the correct answer :

1  $x^{12} + x^4 = \dots$  where  $x \neq 0$

(a)  $x^8$

(b)  $x^3$

(c)  $x^{16}$

(d)  $x^{-8}$

2  $2 \pm \sqrt{\frac{4}{9}} =$

(a)  $\frac{4}{9}$

(b)  $\frac{-2}{9}$

(c)  $\pm \frac{2}{3}$

(d)  $\frac{2}{3}$



## Final Examinations

- 3 If the probability that pupils success is 75 % , then the probability of his failure is  
 (a) -0.75 (b) 0.25 (c) -0.25 (d) 0.75
- 4 If  $-X < 7$  , then  $X$  ..... - 7  
 (a) < (b) > (c) = (d)  $\leq$
- 5  $6000 \times 50 =$   
 (a)  $300 \times 10^2$  (b)  $30 \times 10^5$  (c)  $-3 \times 10^3$  (d)  $3 \times 10^5$
- 6 If  $3X = 6$  , then  $5X =$   
 (a)  $\frac{5}{2}$  (b)  $\frac{2}{5}$  (c) 10 (d) 5

## 2 Complete each of the following :

- 1 The probability of impossible event is
- 2 The additive inverse of the number  $(-\frac{1}{3})^2$  is
- 3  $\sqrt{16+9} =$
- 4  $(\frac{2}{3})^{-2} =$
- 5  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$  (in the same pattern)

## 3 [a] Find in the simplest form :

$$1 \frac{9^{-2} \times 9^3}{9} \quad 2 \left(\frac{3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{5}{2}\right)^0$$

[b] Find in  $\mathbb{Q}$  the solution set of the following equation :  $4X + 1 = 21$

4 [a] If  $X = \frac{1}{2}$  ,  $y = \frac{2}{3}$  ,  $z = \frac{3}{2}$  , then find the value of :  $(X \cdot y \cdot z)^2$ 

[b] Find in  $\mathbb{Q}$  the solution set of the following inequality :  $3X - 1 \leq 2X + 3$

## 5 [a] The sum of three consecutive numbers is 24 , find them

[b] A fair die is rolled once , calculate the probability of appearance :

1, an even number.

2 a number greater than 5

## 6 Alexandria Governorate

Inspectorate of Mathematics



Answer the following questions :

## 1 Complete each of the following :

- 1 The multiplicative identity element in  $\mathbb{Q}$  is

## Algebra and Statistics

2.  $\sqrt{\frac{25}{49}} = \underline{\hspace{2cm}}$

3.  $(X-5)^0 = 1$ , if  $X \neq$

4. If the probability of success of a student is 0.8, then the probability of his failure is

5. If  $X \subset Y$ , then  $X \cap Y =$

2 Choose the correct answer :

1)  $\sqrt{64+36} = 8 +$

(a) 2

(b) 6

(c) 10

(d) -2

2 Half the number  $2^{24}$  equals ..

(a)  $2^{12}$

(b)  $1^{23}$

(c)  $2^{23}$

(d)  $1^{12}$

3.  $0.0000068 =$

(a)  $6.8 \times 10^{-6}$

(b)  $6.8 \times 10^5$

(c)  $6.8 \times 10^{-7}$

(d)  $6.8 \times 10^7$

4 If  $2X = -12$ , then  $X^2 =$

(a) 6

(b) 144

(c) -36

(d) 36

5 The S.S. of the equation  $5 - X = 3$  in  $\mathbb{Q}$  is

(a)  $\{2\}$

(b)  $\{-2\}$

(c)  $\{7\}$

(d)  $\emptyset$

6 The probability of a certain event equals .....

(a) zero

(b) 1

(c) -1

(d)  $\frac{1}{2}$

3 [a] Simplify to the simplest form :  $\frac{X^{-1} \times X^{-2}}{X^{-1} \times X^{-2}}$ , then find the value when  $X = (-2)$

[b] Find the S.S. in  $\mathbb{Q}$  for each of the following :

1  $3X - 5 > 1$

2  $3X + 6 = 30 - 5X$

4 [a] A fair die is rolled once, what is the probability of getting :

1 an even number ? 2 a factor of 6 ?

[b] Find the value of :  $\sqrt{6\frac{1}{4}} \times \left(\frac{2}{7}\right)^0 \times \left(-\frac{2}{5}\right)^2$

5 [a] A rectangle whose length is more than its width by 3 and its perimeter equals 26, find its area.

[b] If  $X = \frac{1}{2}$  and  $y = \frac{4}{3}$  find the value of :  $X^3 y^2$

7

Alexandria Governorate

El-Montaza Educational Zone  
Math's Supervision

Answer the following questions :

## 1 Choose the correct answer :

1  $2^3 \times 2^5 =$

(a)  $2^2$

(b)  $2^8$

(c)  $2^{15}$

(d)  $2^{23}$

2 Which of the following may be a probability of an event ?

(a)  $-0.25$

(b)  $87\%$

(c)  $1.05$

(d)  $130\%$

3 The multiplicative inverse of the number  $\sqrt{\frac{9}{16}}$  is

(a)  $-\frac{4}{3}$

(b)  $-\frac{3}{4}$

(c)  $\frac{4}{3}$

(d)  $\frac{3}{4}$

4  $5x^{-1} =$

(a)  $-5x$

(b)  $5x$

(c)  $\frac{5}{x}$

(d)  $\frac{1}{5x}$

5 If  $-x < 3$ , then

(a)  $x > -3$

(b)  $x > 3$

(c)  $x < -3$

(d)  $x < 3$

6 If  $x + 2 = 5$ , then  $5x =$ 

(a)  $3$

(b)  $35$

(c)  $15$

(d)  $7$

## 2 Complete :

1 The probability of the certain event = ..

2 If  $0.00057 = 5.7 \times 10^n$ , then  $n =$  ..3 The additive inverse of  $(-\frac{1}{2})^2$  is ..

4  $\sqrt{16+9} = 4 +$  ..

5  $2^{10} + 2^{10} = 2$  ..

3 [a] Find in  $\mathbb{Q}$  the solution set of the following : 1  $3x + 1 = 16$  2  $7x - 1 < 13$ [b] \* Find the value of :  $5^2 + [3 \times 8 - 2^2 - 2 \times 3]$ 4 [a] If  $x = \frac{1}{3}$ ,  $y = \frac{1}{6}$ , find the numerical value of :  $(x + y)^{-1}$ 

[b] What is the number that if added to its three times, the result is 24 ?

5 [a] Simplify :  $\frac{a^3 \times a^8}{a^3}$ ,  $a \neq 0$ 

[b] A box contains 4 white, 5 red and 6 blue balls, a ball is drawn randomly from the box, find the probability of getting the following events :

1 the ball is blue.

2 the ball is white or red



## Algebra and Statistics

## 8 El-Kalyoubia Governorate

Directorate of Education  
Math Supervision

Answer the following questions :

## 1 Choose the correct answer :

1 The number which is not in the standard form is

- (a)  $6.2 \times 10^5$  (b)  $7.834 \times 10^{10}$  (c)  $0.8 \times 10^5$  (d)  $6.7 \times 10^{23}$

2 If  $3t = 6$ , then  $6t =$ 

- (a) 16 (b) 14 (c) 12 (d) 10

3  $\frac{1}{4} \times 4^{20} =$ 

- (a)  $4^{15}$  (b)  $4^{10}$  (c)  $2^{10}$  (d)  $2^{30}$

4  $\frac{6a^2x^4}{2a^3x^5} = \dots$  where  $a \neq 0$ ,  $x \neq 0$ 

- (a)  $3ax$  (b)  $3a^3x^7$  (c)  $\frac{3x}{a}$  (d)  $\frac{3}{a^3x}$

5 A class formed from 36 students, 16 of them are girls. If a student selected randomly from the class, then the probability that the student is a boy =

- (a)  $\frac{4}{9}$  (b)  $\frac{5}{9}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{36}$

6 10 % of  $2\frac{1}{2}$  L.E. = ..... L.E.

- (a)  $\frac{1}{2}$  (b)  $\frac{1}{4}$  (c) 1 (d) 25

## 2 Complete :

1 The additive inverse of  $(-\frac{2}{3})^4$  is

2 When a die is tossing twice and observed the upper face in each time the probability of appearance number 5 on the two faces is

3  $\sqrt{10^2 - 6^2} =$ 4 If  $X + 2 > 6$  and the substitution set  $\{-2, -5, -1\}$ , then  $X =$ 5 If  $3a = \sqrt{4b}$ , then  $\frac{a}{b} =$ 3 [a] Find in simplest form :  $(-\frac{1}{3})^2 + \sqrt{\frac{64}{81}} - (\frac{a}{b})^0$  where  $a \neq 0$ ,  $b \neq 0$ [b] If  $X = \frac{1}{2}$ ,  $y = \frac{3}{4}$ , find the value of :  $(\frac{y}{X})^{-2}$ 4 Find in  $\mathbb{Q}$  the S.S. of :

1  $3X + 1 = 25$

2  $15 + 2X < 1$

5 [a] Find with steps the value of :  $(2 \times \sqrt{16} - 2^4) - 4$

[b] If a regular die is thrown once and observing the number on the upper face

Find the probability of getting :

1 a prime even number

2 an odd number less than 4

9 El-Sharkia Governorate

El-Sharkia Educational Administration  
Omar Al-Farouk Formal Language School



Answer the following questions :

1 Complete the following :

1  $\left(\frac{-2}{3}\right)^0 =$

2 The probability of certain (sure) event =

3  $\sqrt{a^4 b^2} =$

4 The number 0.00023 in the standard form is

5  $* 7 (6^2 - 5 \times 6) =$

2 Choose the correct answer :

1] Half the number  $2^{10} =$

(a)  $2^{10}$

(b)  $2^{11}$

(c)  $2^9$

(d) 2

2  $\sqrt{10^2 - 6^2} =$

(a) 4

(b) 8

(c) 1

(d) 0

3 If the probability of success of a student is 0.7 , then the probability of his failure is

(a) 0.7

(b) 0.4

(c) 1

(d) 0.3

4  $7^{-1} =$

(a)  $\frac{1}{7}$

(b) 7

(c)  $-\frac{1}{7}$

(d) -7

5  $\frac{4}{10} + \frac{3}{100} =$

(a) 0.34

(b) 0.43

(c) 4.3

(d) 3.4

6  $5^3 \times 5^4 =$

(a)  $5^3$

(b)  $25^6$

(c)  $5^6$

(d)  $25^6$

3 [a] Find in Q the solution set of :  $2x + 1 = 9$

[b] Find the value of the following in the simplest form :  $\left(\frac{1}{4}\right)^{-1} + \sqrt{\frac{64}{81}} + \left(\frac{3}{7}\right)^0$

## Algebra and Statistics

- 4 [a] Simplify to the simplest form :  $\frac{3^6 \times 3^{-2}}{3^3}$   
 [b] If  $X = -\frac{1}{2}$  ,  $y = \frac{3}{4}$  , find the value of :  $\frac{y}{X^2}$

- 5 [a] Find in  $\mathbb{Q}$  the solution set of :  $3X - 2 \leq 7$

[b] A box contains 5 black balls , 3 white balls and 6 red balls , if a ball is drawn randomly from the box Calculate the probability of :

(1) the ball is white.

(2) the ball is not red

## 10 | El-Monofia Governorate

Monof Educational Directorate



Answer the following questions : (Calculators are Permitted)

- 1 Choose the correct answer :

- 1-  $(3^2)^3 =$  .....  
 (a)  $3^5$  (b)  $2^6$  (c)  $9^3$  (d)  $3^8$   
 2-  $|-3| + 5 =$  .....  
 (a) -8 (b) -2 (c) 2 (d) 8  
 3- If  $A = 7^X$  ,  $B = 7^{-X}$  , then  $A \times B =$  .....  
 (a) 49 (b) 7 (c) 1 (d) 0  
 4- Quarter of  $2^{20} =$  ....  
 (a)  $2^{10}$  (b)  $2^{19}$  (c)  $4^{19}$  (d)  $4^9$   
 5- 0.0037 in the standard form is  $3.7 \times 10^X$  , then  $X =$  .....  
 (a) 3 (b) 4 (c) -3 (d) -4  
 6- Which of the following could be a probability of an event ?  
 (a) 2 (b) 3 (c) -1 (d) 3 %

- 2 Complete each of the following :

1. 1 , 2 , 3 , 5 , 8 , ..... (in the same pattern)  
 2 The S.S. of inequality :  $-X > 0$  in  $\mathbb{R}$  is .....  
 3  $\sqrt{3^2 + 4^2} = 2 + \dots$   
 4  $\frac{8}{27} = \left(\frac{\dots}{\dots}\right)^3$   
 5 The probability of impossible event = .....



3 [a] Find the S.S. of the following in Q :

1  $5x - 2 = 8$

2  $2x + 3 > 4$

(b) \* Calculate the value of :  $2[(5^2 + 1) - (4^2 - 1)]$

4 [a] Simplify :  $\left(\frac{2}{3}\right)^3 \times \sqrt{\frac{81}{64}} \times \left(\frac{1}{3}\right)^{\text{zero}}$

(b) A box contains 3 red balls , 4 yellow balls and 5 green balls . A ball is drawn randomly from the box . Find the probability of the drawn ball is :

1 yellow.

2 not green

5 [a] Evaluate :  $\frac{7^5 \times 7^{-2}}{7^3}$

(b) If  $x = \frac{1}{2}$  ,  $y = \frac{1}{3}$  , find the value of the expression :  $(4x^2 - y)^2$

### 11 El-Dakahlia Governorate

Dakahlia Directorate of Education  
Math supervision



Answer the following questions :

1 Choose the correct answer from those given :

1 The additive inverse of  $(-1)^{10}$  is .....

(a) 0

(b) -1

(c) -10

(d) 1

2 Half milliard =  $5 \times 10$

(a) 6

(b) -9

(c) 8

(d) 9

3 Quarter of the number  $2^{16}$  is

(a)  $1^{16}$

(b)  $4^7$

(c)  $4^{13}$

(d)  $2^4$

4  $\sqrt{144} + \dots = 12 + 5$

(a) 40

(b) 25

(c) 16

(d) 145

5 If the probability of success of a student is 0.7

, then the probability of his failure is

(a) 0.03

(b) 1

(c) 30 %

(d) 3

6 If  $8y = 16$  , then  $y + 5 =$

(a) 10

(b) 13

(c) 21

(d) 7

2 Complete each of the following :

1 The smallest odd prime number is

2 If the area of a square is  $169 \text{ k}^2 \text{ cm}^2$  , then its side length =

cm.

## Algebra and Statistics

3. If  $y = \frac{1}{4}$ ,  $x = \frac{1}{3}$ , then  $(x - y)^{-2} =$

4) ..... is a subset of the sample space

5 The S.S. of the inequality  $5 \leq x \leq 6$  in  $\mathbb{N}$  is

- 3 [a] Divide :  $(25a + 5a) \div 5a$ , then find the numerical value of the expression when  $a = 2$ ,  $b = -1$

[b] Simplify to the simplest form :  $\left(\frac{49}{25}\right)^0 \times \left(\frac{-2}{7}\right)^2 \times \sqrt{12\frac{1}{4}}$

- 4 [a] Find in  $\mathbb{Q}$  the S.S. of the following :

$$1) 2x + 7 < 15$$

$$2) 6x + 6 = 6$$

[b] Simplify to the simplest form :  $\frac{(3)^{-6} \times (3)^{11}}{(3)^4}$

- 5 [a] Find in  $\mathbb{N}$  the S.S. of the inequality :  $5x - 2 \geq 3$ , then represent it on the number line

[b] A ball chosen randomly from a bag contains 5 red balls + 7 blue balls + 3 yellow balls, find the probability of each of the following :

(1) getting black ball.

(2) getting a red ball

12 El-Amalia Governorate

Directorate of Education  
Math's Supervision



Answer the following questions :

- 1 Choose the correct answer :

1) If  $x = y$ , then  $\left(\frac{3}{4}\right)^{x-y} =$

(a) 0

(b) 1

(c)  $\frac{3}{4}$

(d)  $\frac{1}{2}$

2 If  $4790000 = a \times 10^6$ , then  $a =$

(a) 479

(b) 47.9

(c) 4.79

(d) 470

3 If  $\frac{x-3}{x+4} = 0$ , then  $x =$

(a) 3

(b) -3

(c) 4

(d) -4

4 The ratio between two numbers is  $1 : 2$ , if the first is 100, then the second is

(a) 50

(b) 25

(c) 10

(d) 200

5  $7x^2y \div$

(a)  $\frac{7}{x^2y}$

(b)  $\frac{7x^2}{y}$

(c)  $\frac{x^2y^2}{7}$

(d)  $\frac{x^2}{7y^2}$

8 When tossing a die once , the probability of getting an odd number =

- (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c)  $\frac{2}{3}$  (d)  $\frac{5}{6}$

**2 Complete :**

1 If  $2X - 1 = 5$  , then  $10X =$

2  $5y^2 \times 3y^2 =$

3  $\sqrt{\frac{25x^4}{y^4}} =$  where  $y \neq 0$

4 If  $3^{10} + 3^{10} + 3^{10} = 3^x$  , then  $x =$

5  $4 + 4 \times 4 + 4 - 2^2 =$

**3 (a) Find in the simplest form :  $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$**

(b) Simplify :  $\frac{a^7 \times a^5}{a^4 \times a^6}$  , then find the value at  $a = -3$

**4 (a) Find the S.S. in Q of :  $3X + 4 < 25$**

(b) If  $x = \frac{1}{2}$  ,  $y = -\frac{2}{3}$  ,  $z = \frac{3}{4}$

Find the value of :  $x^2 y^2 + z$

**5 (a) One card is selected randomly from 8 cards numbered from 1 to 8 , find the probability of getting :**

- 1 an even number      2 a prime number      3 a number more than 7

(b) Find the S.S. in Q of :  $6X - 8 = 22$



Damietta Governorate

Damietta inspection of Mathematics  
official language schools



Answer the following questions :

**1 Choose the correct answer :**

1 If  $X + 9 = 11$  , then the value of  $7X =$

- (a) 9 (b) 14 (c) 2 (d) 13

2  $\left(\frac{2}{3}\right)^3 =$

- (a)  $\frac{27}{8}$  (b)  $\frac{8}{27}$  (c)  $\frac{8}{27}$  (d)  $\frac{27}{8}$

3  $2^3 + 2^3 =$

- (a)  $2^6$  (b)  $2^4$  (c)  $2^9$  (d) 1



## Algebra and Statistics

4 If  $-x > 4$ , then

- (a)  $x > -4$       (b)  $x > 4$       (c)  $x < -4$       (d)  $x < 4$

5  $x^2 + x^2 =$ 

- (a)  $x^4$       (b)  $x^2$       (c)  $2x^2$       (d)  $2x^4$

6 Three times of a number is 48, then  $\frac{1}{4}$  the number is

- (a) 16      (b) 4      (c) 12      (d) 8

2 Complete each the following :

[1] The probability of the certain event equals

2 The number 0.000053 in the scientific notation =

[3] The multiplicative inverse of  $\sqrt{\frac{9}{25}}$  is

4 A class has 36 pupils, 20 of them are boys. If a pupil is chosen randomly, then the probability that the pupil is a girl =

[5] 1, 2, 3, 5, 8, ..... (in the same pattern).

3 [a] Simplify to the simplest form :  $\left(-\frac{5}{3}\right)^2 \times \left(-\frac{4}{9}\right)^0 \times \sqrt[3]{\frac{6}{25}}$ [b] Find the solution set of each of the following where  $x \in \mathbb{Q}$  :

1  $x + 1 > 25$

2  $5x + 8 = 15 - 2x$

4 [a] Reduce :  $\frac{x^5 \times x^9}{x^6 \times x^8}$  to the simplest form, then find the value of the result when  $x = 3$ 

[b] The sum of three consecutive even numbers is 60, Find them

5 [a] If  $x = 3$  and  $y = -4$ , find the value of :  $\sqrt{x^2 + y^2}$ 

[b] A box contains 4 white, 5 red and 6 blue balls, a ball is drawn randomly from the box. Calculate the probability of getting :

1 a blue ball.

2 a white or red ball

3 a green ball

## 14 El-Fayoum Governorate

Fayoum west Administration



Answer the following questions :

1 Choose the correct answer :

1) The multiplicative inverse of  $\sqrt{\frac{100}{81}}$  =

(a)  $\pm \frac{10}{9}$

(b)  $\pm \frac{9}{10}$

(c)  $\frac{10}{9}$

(d)  $\frac{9}{10}$

2 The probability of the impossible event =

- (a) zero (b) 1 (c)  $\frac{1}{2}$  (d)  $\frac{1}{3}$

3 If  $a = b$ , then  $\left(\frac{2}{3}\right)^a =$

- (a) zero (b) 1 (c)  $\frac{2}{3}$  (d)  $\frac{3}{2}$

4  $3^{10} + 3^{10} + 3^{10} =$

- (a)  $3^{30}$  (b)  $3^{11}$  (c)  $9^{10}$  (d)  $9^{11}$

5 If  $3x + 1 = 16$ , then the value of  $5x =$

- (a) 10 (b) 15 (c) 25 (d) 26

6  $5^{-1} =$

- (a) -5 (b)  $\frac{1}{5}$  (c) 5 (d) 25

2 Complete the following :

1 If  $2a = 10$ , then  $a^2 b^2 = \dots$

2  $* 3 \times 4 - 21 + 3 =$

3  $x = \frac{1}{2}$ ,  $y = \frac{3}{4}$ ,  $y + x =$

4  $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \dots$  (in the same pattern)

5  $(5 \times 10^2) \times (9 \times 10^3) =$  (in the standard form)

3 [a] Write the following in the simplest form :

1  $\frac{10^{-3} \times 10^6}{10^2}$

2  $\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^3$

[b] Find in  $\mathbb{Q}$  the solution set of the following :

1  $8 + 2x = 14$

2  $3x - 1 = -10$

4 [a] Find in  $\mathbb{Q}$  the solution set of each of the following :

1  $2(x - 3) = -x + 12$

2  $5x - 1 = 29$

[b] Calculate the following when  $a = 2$ ,  $b = 5$  : 1  $\frac{9-b}{a^3}$  2  $\frac{6^2}{a+1}$

5 [a] Find in  $\mathbb{Z}$  the solution set of the following :

1  $3x - 4 \geq -10$

2  $x + 2 \geq 2$

[b] A box contains 15 cards numbered from 1 to 15. A card is drawn randomly.

Find the probability of the drawn card carries :

1 an odd prime number.

2 a number less than or equals 1

3 a number more than 15

4 the number 15

## Algebra and Statistics

15

Qena Governorate

Qena Directorate of Education  
Math's supervision

Answer the following questions :

## 1 Choose the correct answer :

- 1 If  $x + 5 = 11$  , then  $3x =$   
(a) 16 (b) 18 (c) 1.8 (d) 6
- 2 The probability of the certain event =  
(a) 1 (b) 2 (c) 0 (d) 0.5
- 3  $\sqrt{13^2 - 5^2} =$   
(a) 11 (b) 12 (c) 13 (d) 14
- 4 If the standard form of 0.00000058 is  $5.8 \times 10^n$  , then  $n =$   
(a) 7 (b) -7 (c) 5 (d) 8
- 5  $10^{-3} =$   
(a) 10 (b) 1000 (c) 0.001 (d) 0.01
- 6  $3^2 \times 6 \div 3 + (2^4 - 6) =$   
(a) 18 (b) 28 (c) 42 (d) 32

## 2 Complete :

- 1 If the age of Amr now is  $x$  years , then his age after 5 years is \_\_\_\_\_ years.
- 2  $\sqrt{\left(\frac{4}{9}\right)^2} =$
- 3 The probability of the impossible event
- 4  $\left(\frac{4}{9}\right)^{-2} = \left(\frac{9}{4}\right)^n$  , then  $n =$
- 5 If half of  $2^{40} = 2^n$  , then  $n =$

3 (a) Find the S.S. in Q for :  $\frac{3}{5}x + 4 < 28$ 

- (b) If  $a = \frac{2}{3}$  ,  $b = \frac{3}{4}$  find the value of :  $a^3 b^3$

4 (a) Simplify :  $\left(\frac{4}{9}\right)^{-2} \times \left(\frac{4}{9}\right)^6$ 

- (b) Three consecutive even numbers their sum is 156 Find the numbers

5 (a) If  $3^x = 7$  ,  $3^y = 5$  Find :  $3^{x+y}$ 

- (b) A fair die is rolled once Calculate the probability of appearance of :

- 1 an even number      2 a number greater than 3      3 the number 5

# Final Examinations

on Algebra and Statistics





## Model Examinations of the School Book

on Algebra and Statistics

## Model

1

Answer the following questions :

## 1 Complete :

1  $\frac{81}{625} = \left(\frac{25}{9}\right)^{\dots}$

2 If  $7 - 2X = 3$ , then  $X = \dots$  where  $X \in \mathbb{R}$

3  $3^{-1} + 4^{-1} = \dots$

4 The standard form of the number  $0.7 \times 0.005 = \dots$

5 The probability of the certain event = ...

## 2 Choose the correct answer :

1 The sum of the probabilities for all possible outcomes of a randomly experiment is ...

(a) zero

(b) 1

(c)  $> 1$ (d)  $< 1$ 

2 If  $3a = \sqrt{4}b$ , then  $\frac{a}{b} = \dots$

(a)  $2:3$ (b)  $3:2$ (c)  $3:4$ (d)  $4:3$ 

3  $\left(-\frac{2}{3}\right)^{-3}$  equals

(a)  $-\frac{27}{8}$ (b)  $-\frac{8}{27}$ (c)  $\frac{8}{27}$ (d)  $\frac{27}{8}$ 

4 There are 21 boys and 15 girls in a classroom, one pupil is chosen randomly, the probability that the chosen pupil is a girl =

(a)  $\frac{5}{12}$ (b)  $\frac{7}{12}$ (c)  $\frac{4}{7}$ (d)  $\frac{5}{6}$ 

5  $\sqrt{(-8)^2 + (-6)^2} = \dots$

(a)  $|-10|$ (b)  $\pm 10$ 

(c) 14

(d) -14

6 10 % of L.E.  $2\frac{1}{2}$  = L.E. ...

(a)  $\frac{1}{4}$ (b)  $\frac{1}{2}$ 

(c) 1

(d) 25

## 3

(a) Simplify to the simplest form :  $\left(-\frac{3}{7}\right)^0 \times \left(-\frac{2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

(b) Find the numerical value of the expression :

$3ab + 8a + (4b)$  when  $a = 4$ ,  $b = -2$

## Algebra and Statistics

- 4 (a) Find in Q the S.S. of :  $3x + 1 = 25$

(b) Find the value of :  $\frac{8 \times 8^{-3}}{8^{-4}}$

- 5 (a) A factory of a tire record the distance that traveled by a certain type of them before damage for 800 units of this type as following.

The distance in thousand (km.)	Less than 50	50 to 100	More than 100 till 150	More than 150
The number of damage tire	80	120	280	320

If you bought a tyre of this type , what is the probability of change it :

- Before traveled 50 thousand km.
- After traveled more than 100 thousand km.

- (b) Find in Q the S.S. of :  $2x + 5 < 16$

## Model

2

Answer the following questions :

- 1 Complete :

(1)  $\left(-\frac{2}{3}\right)^0 = \dots\dots\dots$

(2)  $\sqrt{\frac{16}{49}} = \dots\dots\dots$

- 3 The probability of the impossible event = .....

- 4  $1, 2, 3, 5, 8, \dots\dots\dots$  (In the same pattern)

- 5 If the probability that the student is absent in a school is 0.15 , if the number of students of this school is 600 , then the number of the present students that day is .....

- 2 Choose the correct answer :

(1)  $2^3 \times 2^3 = \dots\dots\dots$

(a)  $2^6$

(b)  $2^8$

(c)  $2^{15}$

(d)  $2^{53}$

- (2) Which of the following is the greatest ?

(a)  $2.3 \times 10^4$

(b)  $2.3 \times 10^5$

(c)  $3.2 \times 10^4$

(d)  $3.2 \times 10^5$

3  $(x^2)^{-3} \times x^6 = \dots\dots$

(a)  $x^{12}$

(b)  $x^{-12}$

(c)  $x$

(d) 1

4 Which of the following may be probability of an event ?

(a) -0.35

(b) 87 %

(c) 1.05

(d) 130 %

5 If  $-X > 4$ , then .....

(a)  $X > -4$

(b)  $X > 4$

(c)  $X < -4$

(d)  $X < 4$

6 Area of a rectangle of length 120 cm. and width 80 cm. equals  $m^2$

(a) 9600

(b) 400

(c) 9.6

(d) 0.96

3 [a] Two integers numbers , the smaller one is  $2X$  and the greater is  $5X$  , if the difference between them is 30 Find the two numbers.

[b] Find the value of :  $\frac{5^{-4} \times 5^7}{5^3}$  in the simplest form.

4 [a] Find in  $\mathbb{Q}$  the S.S. of each of the following :

1)  $(3X + 2) + 5 = 13$

2)  $2X + 15 < 19$

[b] Find the value of the expression in the simplest form :

$$\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$$

5 [a] If a regular die is thrown once and observed the number on upper face ,

find the probability of each of the following :

1 Getting a prime even number.

2 Getting an odd number less than 4

[b] If  $X = -\frac{1}{2}$  ,  $y = -\frac{3}{4}$  , find in the simplest form :  $\left(\frac{y}{x^2}\right)^{-2}$

## Model examination for the merge students

Answer the following questions :

1 Choose the correct answer :

1  $\left(\frac{-2}{3}\right)^2 =$

(a)  $\frac{4}{9}$

(b)  $\frac{-4}{9}$

(c)  $\frac{4}{6}$

(d)  $\frac{-4}{6}$

2  $\left(\frac{4}{7}\right)^0 =$

(a) 0

(b) 1

(c)  $\frac{4}{7}$

(d) -1

3  $2 \times 6 - 4 \times 2 =$

(a) 4

(b) 8

(c) 10

(d) 2

4  $(7)^{-2} =$

(a) 49

(b)  $\frac{1}{49}$

(c) 14

(d) -14

5  $\sqrt{9+16} =$

(a) 7

(b) 5

(c) 25

(d) -7

2 Complete each of the following :

(1) If  $X + 2 = 6$ , then  $X = \dots\dots$

2 When tossing a coin once, then the probability of the appearance of a tail =  $\dots\dots$

3 The probability of the impossible event =  $\dots\dots$

4  $\sqrt{\left(\frac{2}{5}\right)^2} =$

5  $7(6^2 - 5 \times 6) =$

3 Complete the solution to find the result :

1  $12 \times 2^2 + 24 + 3^2 = 12 \times \dots\dots + 24 +$

$= \dots\dots + 24 + \dots\dots = \dots\dots + \dots\dots =$

2  $\frac{8+20-4}{8-4} = \frac{-4}{\dots\dots} = \dots\dots =$



4 Put (✓) or (X) :

1 If  $2X + 3 = 7$ , then  $X = 2$  ( )

2  $\left(\frac{2}{3}\right)^2 \times \left(\frac{2}{3}\right)^5 = \left(\frac{2}{3}\right)^6$  ( )

3  $(X^2)^3 = X^6$  ( )

4  $\left(\frac{3}{2}\right)^2 = -\frac{9}{4}$  ( )

5  $\sqrt{100-64} = 2$  ( )

5 A card is drawn randomly from 8 cards numbered from 1 to 8

, join from column (A) to column (B) :

Column (A)	Column (B)
1 The event of getting an even number equals .....	• $\frac{1}{2}$
2 The probability of getting an even number equals .....	• $\{8, 6, 4, 2\}$
3 The event of getting a number $> 6$ equals .....	• 1
4 The probability of getting a number $< 9$ equals .....	• $\frac{1}{8}$
5 The probability of getting a number 8 equals .....	• $\{8, 7\}$

## Schools Examinations

## on Algebra and Statistics

1

Cairo Governorate

Ministry of Education  
Matriculation Inspection

Answer the following questions :

1 Choose the correct answer from the given :

1  $\sqrt{9} =$  \_\_\_\_\_

- (a) 3 (b) -3  
(c)  $\pm 3$  (d) 81

2 If the probability of success of a student is 0.8 , then the probability of his failure is \_\_\_\_\_

- (a) 1 (b) -1 (c) 0.2 (d) 0.8

3  $5^{-1} =$  \_\_\_\_\_

- (a) -5 (b)  $\frac{1}{5}$  (c)  $-\frac{1}{5}$  (d) -15

4 If  $X > y$  , then  $X + z$  \_\_\_\_\_  $y + z$ 

- (a) < (b) > (c) = (d)  $\leq$

5  $\frac{4}{10} + \frac{3}{100} =$  \_\_\_\_\_

- (a) 0.34 (b) 0.43 (c) 4.3 (d) 3.4

6  $6000 \times 50 =$  \_\_\_\_\_

- (a)  $300 \times 10^2$  (b)  $3 \times 10^5$  (c)  $30 \times 10^3$  (d)  $-3 \times 10^3$

2 Complete each of the following :

1 The probability of the impossible event = \_\_\_\_\_

2  $(3^2)^{-1} =$  \_\_\_\_\_

3 Twice the number  $\frac{1}{2}$  = \_\_\_\_\_4 Quarter of the number  $4^{20}$  is 45  $3x + x + 2y + y$  in the simplest form is . .3 [a] Find in  $\mathbb{Q}$  the S.S. of the equation :  $2x + 1 = 9$ 

[b] Simplify to the simplest form :  $\frac{5^4 \times 5^{-2}}{5^2}$

[c] If  $x = 6$  ,  $y = 3$  , then find the value of :  $\left(\frac{x}{y}\right)^{-2}$

4 [a] Find the S.S. of the following inequality in  $\mathbb{Q}$ :  $4x - 3 < 7$

[b] Find the value of:  $12 \times 2 + 24 \div 9$

5 [a] Find the value of the expression in the simplest form:  $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

[b] A box contains 4 white, 5 red and 6 blue balls. A ball is drawn randomly from the box. Calculate the probabilities of the following events:

1 The ball is red.

2 The ball is blue or white.

3 The ball is not blue.

2

Cairo Governorate

Answer the following questions:

1 Choose the correct answer:

1 If  $4x = 20$ , then  $3x - 1 =$

(a) 13

(b) 14

(c) 15

(d) 16

2  $3^{10} + 3^{10} + 3^{10} = \dots\dots\dots$

(a)  $9^{30}$

(b)  $3^{30}$

(c)  $3^{10}$

(d)  $3^{11}$

3 If  $\frac{4x-1}{2x+3} = 0$  where  $x \in \mathbb{Q}$ , then  $x = \dots\dots\dots$

(a)  $-\frac{3}{2}$

(b)  $\frac{2}{3}$

(c)  $\frac{1}{4}$

(d) 0

4 A class contains 40 students, 36 of them are succeed in a test, then the probability of failed is ...

(a) 0.1

(b) 0.9

(c) 0.3

(d)  $\frac{2}{3}$

5  $\sqrt{(8)^2 + (6)^2} = 8 + \dots\dots\dots$

(a) 14

(b) 10

(c) 2

(d) 6

6  $\frac{|-5|+1}{3} \dots\dots\dots \mathbb{Z}$

(a)  $\in$

(b)  $\notin$

(c)  $\subset$

(d)  $\not\subset$

2 Complete the following:

1 If  $0.0000016 = 1.6 \times 10^n$ , then  $n = \dots\dots\dots$

2 The multiplicative invers of  $\sqrt{5\frac{4}{9}}$  = .....

3 When a fair die is tossed once, then the probability of getting an even prime number = ..

4 If  $\frac{3}{4}x = 75$ , then  $\sqrt{x} = \dots\dots\dots$

5 25 % from ..... = 8

## Algebra and Statistics

3 [a] Find in the simplest form :  $\left(\frac{5^3 \times 5^{-2}}{5^4}\right)^2$

[b] Find the S.S. in  $Q : -2 < 4X + 1 < 6$

4 [a] Simplify :  $\left(\frac{-2}{7}\right)^2 \times \sqrt{\frac{16}{49}} \times \left(\frac{-1}{7}\right)^0$

[b] If the length of a rectangle is 5 cm, more than its width and its perimeter is 42 cm.  
Find the area of this rectangle.

5 [a] Simplify :  $\frac{(4x^3y^2)^2}{(2xy^2)^3}$  where  $x \neq 0, y \neq 0$ , then find the value when  $x = \frac{1}{2}, y = 3$

[b] A team plays 30 matches in national league its drawn probability is 0.2 and its win probability is 0.7 Calculate the number of loss matches

3

Cairo Governorate

31 Model Questions  
of Orissa School Exam

Answer the following questions :

1 Choose the correct answer :

1 If  $0 \in \{5, x-3\}$ , then  $x =$

(a) 0

(b) -5

(c) 3

(d) -3

2  $3^4 \times 3^3 =$

(a)  $3^{12}$

(b)  $3^7$

(c) 3

(d)  $3^{-1}$

3 The S.S. of the inequality :  $x < 3$  in  $\mathbb{N}$  is

(a)  $\{0\}$

(b)  $\{0, 1, 2\}$

(c)  $\{1, 2\}$

(d)  $\emptyset$

4  $\frac{6a^3x^4}{3a^2x^3} =$  where  $a \neq 0, x \neq 0$

(a)  $2a^2x$

(b)  $2a^2x^7$

(c)  $\frac{3x}{a}$

(d)  $\frac{3}{a^2x}$

5  $\left(\frac{-2}{3}\right)^3 =$

(a)  $\frac{27}{8}$

(b)  $\frac{8}{27}$

(c)  $\frac{8}{27}$

(d)  $-\frac{27}{8}$

6 If the probability of success of a student is 0.6, then the probability of his failure is

(a) 1

(b)  $\frac{1}{10}$

(c)  $\frac{4}{10}$

(d)  $\frac{6}{10}$

2 Complete the following :

1 The multiplicative inverse of 7 is

2 0.00025 in scientific notation =



$$3 \left( \frac{3}{4} \right)^2 + \left( \frac{3}{4} \right)^3 =$$

$$4 \sqrt{16+9} = 4 +$$

- 5 A class has 36 pupils , 25 of them are boys , if a pupil is chosen randomly , then the probability that the pupil is a girl =

3 [a] Find the simplest form of :  $\left( \frac{7^{-2} \times 7^5}{7^3} \right)^2$

[b] Simplify and find the value of :  $\left( -\frac{3}{2} \right)^2 \times \sqrt{\frac{64}{9}} \times \left( \frac{2}{5} \right)^0$

4 [a] 1 Calculate :  $\sqrt{100 - (-8)^2}$

2 If  $x = \frac{1}{2}$  ,  $y = \frac{2}{3}$  , then find the value of :  $(x^2 y^2)^{-3}$

[b] Find in Q the S.S. of the inequality :  $3x + 6 > 3$

5 [a] Find the solution set in Q :  $4x - 5 = 27$

[b] A fair die is rolled once Calculate the probability of appearance :

- 1 an even number.      2 a number greater than 4

4

Giza Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1  $\left( \frac{2}{3} \right)^2 \times \frac{2}{3} =$

(a)  $\frac{4}{9}$

(b)  $\frac{2}{3}$

(c)  $\frac{4}{9}$

(d)  $\frac{8}{27}$

2  $5^{-1} =$

(a) 5

(b) -5

(c)  $\frac{1}{5}$

(d)  $\frac{1}{3}$

3  $\sqrt{\frac{4}{9}} =$

(a)  $\frac{2}{3}$

(b)  $\frac{3}{2}$

(c)  $\frac{2}{9}$

(d)  $\frac{4}{3}$

4 Quarter of 16 =

(a) 1

(b) 4

(c) 8

(d) 16

5 The age of Omar is X year , then his age 5 years ago is

(a) 5X

(b) 5 + X

(c) 5 - X

(d) X - 5

## Algebra and Statistics

6 A letter is selected at random from the name (ZAMALEK) the probability of selecting the letter A is

(a)  $\frac{1}{7}$

(b)  $\frac{2}{7}$

(c)  $\frac{3}{7}$

(d)  $\frac{4}{7}$

2 Complete the following :

1  $\left(\frac{3}{4}\right)^4 + \left(\frac{3}{4}\right)^3 =$

2  $0.000735 =$   $\times$  (In standard form)

3 If  $2x - 7 = 3$ , then  $x =$

4 If  $a = b$ , then  $\left(\frac{4}{3}\right)^{a-b} =$

5 The probability of any event not less than  $\quad$  and not more than  $\quad$

3 (a) Calculate the following :

1  $6 \times (2)^2 + 24 + 3^2$

2  $\sqrt{100 - (-6)^2}$

(b) Find the result in the simplest form : 1  $\frac{7^{-2} \times 7^5}{7^3}$  2  $\left(\left(\frac{1}{3}\right)^2\right)^2$

4 (a) Find in Q the S.S. of the following :

1  $x + 4 = 14$

2  $3x + 1 = 25$

(b) If  $x = \frac{1}{2}$ ,  $y = \frac{2}{3}$ ,  $z = \frac{-3}{2}$ , then find the value of :  $(xyz)^2$

5 (a) Find in Q the S.S. of :  $2x + 5 < 15$

(b) A box contains 3 white, 5 red and 7 blue balls, a ball is drawn randomly from the box. Calculate the probabilities of the following :

1 The ball is red.

2 The ball is white or blue.

5

Giza Governorate

Double Zero

Giza Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1  $3^3 \times 3^4 =$

(a)  $3^{12}$

(b) 3

(c)  $3^7$

(d)  $3^{-1}$

2  $3x^{-1} =$  where  $x \neq 0$

(a)  $-3x$

(b)  $\frac{3}{x}$

(c)  $3x$

(d)  $\frac{1}{3x}$

## Final Examinations

- 3 The multiplicative inverse of the number  $\sqrt{\frac{4}{9}}$  = \_\_\_\_\_  
 (a)  $-\frac{3}{2}$  (b)  $\frac{2}{3}$  (c)  $-\frac{2}{3}$  (d)  $\frac{3}{2}$
- 4 If  $2a = \sqrt{9}b$ , then  $\frac{a}{b}$  = \_\_\_\_\_  
 (a)  $\frac{3}{2}$  (b)  $-\frac{2}{3}$  (c)  $-\frac{3}{2}$  (d)  $\frac{2}{3}$
- 5 If  $-X < 2$ , then .....  
 (a)  $X > -2$  (b)  $X > 2$  (c)  $X < -2$  (d)  $X < 2$

## 2 Complete each of the following :

- 1  $\sqrt{16+9} = 4 +$  \_\_\_\_\_
- 2 The additive inverse of  $(-\frac{2}{3})^2$  is \_\_\_\_\_
- 3  $3^5 + 3^5 + 3^5 = 3$  \_\_\_\_\_
- 4  $\frac{6a^2x^4}{2a^3x^3} =$  \_\_\_\_\_ where  $a \neq 0, x \neq 0$
- 5  $(2x)^2 \times \frac{1}{x} =$  \_\_\_\_\_ where  $x \neq 0$
- 6  $(2\sqrt{3})^2 =$  \_\_\_\_\_

## 3 Find in Q the solution set of the following :

- 1  $8X + 4 = 12$  2  $3X - 1 \leq 2X + 3$

- 4 (a) If the length of a rectangle exceeds its width by 4 metres and its perimeter is 108 metres. Find the dimensions of the rectangle.

(b) Simplify :  $\frac{a^4 \times a^{-7}}{a^{-6}}, a \neq 0$

- 5 (a) Write the following number in the standard form :  $0.75 \times 10^8$

(b) If  $X = -\frac{1}{2}$ ,  $y = \frac{3}{4}$ ,  $z = -\frac{3}{2}$ , find the numerical value of :  $X^3 + yz^2$

6

Giza Governorate

Answer the following questions :

## 1 Choose the correct answer :

- 1  $X^{12} + X^4 =$  \_\_\_\_\_ where  $X \neq 0$   
 (a)  $X^8$  (b)  $X^3$  (c)  $X^{16}$  (d)  $X^{-8}$

## Algebra and Statistics

2.  $\pm\sqrt{\frac{4}{9}} =$

(a)  $-\frac{4}{9}$

(b)  $-\frac{2}{3}$

(c)  $\pm\frac{2}{3}$

(d)  $\frac{2}{3}$

- 3 If the probability that pupils success is 75 % , then the probability of his failure is .....

(a) -0.75

(b) 0.25

(c) -0.25

(d) 0.75

- 4 If
- $-X < 7$
- , then
- $X$
- ..... - 7

(a)  $<$

(b)  $>$

(c)  $=$

(d)  $\leq$

- 5
- $6000 \times 50 =$
- ..

(a)  $300 \times 10^2$

(b)  $30 \times 10^5$

(c)  $3 \times 10^3$

(d)  $3 \times 10^5$

- 6 If
- $3X = 6$
- , then
- $5X =$
- .....

(a)  $\frac{5}{2}$

(b)  $\frac{2}{3}$

(c) 10

(d) 5

## 2 Complete each of the following :

- 1 The probability of impossible event is ..

- 2 The additive inverse of the number
- $(-\frac{1}{3})^2$
- is ..

3  $\sqrt{16+9} =$  ..

4  $(\frac{2}{3})^{-2} =$  ..

5  $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots$  (in the same pattern)

## 3 [a] Find in the simplest form :

1  $\frac{9^{-2} \times 9^4}{9^1}$

2  $(-\frac{3}{2})^2 \times \sqrt{\frac{64}{9}} \times (\frac{5}{2})^0$

- [b] Find in
- $\mathbb{Q}$
- the solution set of the following equation :
- $4X + 1 = 21$

4 [a] If  $X = \frac{1}{2}$  ,  $y = \frac{2}{3}$  ,  $z = \frac{-3}{2}$  , then find the value of :  $(X y z)^2$ 

- [b] Find in
- $\mathbb{Q}$
- the solution set of the following inequality :
- $3X - 1 \leq 2X + 3$

## 5 [a] The sum of three consecutive numbers is 24 , find them

- [b] A fair die is rolled once , calculate the probability of appearance :

- 1 An even number.

- 2 A number greater than 5



## 7 Alexandria Governorate

Maths Educational Year  
Preparation for the Ministry

Answer the following questions :

## 1 Complete each of the following :

1 The multiplicative identity element in  $\mathbb{Q}$  is

2  $\sqrt{\frac{25}{49}} =$

3  $(X-5)^0 = 1$ , if  $X \neq$  .

4 If the probability of success of a student is 0.8 , then the probability of his failure is .....

5 If  $X \subset Y$ , then  $X \cap Y =$  .....

## 2 Choose the correct answer :

1  $\sqrt{64+36} = 8 +$  .....

(a) 2

(b) 6

(c) 10

(d) -2

2 Half the number  $2^{24}$  equals .....(a)  $2^{12}$ (b)  $1^{23}$ (c)  $2^{23}$ (d)  $1^{12}$ 3  $0.0000068 =$ (a)  $6.8 \times 10^{-6}$ (b)  $6.8 \times 10^5$ (c)  $6.8 \times 10^{-7}$ (d)  $6.8 \times 10^7$ 4 If  $2X = -12$ , then  $X^2 =$ 

(a) 6

(b) 144

(c) -36

(d) 36

5 The S.S. of the equation :  $5 - X = 3$  in  $\mathbb{Q}$  is ....(a)  $\{2\}$ (b)  $\{-2\}$ (c)  $\{7\}$ (d)  $\emptyset$ 

6 The probability of a certain event equals .

(a) zero

(b) 1

(c) -1

(d)  $\frac{1}{2}$ 3 [a] Simplify to the simplest form :  $\frac{X^3 \times X^{-2}}{X^{-3} \times X}$ , then find the value when  $X = -2$ [b] Find the S.S. in  $\mathbb{Q}$  for each of the following :

1  $3X - 5 > 1$

2  $3X + 6 = 30 - 5X$

## 4 [a] A fair die is rolled once , what is the probability of getting :

1 an even number ? 2 a factor of 6 ?

[b] Find the value of :  $\sqrt{6\frac{1}{4}} \times \left(\frac{2}{3}\right)^0 \times \left(-\frac{2}{5}\right)^2$

## Algebra and Statistics

- 5 [a] A rectangle whose length is more than its width by 3 cm. and its perimeter equals 26 cm.  
 find its area.  
 [b] If  $X = \frac{1}{2}$  and  $y = \frac{4}{3}$  find the value of :  $X^3 y^2$

8

Alexandria Governorate

Mid Educational Exam  
Math Inspection

Answer the following questions :

- 1 Choose the correct answer :

1  $X^9 + X^3 = \dots\dots\dots$  ,  $X \neq 0$

- (a)  $X^{12}$  (b)  $X^3$  (c)  $X^6$  (d)  $X^{-3}$

2 If  $0.0035 = 3.5 \times 10^a$  , then  $a =$

- (a) 2 (b) -3 (c) -2 (d) 3

3  $\left(\frac{4}{7}\right)^0 =$

- (a) 1 (b)  $\frac{4}{7}$  (c) 0 (d) -1

4  $5^2 + 5^2 = \dots\dots\dots$

- (a)  $10^4$  (b) 50 (c)  $5^4$  (d)  $10^2$

5 If  $-X > 4$  , then  $\dots\dots\dots$

- (a)  $X > -4$  (b)  $X < 4$  (c)  $X > 4$  (d)  $X < -4$

6 The sum of all probabilities of all possible events of a random experiment  $\dots\dots\dots$

- (a) = 0 (b) = 1 (c) > 1 (d) < 1

- 2 Complete each of the following :

1 The probability of the certain event =  $\dots\dots\dots$

2 1 , 2 , 3 , 5 , 8 ,  $\dots\dots\dots$  (in the same pattern)

3  $(a^2)^4 = \dots\dots\dots$

4  $\sqrt{9+16} = \dots\dots\dots$

5  $2 \times 6 - 4 + 2 = \dots\dots\dots$

- 3 Find in the simplest form :

1  $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$  2  $\frac{7^{-2} \times 7^7}{7^3}$

- 4 [a] Find in Q the solution set of :  $3X + 1 = 25$

[b] If  $X = \frac{3}{2}$  ,  $y = \frac{4}{3}$  , find the value of :  $X^2 y^2$

34

5 [a] Find in  $\mathbb{Q}$  the solution set of :  $2x + 15 < 19$

[b] A bag contains 5 red , 3 yellow and 2 black balls. A ball is drawn randomly from the box. Calculate the probability of getting :

1 A yellow ball.

2 A green ball.

9

El-Kalyoubia Governorate

Directorate of Education  
Math Supervision

Answer the following questions :

1 Choose the correct answer :

1 The number which is not in the standard form is

(a)  $6.2 \times 10^5$

(b)  $7.834 \times 10^{16}$

(c)  $0.8 \times 10^5$

(d)  $6.7 \times 10^{25}$

2 If  $3t = 6$ , then  $6t =$

(a) 16

(b) 14

(c) 12

(d) 10

3  $\frac{1}{4} \times 4^{20} =$

(a)  $4^{15}$

(b)  $4^{19}$

(c)  $2^{19}$

(d)  $2^{39}$

4  $\frac{6a^2x^4}{2a^3x^7} =$  where  $a \neq 0$  ,  $x \neq 0$

(a)  $3a^2x$

(b)  $3a^5x^7$

(c)  $\frac{3x}{a}$

(d)  $\frac{3}{a^2x}$

5 A class formed from 36 students , 16 of them are girls. If a student selected randomly from the class , then the probability that the student is a boy =

(a)  $\frac{4}{9}$

(b)  $\frac{5}{9}$

(c)  $\frac{1}{2}$

(d)  $\frac{1}{36}$

6 10 % of  $2\frac{1}{2}$  L.E. = ..... L.E.

(a)  $\frac{1}{2}$

(b)  $\frac{1}{4}$

(c) 1

(d) 25

2 Complete :

1 The additive inverse of  $\left(\frac{-2}{3}\right)^4$  is

2 When a die is tossing twice and observed the upper face in each time the probability of appearance number 5 on the two faces is

3  $\sqrt{10^2 - 6^2} =$

4 If  $-x + 2 > 6$  and the substitution set =  $\{-2, -5, -1\}$  , then  $x =$

5 If  $3a = \sqrt{4b}$  , then  $\frac{a}{b} =$

## Algebra and Statistics

3 [a] Find in the simplest form :  $\left(-\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{a}{b}\right)^0$  where  $a \neq 0$  ,  $b \neq 0$

[b] If  $x = -\frac{1}{2}$  ,  $y = -\frac{3}{4}$  , find the value of :  $\left(\frac{y}{x}\right)^{-2}$

4 Find in  $\mathbb{Q}$  the S.S. of :

1  $3x + 1 = 25$

2  $15 + 2x < 1$

5 [a] Find with steps the value of :  $(2 \times \sqrt{36} - 2^4) \div 4$

[b] If a regular die is thrown once and observing the number on the upper face  
Find the probability of getting :

1 a prime even number.

2 an odd number less than 4

10

El-Kalyoubia Governorate



Answer the following questions :

1 Choose the correct answer :

1 The side length of a square whose area  $9x^2$  is

(a)  $3x$

(b)  $3x^2$

(c)  $9x$

(d)  $9x^2$

2  $\left(-\frac{2}{3}\right)^3 =$

(a)  $-\frac{27}{8}$

(b)  $-\frac{8}{27}$

(c)  $\frac{8}{27}$

(d)  $\frac{27}{8}$

3  $2^3 \times 2^3 =$

(a) 1

(b)  $2^9$

(c)  $2^4$

(d)  $2^6$

4  $\sqrt{9+16} =$

(a) 7

(b) 5

(c) 25

(d) -7

5 If  $-x > 4$  , then

(a)  $x > -4$

(b)  $x > 4$

(c)  $x < 4$

(d)  $x < -4$

6 There are 21 boys and 15 girls in a classroom , one pupil is chosen randomly , then the probability that the chosen pupil is a girl = ...

(a)  $\frac{5}{12}$

(b)  $\frac{7}{12}$

(c)  $\frac{4}{7}$

(d)  $\frac{5}{6}$

2 Complete :

1  $7(6^2 - 5 \times 6) =$

2 1 , 2 , 3 , 5 , 8 , , (in the same pattern).



3 The additive inverse of  $\left(-\frac{2}{5}\right)^2$  is .

4  $\left(-\frac{2}{5}\right)^{\text{zero}} =$

5 The probability of the certain event =

3 [a] Simplify to the simplest form :  $\left(-\frac{3}{7}\right)^0 \times \left(-\frac{2}{5}\right)^2 \times \sqrt{6\frac{1}{4}}$

[b] Write the following number in the standard form  $720 \times 10^6$

4 [a] Find in  $\mathbb{Q}$  the S.S. of :  $2x + 15 < 19$

[b] Simplify to the simplest form :  $\frac{5^{-4} \cdot 5^7}{5^3}$

5 [a] Find in  $\mathbb{Q}$  the S.S. of :  $3x + 1 = 25$

[b] The set  $\{2, 3, 5\}$  is used in writing a 2-digit number.

Find the probability of the following events :

1. The tens digit is odd.      2. The units digit is even.

11

El-Monofia Governorate

El-Monofia Education System  
El-Monofia Final Exam

Answer the following questions :

1 Choose the correct answer :

1 The S.S. of the inequality :  $x < 2$  in  $\mathbb{N}$  is

(a)  $\{0\}$

(b)  $\{1\}$

(c)  $\{0, 1\}$

(d)  $\emptyset$

2  $3^{10} + 3^{10} + 3^{10} =$

(a)  $3^{10}$

(b)  $3^{30}$

(c)  $9^{10}$

(d)  $3^{11}$

3  $\sqrt{(-8)^2 + (-6)^2} =$

(a)  $|-10|$

(b)  $\pm 10$

(c) 14

(d) -14

4  $\frac{6a^2x^4}{2a^3x^3} =$  where  $a \neq 0$

(a)  $3a^2x^2$

(b)  $3a^5x^7$

(c)  $\frac{3x}{a}$

(d)  $\frac{3}{ax}$

5 Which of the following is the greatest ?

(a)  $2.3 \times 10^4$

(b)  $2.3 \times 10^3$

(c)  $3.2 \times 10^4$

(d)  $3.2 \times 10^5$

6  $2 \times 6 - 4 + 2 =$

(a) 10

(b) 2

(c) 12

(d) 6

## Algebra and Statistics

## 2 Complete the following :

- 1 A class has 36 pupils , 20 of them are boys. If a pupil is chosen randomly , then the probability that the pupil is a girl = ...
- 2 If  $7 - 2X = 3$  , then  $3X =$  ...
- 3  $2\frac{1}{4} = \left(\frac{3}{2}\right)^{\quad}$
- 4  $\sqrt{100 - 64} = 10 -$
- 5 The standard form of the number  $0.7 \times 0.005 =$

3 [a] Find in  $\mathbb{Q}$  the S.S. of the inequality :  $3X + 5 > 2$ [b] Find in  $\mathbb{Q}$  the S.S. of the equation :  $3X + 4 = 2X + 5$ 4 [a] Find the value of :  $12 \times (2)^2 + 24 + 3^2$ [b] Find in the simplest form :  $\left(7^4 \times 7^{-7}\right)^{-2}$ 5 [a] Simplify to the simplest form :  $\left(\frac{-5}{3}\right)^2 \times \left(\frac{-4}{9}\right)^0 \times \sqrt{3\frac{6}{25}}$ [b] The set  $\{2, 3, 5\}$  is used in writing a 2-digit number.

Find the probability of each of the following events :

- 1 The sum of the two digits is 7
- 2 Both digits are equal.

12

El-Dakahlia Governorate

Dakahlia Directorate of Education  
Maths supervision

Answer the following questions :

## 1 Choose the correct answer from those given :

1 The additive inverse of  $(-1)^{10}$  is

- (a) 0                      (b) -1                      (c) -10                      (d) 1

2 Half milliard =  $5 \times 10^{\quad}$ 

- (a) 6                      (b) -9                      (c) 8                      (d) 9

3 Quarter of the number  $2^{16}$  is ..

- (a)  $1^{16}$                       (b)  $4^7$                       (c)  $4^{15}$                       (d)  $2^4$

4  $\sqrt{144 + \dots} = 12 + 5$ 

- (a) 40                      (b) 25                      (c) 16                      (d) 145

5 If the probability of success of a student is 0.7 , then the probability of his failure is

- (a) 0.03 (b) 1 (c) 30 % (d) 3

6 If  $8y = 16$  , then  $y + 5 =$

- (a) 10 (b) 13 (c) 21 (d) 7

2 Complete each of the following :

1 The smallest odd prime number is ----

2 If the area of a square is  $169 \text{ k}^2 \text{ cm}^2$  , then its side length = ---- cm.

3 If  $y = \frac{1}{4}$  ,  $x = \frac{1}{3}$  , then  $(x - y)^{-2} = \dots$

4 ---- is a subset of the sample space.

5 The S.S. of the inequality  $5 \leq x \leq 6$  in  $\mathbb{N}$  is ----

3 (a) Divide :  $(25a + 5a) \div 5a$  where  $a \neq 0$  , then find the numerical value of the expression when  $a = 2$  ,  $b = -1$

(b) Simplify to the simplest form :  $\left(-\frac{49}{25}\right)^0 \times \left(-\frac{2}{7}\right)^2 \times \sqrt{12\frac{1}{4}}$

4 (a) Find in  $\mathbb{Q}$  the S.S. of the following :

1  $2x + 7 < 15$

2  $6x + 6 = 6$

(b) Simplify to the simplest form :  $\frac{(3)^{-6} \times (3)^{11}}{(3)^7}$

5 (a) Find in  $\mathbb{N}$  the S.S. of the inequality :  $5x - 2 \geq 3$  , then represent it on the number line.

(b) A ball chosen randomly from a bag contains 5 red balls , 7 blue balls , 3 yellow balls , find the probability of each of the following :

1 getting black ball.

2 getting a red ball.

13

Port Said Governorate

Educational Supervisor  
Math Inspection

Answer the following questions :

1 Choose the correct answer :

1  $6 \times 2 - 4 \div 2 =$

- (a) 1 (b) 2 (c) 10 (d) 12

## Algebra and Statistics

2  $|-3| + |5| = \dots$

(a) -8

(b) -2

(c) 2

(d) 8

3 There are 21 boys and 15 girls in a classroom, one pupil is chosen randomly, the probability that the chosen pupil is a girl =  $\dots$

(a)  $\frac{5}{12}$

(b)  $\frac{7}{12}$

(c)  $\frac{4}{7}$

(d)  $\frac{5}{6}$

4 The S.S. of the inequality :  $X < 0$  in  $\mathbb{N}$  is  $\dots$

(a)  $\{0\}$

(b)  $\{1\}$

(c)  $\{0, 1\}$

(d)  $\emptyset$

5 Quarter of  $4^{20} = \dots$

(a)  $4^5$

(b)  $4^{10}$

(c)  $4^{19}$

(d)  $2^{10}$

6  $\frac{6a^2x^4}{2a^3x^3} = \dots$  where  $a \neq 0$ ,  $x \neq 0$

(a)  $3 \neq x$

(b)  $3a^5x^7$

(c)  $\frac{3x}{a}$

(d)  $\frac{3}{ax}$

2 Complete :

1  $\sqrt{16+9} = 2 + \dots$

2  $10^{-3} = \frac{1}{\dots}$

3 The probability of the impossible event =  $\dots$

4 1, 8, 27,  $\dots$  (in the same pattern).

5 The multiplicative inverse of 7 is  $\dots$

3 [a] Solve :  $X + 2 = 8$  in  $\mathbb{Z}$

[b] Evaluate :  $\frac{7^{-2} \times 7^3}{7^3}$

4 [a] Find the S.S. in  $\mathbb{Q}$  :  $3X - 1 \leq 2X + 3$

[b] Find the value of the expression in the simplest form :  $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0$

5 [a] A fair die is rolled once, calculate the probability of getting :

1 An even number.

2 A prime number.

3 A number greater than 3

[b] If  $X = 3$  find the numerical value of the expression :  $2\left(\frac{5X+3}{4X-3}\right)$



14

**Danietta Governorate**

**Answer the following questions :**

**Choose the correct answer :**

- 1 If  $X + 9 = 11$ , then the value of  $7X =$  \_\_\_\_\_
- (a) 9 (b) 14 (c) 2 (d) 13
- 2  $\left(\frac{-2}{3}\right)^{-3} =$
- (a)  $\frac{-27}{8}$  (b)  $\frac{-8}{27}$  (c)  $\frac{8}{27}$  (d)  $\frac{27}{8}$
- 3  $2^3 + 2^3 =$  .....
- (a)  $2^6$  (b)  $2^4$  (c)  $2^9$  (d) 1
- 4 If  $-X > 4$ , then ...
- (a)  $X > -4$  (b)  $X > 4$  (c)  $X < -4$  (d)  $X < 4$
- 5  $X^2 + X^2 =$  .....
- (a)  $X^4$  (b)  $X^2$  (c)  $2X^2$  (d)  $2X^4$
- 6 Three times of a number is 48, then  $\frac{1}{4}$  the number is \_\_\_\_\_
- (a) 16 (b) 4 (c) 12 (d) 8

**2** Complete each the following :

- 1 The probability of the certain event equals .....
- 2 The number 0.000053 in the scientific notation = .....
- 3 The multiplicative inverse of  $\sqrt{\frac{9}{25}}$  is .....
- 4 A class has 36 pupils , 20 of them are boys. If a pupil is chosen randomly , then the probability that the pupil is a girl = .....
- 5 1 , 2 , 3 , 5 , 8 , ..... (in the same pattern).

**3** (a) Simplify to the simplest form :  $\left(\frac{5}{3}\right)^2 \times \left(\frac{4}{9}\right)^0 \times \sqrt{3\frac{6}{25}}$

**[b]** Find the solution set of each of the following where  $x \in \mathbb{Q}$ :

- $1 \quad 3x + 1 > 25$

**Q 10** Reduce :  $\frac{x^7 \times x^9}{x^6 \times x^1}$  to the simplest form , then find the value of the result when :  $x = -3$

**[b]** The sum of three consecutive even numbers is 60, find them.

## Algebra and Statistics

- 5 [a] If  $X = 3$  and  $y = -4$ , find the value of:  $\sqrt{x^2 + y^2}$
- [b] A box contains 4 white, 5 red and 6 blue balls, a ball is drawn randomly from the box. Calculate the probability of getting:
- 1 a blue ball.                      2 a white or red ball.                      3 a green ball.

## 15 El-Fayoum Governorate

Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer :

- 1 The multiplicative inverse of  $\sqrt{\frac{100}{36}}$  =
- (a)  $\pm \frac{10}{6}$                       (b)  $\pm \frac{6}{10}$                       (c)  $\frac{10}{6}$                       (d)  $\frac{6}{10}$
- 2 The probability of the certain event =
- (a) zero                      (b) 1                      (c)  $\frac{1}{2}$                       (d)  $\frac{1}{3}$
- 3 If  $a = b$ , then:  $\left(\frac{2}{3}\right)^{(a-b)} =$
- (a) zero                      (b) 1                      (c)  $\frac{2}{3}$                       (d)  $\frac{3}{2}$
- 4  $3^{10} + 3^{10} + 3^{10} =$
- (a)  $3^{30}$                       (b)  $3^{11}$                       (c)  $9^{10}$                       (d)  $9^{11}$
- 5 If  $3X + 1 = 16$ , then the value of  $5X =$
- (a) 10                      (b) 15                      (c) 25                      (d) 26
- 6  $5^{-1} =$
- (a) -5                      (b) 25                      (c) 5                      (d)  $\frac{1}{5}$

- 2 Complete the following :

- 1 The probability of the impossible event =
- 2  $(3 \times 10^2) \times (15 \times 10^3) =$  in the standard form.
- 3 If  $\frac{x}{5} = 30\%$ , then  $x =$
- 4  $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \dots$  (in the same pattern)
- 5 If  $x = \frac{1}{2}$ ,  $y = \frac{3}{4}$ , then  $y + x =$

3 [a] Write the following in the simplest form :  $\frac{7^3 \times 7^6}{7^2}$

[b] Find in  $\mathbb{Q}$  the solution set of the following :  $8 + 2x = 14$

4 [a] Find the result of the following :  $10 \times 4 - (2 \times 6 - 8)$

[b] Calculate the following when :  $a = 2$  ,  $b = 5$

1  $\frac{b-a}{b^2}$

2  $\frac{a^2}{b-1}$

5 [a] A box contains 15 cards numbered from 1 to 15 , a card is drawn randomly , find the probability of :

- 1 The drawn card carries a prime number
- 2 The drawn card carries a number divisible by 3

[b] Find in  $\mathbb{Q}$  the solution set of the following :  $3x - 4 \geq -10$

Second

# Geometry and Measurement



- 12 quizzes ..... 44
- Final revision ..... 51
- Final examinations : ..... 59
  - School book examinations  
(2 models + model for the merge students)
  - 15 schools examinations



# Quizzes

on Geometry and  
Measurement



## Quiz 1

on lesson 1 - unit 3

time  
15 min

## 1 Complete :

- If two straight lines intersect , then the measures of each two vertically opposite angles are
- The sum of the measures of the accumulative angles at a point is equal to
- If two straight lines are parallel to a third , then the two straight lines are

## 2 (a) In the opposite figure :

$$\overline{AE} \cap \overline{BD} = \{C\}$$

$$BC = CE$$

$$m(\angle B) = m(\angle E)$$

Prove that :  $AB = DE$ 

## (b) In the opposite figure :

Find by proof the value of  $x$ 

## Quiz 2

till lesson 2 - unit 3

time  
15 min

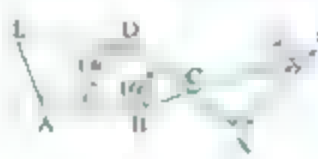
## 1 Complete the following :

- The sum of measures of the interior angles of heptagon
- The sum of measures of the exterior angles of the triangle =
- If the measure of an interior angle of a regular polygon =  $120^\circ$  then the number of its sides =

## 2 (a) In the opposite figure :

$$\overline{BF} \cap \overline{DX} = \{C\}, \overline{FX} \perp \overline{DX}, m(\angle F) = 30^\circ$$

$$m(\angle B) = 160^\circ, m(\angle D) = 140^\circ, m(\angle A) = 110^\circ$$

Prove that :  $\overline{AB} \parallel \overline{ED}$ 

## (b) In the opposite figure :

$$\overline{AB} \parallel \overline{CD}, m(\angle EAC) = 150^\circ$$

$$\overline{AB} \perp \overline{AE}$$

Find : 1  $m(\angle BAC)$ 2  $m(\angle C)$ 

## Geometry and Measurement

## Quiz 3

till lesson 3 – unit 3



## 1 Complete the following :

- If ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 110^\circ$ , then  $m(\angle D) = \dots\dots\dots$
- The quadrilateral in which only two sides are parallel is called
- The measure of the interior angle of the regular hexagon =

## 2 (a) In the opposite figure :

$E \in \overline{BC}$ ,  $m(\angle BAE) = 50^\circ$ ,  $m(\angle AEB) = 75^\circ$   
 $m(\angle D) = 55^\circ$ ,  $m(\angle C) = 125^\circ$

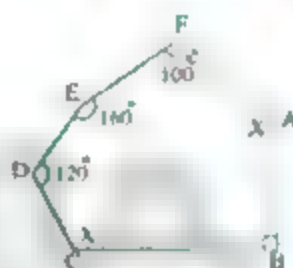
Prove that : the figure ABCD is a parallelogram.



## (b) The opposite figure :

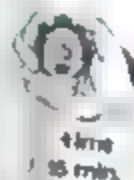
ABCDEF is a hexagon and  
 $m(\angle A) = m(\angle C) = X$

Find : the value of X



## Quiz 4

till lesson 4 – unit 3

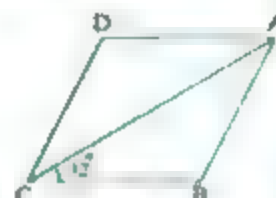


## 1 Complete the following :

- If two adjacent sides in a rectangle are equal in length, then it will be
- If ABCD is a square, then  $m(\angle CAB) =$
- The polygon which has an interior reflex angle is called

## 2 (a) In the opposite figure :

ABCD is a rhombus,  
 $\overline{AC}$  is a diagonal in it  $m(\angle ACB) = 32^\circ$   
 Find :  $m(\angle D)$



- (b) If the measure of the exterior angle of a regular polygon equals  $30^\circ$ ,  
 what is the number of sides of this polygon and what is the sum of measures of its interior angles ?

## Quiz 5

till lesson 5 - unit 3



## 1 Complete the following :

- The sum of measures of the interior angles of the triangle =
- The measure of the exterior angle of the equilateral triangle =
- In the parallelogram ABCD , if  $m(\angle A) = \frac{1}{2} m(\angle B)$  , then  $m(\angle B) =$

## 2 (a) In the opposite figure :

 $\overline{DE} \parallel \overline{FR} \parallel \overline{BC}$  ,  $m(\angle D) = 140^\circ$  $m(\angle F) = 125^\circ$ Calculate the measures of the angles of :  $\triangle ABC$ 

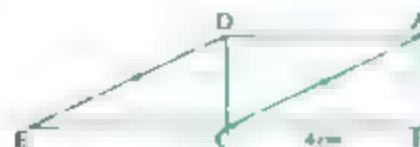
## (b) In the opposite figure :

ABCD is a rectangle ,

 $\overline{AC} \parallel \overline{DE}$  ,  $E \in \overline{BC}$ 

1 Prove that : ACED is a parallelogram.

2 Find : The length of CE



## Quiz 6

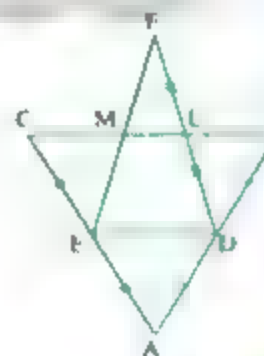
till lesson 6 - unit 3



## 1 Complete the following :

- The ray which is drawn from the midpoint of a side of a triangle parallel to one of the other sides .....
- If the measure of an angle in a triangle is greater than the sum of measure of the other two angles , then the triangle is .....
- The length of the line segment joining the midpoints of two sides of a triangle equals ..... the length of the third side

## 2 (a) In the opposite figure :

D is the midpoint of  $\overline{AB}$ E is the midpoint of  $\overline{AC}$ L is the midpoint of  $\overline{FD}$  $BC = 20$  cmFind : The length of  $\overline{LM}$ 

## (b) In the opposite figure :

 $\overline{AD} \cap \overline{CE} = \{F\}$  ,  $m(\angle B) = 90^\circ$  $m(\angle C) = 120^\circ$  ,  $m(\angle E) = 60^\circ$  ,  $m(\angle D) = 70^\circ$ Find :  $m(\angle A)$ 



## Geometry and Measurement

## Quiz 7

till lesson 7 – unit 3

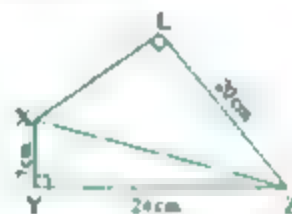
Time  
15 min.

## 1 Complete the following :

- A rectangle whose length = 4 cm and width = 3 cm , then the length of its diagonal = ..... cm
- In  $\triangle ABC$  , if  $m(\angle B) = m(\angle A) + m(\angle C)$  , then  $\angle B$  is
- In the right-angled triangle , the area of the square on the hypotenuse equals

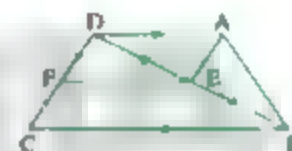
## 2 (a) In the opposite figure :

$m(\angle Y) = m(\angle L) = 90^\circ$   
 $XY = 7$  cm. ,  $YZ = 24$  cm.  
 and  $LZ = 20$  cm.

Find : The length of  $\overline{XL}$ .

## (b) In the opposite figure :

$\overline{AD} \parallel \overline{BC}$  ,  $AD = \frac{1}{2} BC$   
 $E$  is the midpoint of  $\overline{BD}$   
 and  $F$  is the midpoint of  $\overline{CD}$

Prove that :  $AEDF$  is a parallelogram.

## Quiz 8

till lesson 8 – unit 3

Time  
15 min.

## 1 Complete the following :

- The image of the point  $(1, 2)$  by the transformation  $(X, y) \rightarrow (X, -y)$  is .....
- If  $\triangle XYZ$  is a right-angled triangle at  $Z$  , then  $(XZ)^2 =$
- In the opposite figure :  
 $m(\angle A) = m(\angle D) = 50^\circ$   
 $\overline{AB}$  and  $\overline{DC}$  intersect at  $E$   
 not shown in the figure  
 , then  $m(\angle E) =$

2 (a) If the ratio among the measures of the interior angles of a quadrilateral is  $2 : 2 : 3 : 5$  , find the measure of the greatest angle in the quadrilateral.

- (b) On lattice , draw  $\triangle ABC$  where  $A(1, 2)$  ,  $B(4, 2)$  ,  $C(4, 4)$  , then map its image by the transformation  $(X, y) \rightarrow (y, -X)$

## Quiz 9

till lesson 9 – unit 3

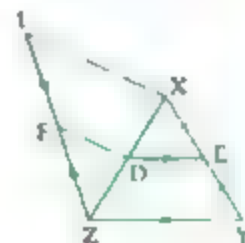


## 1 Complete :

- The image of the point  $(3, -5)$  by the reflection in  $X$ -axis is
- The triangle contains two ..... angles at least.
- The image of the point  $(2, 3)$  by the reflection in  $y$ -axis is

## 2 [a] In the opposite figure :

$XE = EY$ ,  $ED \parallel YZ$   
 $F$  is the midpoint of  $ZL$   
 Prove that :  $DF \parallel XL$

[b] On lattice , Draw the image of  $\Delta ABO$  where

$A(2, 2)$  ,  $B(4, 2)$  ,  $O(0, 0)$  by reflection in  $y$ -axis.

## Quiz 10

till lesson 10 – unit 3

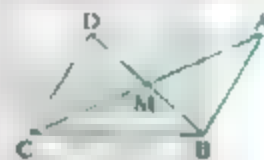


## 1 Complete the following :

- The point  $(2, -4)$  is the image of the point ..... by reflection in the origin point.

## 2 In the opposite figure :

If  $ABCD$  is a parallelogram , then the image of  
 $\Delta AMD$  by reflection in the point  $M$  is  $\Delta$  .....

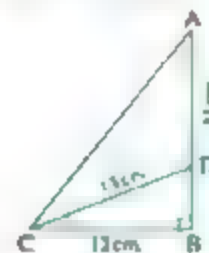


- If the reflection in a straight line transforms the figure to itself , then this straight line is called ...

## 2 [a] In the opposite figure :

$ABC$  is a triangle in which  $m(\angle B) = 90^\circ$   
 $D \in \overline{AB}$  , where  $AD = 11$  cm. ,  $BC = 12$  cm.  
 and  $DC = 13$  cm.

Find : The length of each of  $\overline{BD}$  and  $\overline{AC}$



- Draw on a lattice  $\Delta ABC$  , where  $A(5, 1)$  ,  $B(2, 1)$  and  $C(5, 3)$  , then draw its image by reflection in the origin point.

## Geometry and Measurement

## Quiz 11

till lesson 11 – unit 3

time  
15 min.

## 1 Complete the following :

- The translation reserves .....
- The translation is determined by two things , they are ..... and .....
- If  $\hat{A}(3, 5)$  is the image of the point A by the translation  $(X, y) \longrightarrow (X + 4, y - 2)$  , then the point A is .....

2 On lattice , draw  $\Delta ABC$  where  $A(-3, 2)$  ,  $B(-1, 1)$  and  $C(-2, 4)$  then draw its image

- by the translation  $(X, y) \longrightarrow (X + 4, y - 3)$
- by reflection in X-axis.

## Quiz 12

till lesson 12 – unit 3

time  
15 min.

## 1 Complete the following :

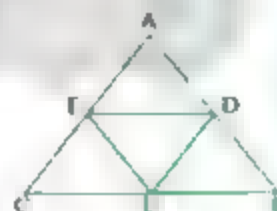
- The point  $(3, 4)$  is the image of the point  $(4, -3)$  by rotation about the origin point with an angle of measure .....
- The image of the point  $(2, -1)$  by a translation of magnitude 3 units in the negative direction of y-axis is .....
- If M is the midpoint of  $\overline{LN}$  , then the image of  $\overline{LN}$  by a rotation  $R(M, 180^\circ)$  is .....

## 2 [a] In the opposite figure :

Each of the triangles ADF , DBE , DEF , FEC is equilateral

Find the image of  $\Delta ADF$  :

- By the translation AD in the direction of  $\overline{AD}$
- By reflection in  $\overline{DF}$
- By a rotation  $R(D, 60^\circ)$

[b] On lattice , draw  $\Delta ABC$  where  $A(1, 1)$  ,  $B(3, 1)$  ,  $C(2, 3)$  , then draw :

- The image of  $\Delta ABC$  by reflection in X-axis
- The image of  $\Delta ABC$  by rotation  $R(O, 180^\circ)$

# Final Revision

of Geometry and  
Measurement





## Revision for the important theorems, corollaries and rules of geometry and measurement

### Some relations between the angles

If two straight lines intersect, then the measures of each two vertically opposite angles are equal.



If  $\overline{AB} \cap \overline{CD} = \{M\}$ , then

$$m(\angle AMC) = m(\angle BMD)$$

"Vertically opposite angles"

$$m(\angle AMD) = m(\angle CMB)$$

"Vertically opposite angles"

The sum of the measures of the accumulative angles at a point is equal to  $360^\circ$



$$m(\angle AMD) + m(\angle DMC)$$

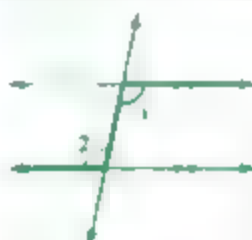
$$+ m(\angle CMB) + m(\angle BMA)$$

$$= 360^\circ$$

### Parallelism

If a straight line intersects two parallel straight lines, then

Each two alternate angles are equal in measure.

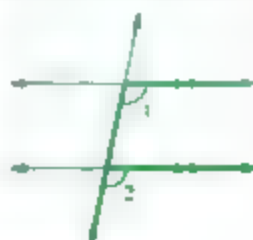


For example :

$$m(\angle 1) = m(\angle 2)$$

(alternate angles)

Each two corresponding angles are equal in measure.

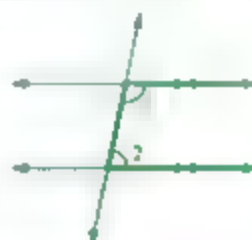


For example :

$$m(\angle 1) = m(\angle 2)$$

(corresponding angles)

Each two interior angles in the same side of the transversal are supplementary.



For example :

$$m(\angle 1) + m(\angle 2) = 180^\circ$$

**Remember** How to prove the parallelism of two straight lines

The two straight lines are parallel if a third straight line intersects them (as a transversal) and one of the following cases is satisfied

- 1 Two alternate angles have the same measure.
- or 2 Two corresponding angles have the same measure.
- or 3 Two interior angles in the same side of the transversal are supplementary

**Congruence of triangles****First case**

**Two sides and the included angle (S.A.S.)**

Two triangles are congruent if two sides and the included angle of one triangle are congruent to the corresponding parts of the other triangle

**Second case**

**Two angles and one side (A.S.A.)**

Two triangles are congruent if two angles and the side drawn between their vertices of one triangle are congruent to the corresponding parts of the other triangle



The cases of congruence  
of two triangles

**Third case**

**Three sides (S.S.S.)**

Two triangles are congruent if each side of one triangle is congruent to the corresponding side of the other triangle.

**Fourth case**

**Hypotenuse and one side in the right-angled triangle (R.H.S.)**

Two right-angled triangles are congruent if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle



## Geometry and Measurement

## The polygon

## First rule

The sum of measures of the interior angles of a polygon of  $n$  sides  
 $= (n - 2) \times 180^\circ$

## Second rule

The measure of each interior angle of the regular polygon of  $n$  sides  
 $= \frac{(n - 2) \times 180^\circ}{n}$

## Third rule

The sum of measures of the exterior angles of any convex polygon  $= 360^\circ$

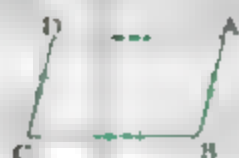
## Fourth rule

The number of sides of the regular polygon in which the measure of one of its interior is  $X^\circ = \frac{360^\circ}{X}$

Rules  
of the convex  
polygon

## The parallelogram and its properties

- 1 Each two opposite sides are equal in length



- $AB = DC$
- $AD = BC$

- 2 Each two opposite angles are equal in measure.



- $m(\angle A) = m(\angle C)$
- $m(\angle B) = m(\angle D)$
- $m(\angle A) + m(\angle B) = 180^\circ$
- $m(\angle B) + m(\angle C) = 180^\circ$
- $m(\angle C) + m(\angle D) = 180^\circ$
- $m(\angle D) + m(\angle A) = 180^\circ$

- 3 The sum of measures of each two consecutive angles is  $180^\circ$



- $AM = CM$
- $BM = DM$

- 4 The two diagonals bisect each other

## When does a quadrilateral represent a parallelogram ?

A quadrilateral represents a parallelogram if one of the following conditions satisfies

Each two opposite sides are parallel



Each two opposite sides are equal in length.



Two opposite sides are parallel and equal in length.



Each two opposite angles are equal in measure



The two diagonals bisect each other



## The parallelogram is

a rectangle

If :

One of its angles is a right angle  
or

Its two diagonals are equal in length.

a rhombus

If :

Two adjacent sides are equal in length  
or

Its two diagonals are perpendicular.

a square

If :

One of its angles is right and two adjacent sides are equal in length  
orOne of its angles is right and its two diagonals are perpendicular.  
orIts two diagonals are perpendicular and equal in length.  
or

Two adjacent sides are equal in length and its two diagonals are equal in length.

## Notice that

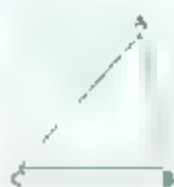
- A square is a rectangle with two adjacent sides equal in length or with two perpendicular diagonals.
- A square is a rhombus with a right angle or with two diagonals equal in length
- To prove that the quadrilateral is a rectangle , a rhombus or a square , we must first prove that it is a parallelogram.



## Geometry and Measurement

## The triangle -

- The sum of the measures of the interior angles of a triangle is  $180^\circ$



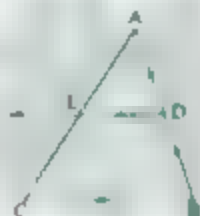
$$m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

- The measure of the exterior angle of a triangle is equal to the sum of the measures of its non adjacent interior angles.



If  $D \in \overline{CB}$ , then  $\angle ABD$  is an exterior angle of  $\triangle ABC$   
 $m(\angle ABD) = m(\angle A) + m(\angle C)$

- The ray drawn from the midpoint of a side of a triangle parallel to another side bisects the third side.



If D is the midpoint of  $\overline{AB}$ ,  
 $\overline{DE} \parallel \overline{BC}$ , then E is the midpoint of  $\overline{AC}$

- The line segment joining the midpoints of two sides of a triangle is parallel to the third side



If D is the midpoint of  $\overline{AB}$ ,  
 E is the midpoint of  $\overline{AC}$ ,  
 then  $\overline{DE} \parallel \overline{BC}$

- The length of the line segment joining the midpoints of two sides of a triangle is equal to half the length of the third side

If D the midpoint of  $\overline{AB}$ ,  
 E is the midpoint of  $\overline{AC}$ ,  
 then  $DE = \frac{1}{2} BC$

## Pythagoras' theorem :

The sum of areas of the squares on the sides of the right angle of a right-angled triangle is the same as the area of the square on the hypotenuse.



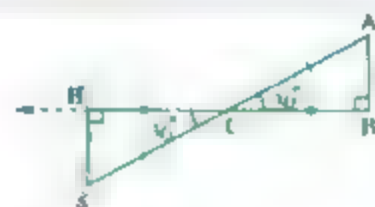
In  $\triangle ABC$  :

If  $m(\angle B) = 90^\circ$   
 then  $(AC)^2 = (AB)^2 + (BC)^2$   
 $(AB)^2 = (AC)^2 - (BC)^2$   
 $(BC)^2 = (AC)^2 - (AB)^2$

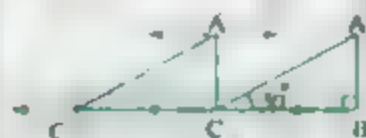
## Geometric transformations



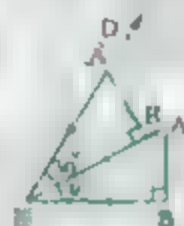
- Image of each of A and B are them
- To find the image of C : We place the compasses at C , then we draw an arc cuts  $\overline{AB}$  at two points , then we place at each of them and draw two arcs intersect at  $C'$  , then  $\triangle ABC'$  is the image of  $\triangle ABC$  by reflection in  $\overline{AB}$



- Image of C is itself
- To find the image of A : We draw  $\overline{AC}$  , and by using the compasses take the point  $A'$  on  $\overline{AC}$  where  $AC = A'C$
- Similarly , we find image of B , then  $\triangle A'B'C$  is the image of  $\triangle ABC$  by reflection in the point C

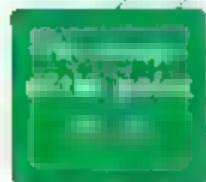


- Image of B is C
- To find the image of C : We draw  $\overline{BC}$  and by using the compasses , we take point  $C'$  on  $\overline{BC}$  where  $CC' = BC$
- To find the image of A : We draw from A a ray parallels to  $\overline{BC}$  , then by using the compasses we determine on it  $A'$  where  $AA' = BC$  , then  $\triangle A'CC'$  is the image of  $\triangle ABC$  by translation BC in the direction  $\overline{BC}$



- Image of C is itself.
- To find the image of A : Put the protractor with its straight edge on  $\overline{CA}$  and in the antic lockwise direction , draw  $\overline{CD}$  where  $m(\angle ACD) = 30^\circ$  , then take the point  $A'$  on  $\overline{CD}$  where  $CA' = CA$
- Similarly , we find  $B'$  which is the image of B ,  $\triangle A'B'C$  is the image of  $\triangle ABC$  by rotation around C with angle of measure  $30^\circ$

## Geometry and Measurement

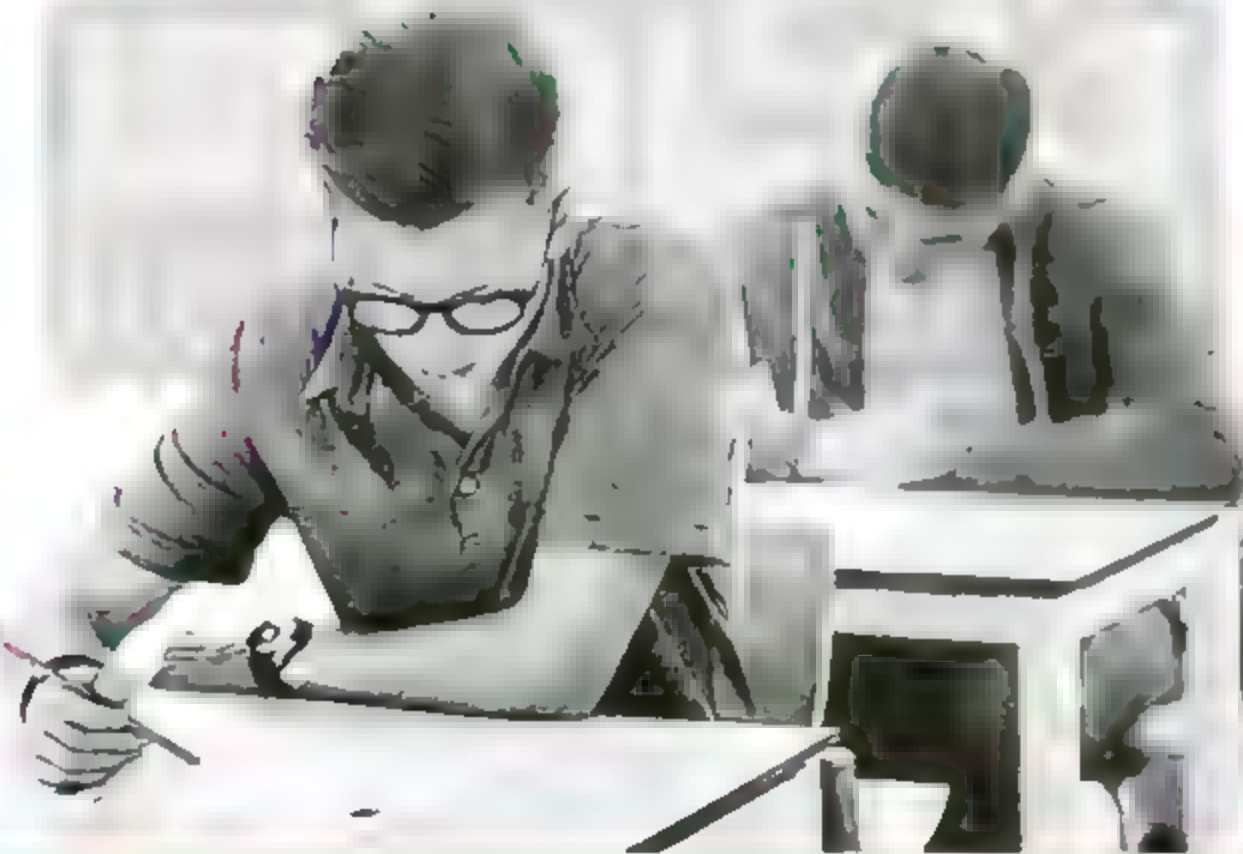
by reflection in the  $X$ -axis→ The point  $(X, y)$ by reflection in the  $y$ -axis→ The point  $(-X, y)$ 

by reflection in the origin point

→ The point  $(-X, -y)$ by translation  $(X, y) \Rightarrow (X + k, y + l)$ → The point  $(X + k, y + l)$ by rotation  $R(O, 90^\circ)$  ( $\frac{1}{4}$  turn)→ The point  $(-y, X)$ by rotation about  $O$  with an angle of measure  $(-90^\circ)$  or  $(270^\circ)$ → The point  $(y, -X)$ by rotation  $R(O, \pm 180^\circ)$  ( $\frac{1}{2}$  turn)→ The point  $(-X, -y)$ by rotation  $R(O, \pm 360^\circ)$   
(identity rotation)→ The point  $(X, y)$

# Final Examinations

on Geometry and  
Measurement





## Model Examination of the School Book



## on Geometry and Measurement

## Model

1

Answer the following questions :

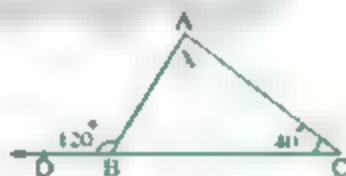
1 Choose the correct answer from those given :

- 1 Circumference of a circle of radius 7 cm. = ..... cm ( $\pi = \frac{22}{7}$ )
- (a) 11 (b) 22 (c) 44 (d) 88
- 2 The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is
- (a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$
- 3 The measure of the exterior angle of the equilateral triangle is
- (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $120^\circ$
- 4 In a parallelogram if the adjacent sides are equal in the length, then the shape is
- (a) square (b) rhombus (c) rectangle (d) trapezium.
- 5 The number of the diagonals of a pentagon is
- (a) 3 (b) 5 (c) 7 (d) 9
- 6 The number of axes of symmetry of an isosceles triangle =
- (a) zero (b) 1 (c) 2 (d) 3

2 Complete the following :

1 The image of the point  $(2, 1)$  by reflection in X-axis is

2 In the opposite figure :

 $X = \dots\dots\dots^\circ$ 3  $\triangle XYZ$  is a triangle in which  $m(\angle Y) = 90^\circ$ ,  $XY = 3$  cm,  $XZ = 5$  cm, then  $YZ = \dots\dots\dots$  cm.4 ABCD is a parallelogram in which  $m(\angle A) = 100^\circ$ , then  $m(\angle B) + m(\angle D) = \dots\dots\dots$ 5 The sum of the measures of the interior angles of a triangle =  $\dots\dots\dots$ 

3 [a] In the opposite figure :

 $m(\angle A) = m(\angle B) = 25^\circ$ Find :  $m(\angle ACD)$ 

- [b] Draw a triangle ABC in which  $AB = 5$  cm,  $AC = 3$  cm and  $m(\angle A) = 40^\circ$   
 , then draw  $\hat{C}$  is the image of C under rotation  $R(A, 40^\circ)$  ,  $\hat{B}$  is the image  
 of B under rotation  $R(A, -40^\circ)$

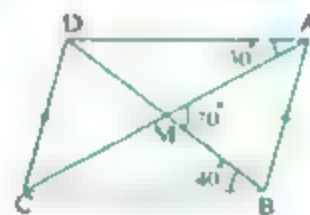
**4 [a] In the opposite figure :**

$$\overline{AB} \parallel \overline{DC}, \overline{AC} \cap \overline{BD} = \{M\},$$

$$m(\angle DAC) = 30^\circ, m(\angle DBC) = 40^\circ$$

$$\text{and } m(\angle AMB) = 70^\circ$$

Prove that : ABCD is a parallelogram.



- [b] Use the translation :  $(x, y) \longrightarrow (x + 2, y + 3)$   
 to find the point whose image is  $(2, 3)$

**5 [a] In the opposite figure :**

$$\overline{AD} \perp \overline{BC}, \text{ if } AD = 24 \text{ cm, } AB = 26 \text{ cm, } AC = 30 \text{ cm}$$

1 Find : The length of  $\overline{BC}$

2 Find : The area of  $\triangle ABC$



[b] In the opposite figure :

$$ABCD \text{ is a square, } E \in \overline{BC}, \overline{AC} \parallel \overline{DE}$$

Prove that : ACED is a parallelogram.



## Model 2

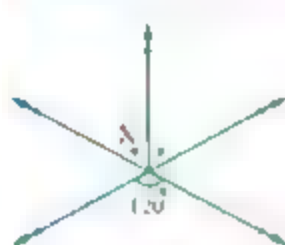
Answer the following questions :

**1 Choose the correct answer from those given :**

- ABC is a right-angled triangle at B ,  $AB = 6$  cm. ,  $BC = 8$  cm , then  $AC =$  cm.  
 (a) 10 (b) 28 (c) 100 (d) 160
- The measure of each angle of regular hexagon equals \_\_\_\_\_  
 (a)  $60^\circ$  (b)  $108^\circ$  (c)  $120^\circ$  (d)  $135^\circ$
- The two diagonals are equal in length and not perpendicular in  
 (a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

# Geometry and Measurement

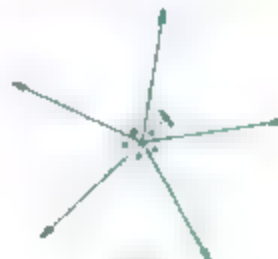
4 In all the following shapes  $m(\angle X) = 60^\circ$  except the shape



(a)



(b)



(c)



(d)

5 In the opposite figure :

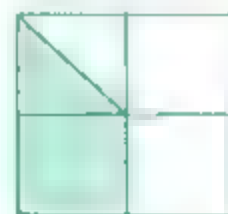
The area of the shaded part from the area of all shape equals .....

(a)  $\frac{1}{8}$

(b)  $\frac{1}{4}$

(c)  $\frac{3}{8}$

(d)  $\frac{3}{4}$



6 In the opposite figure :

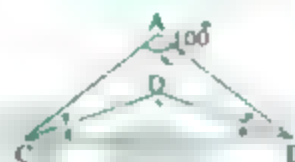
$m(\angle BDC) =$  .....

(a) 60

(b) 80

(c) 100

(d) 140



2 Complete the following :

1 In the opposite figure :

Semicircle of diameter 14 cm. and two semicircles the diameter of each is 7 cm

then the perimeter of the figure equals ..... cm  $(\pi = \frac{22}{7})$



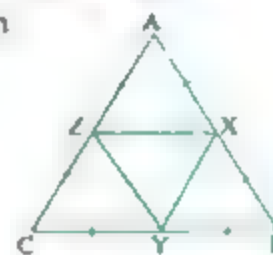
2 The image of the point (2, 3) by translation MN, in direction  $\vec{MN}$ , where M (2, -1), N (5, 1) is .....

3 The volume of a cube of side length 1.2 m = .....  $\text{cm}^3$

4 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side

5 In the opposite figure :

The image of the triangle XBY by translation XZ in direction  $\vec{XZ}$  is .....



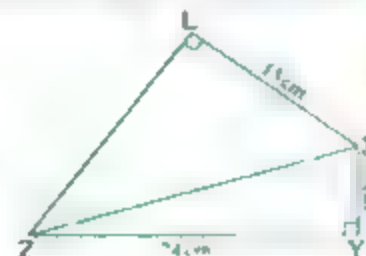
3 [a] In the opposite figure :

XYZL is a quadrilateral in which

$m(\angle Y) = m(\angle L) = 90^\circ$ , XY = 7 cm,

YZ = 24 cm, XL = 15 cm

Find : The length of each of  $\overline{XZ}$  and  $\overline{LZ}$



- [h] Using the square lattice, draw  $\overline{AB}$  where  $A(4, 3)$ ,  $B(-1, 1)$   
then find the image of  $\overline{AB}$  by translation  $(X, y) \rightarrow (X + 2, y - 1)$

- 4 [a] Draw the image of triangle ABC where  $A(1, 1)$ ,  $B(3, 4)$ ,  $C(5, 2)$   
by reflection in X-axis

- [b] In the opposite figure :

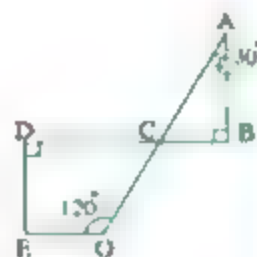
$\overline{AB}$  and  $\overline{ED}$  are perpendicular to  $\overline{BD}$

$$\overline{BD} \cap \overline{AO} = \{C\}$$

$$m(\angle A) = 30^\circ$$

$$m(\angle EOC) = 120^\circ$$

Find :  $m(\angle E)$



- 5 [a] In the opposite figure :

$$\overline{EO} \parallel \overline{CD}, m(\angle E) = 50^\circ$$

$$m(\angle C) = 30^\circ$$

Find the measures of angles of  $\triangle ABC$ ,  $m(\angle ABD)$



- [b] In the opposite figure :

X is the midpoint of AB

$$Y \in \overline{CD}, Z \in \overline{CE}$$

$$\overline{AD} \parallel \overline{XY} \parallel \overline{BC}, \overline{YZ} \parallel \overline{DE}$$

Is  $CZ = ZE$  ? giving reason





## Model examination for the merge students

Answer the following questions :

## 1 Choose the correct answer :

- The sum of the measures of the interior angles of a triangle = .....  
 (a) 90 (b) 360 (c) 180 (d) 540
- The image of the point  $(3, -2)$  by reflection in the y axis is the point  
 (a)  $(3, 2)$  (b)  $(-3, -2)$  (c)  $(-3, 2)$  (d)  $(-2, 3)$
- The diagonals are equal and perpendicular in  
 (a) rhombus (b) square (c) rectangle (d) parallelogram.

## 4 In the opposite figure :

AC = ..... cm.

- (a) 5 (b) 7  
(c) 25 (d) 625

## 5 In the opposite figure :

 $m(\angle ACD) = \dots\dots\dots^\circ$ 

- (a) 40 (b) 140  
(c) 90 (d) 50



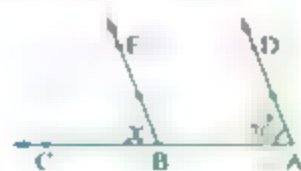
## 2 Complete each of the following :

- The length of the line segment that joins two midpoints of two sides of a triangle equals ..... the length of the third side.
- The rectangle is a parallelogram in which one of it's angles is .....
- The length of the side of a rhombus whose perimeter is 24 cm equals ..... cm
- The image of the point A  $(-3, 2)$  by reflection in the origin point is the point  $\hat{A}$  ( ..... , ..... )

## Final Examinations

2 In the opposite figure :

$X = \dots\dots\dots^\circ$



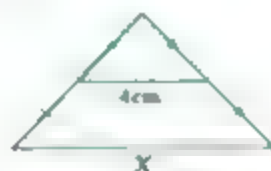
3 Put (✓) or (X) :

- 1 The image of the point  $(4, 3)$  by reflection in the X-axis is the point  $(3, -4)$  ( )
- 2 If ABC is a right-angled triangle at B , then  $(AB)^2 = (BC)^2 + (AC)^2$  ( )
- 3 The pentagon has 5 diagonals. ( )
- 4 ABCD is a parallelogram , in which  $m(\angle A) = 70^\circ$  , then  $m(\angle C) = 110^\circ$  ( )
- 5 Any triangle contains at least two acute angles. ( )

4 Join from the column (A) to the suitable in the column (B) :

Column (A)	Column (B)
1 The sum of the measures of the interior angles of a quadrilateral = .....	• $120^\circ$
2 The measure of each angle of a regular hexagon = .....	• $160^\circ$
3 The image of the point $(3, 2)$ by translation $(1, -2)$ is the point .....	• $(-1, 3)$
4 The image of the point $(1, 3)$ by rotation about the origin point , of angle of measure $180^\circ$ is the point ( ..... )	• 45
5 The diagonal of the square divides the angle of the vertex into two angles , the measure of each = .....	• $(4, 0)$

5 Find the value of X :



$X = \dots\dots\dots$  cm

Fig. (1)



$X = \dots\dots\dots^\circ$

Fig. (2)

## School Examinations

on Geometry and  
Measurement

1

Cairo Governorate

East Nour City Zone  
Menaret Helwan School

Answer the following questions :

## 1 Complete :

- The sum of measures of the exterior angles of the heptagon is .....
- If two lines intersect , then each two vertically opposite angles are .....
- In  $\triangle ABC$  , if  $m(\angle A) + m(\angle C) = m(\angle B)$  , then  $m(\angle B) =$  .....
- The image of  $(2, 3)$  by translation  $(x, y) \rightarrow (x + 4, y - 2)$  is .....
- The length of the line segment joining two midpoints of two sides of a triangle equals the length of the third side .....

## 2 Choose the correct answer :

- ABCD is a parallelogram in which  $m(\angle A) = 80^\circ$  , then  $m(\angle C) =$  .....  
(a)  $80^\circ$  (b)  $100^\circ$  (c)  $120^\circ$  (d)  $60^\circ$
- If the image of the point  $(5, -3)$  by rotation about the origin point is itself , then the measure of the rotation angle is .....  
(a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$
- The line segment joining two midpoints of two sides of a triangle is ..... the third side.  
(a) intersecting (b) parallel to (c) perpendicular to (d) congestive
- The sum of measures of the accumulative angles at a point is .....  
(a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$
- If ABC is a right-angled triangle at B and  $AB = 4$  cm ,  $BC = 3$  cm , then  $AC =$  ..... cm.  
(a) 16 (b) 25 (c) 9 (d) 5
- The sum of measures of the interior angles of a hexagon is .....  
(a)  $360^\circ$  (b)  $540^\circ$  (c)  $720^\circ$  (d)  $120^\circ$

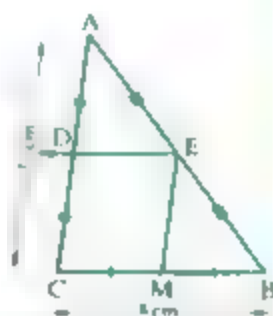
## 3 [a] In the opposite figure :

ABE is a triangle ,  $\overline{AB} \parallel \overline{GH} \parallel \overline{CD}$  ,  $m(\angle CDB) = 145^\circ$ ,  $m(\angle AGH) = 135^\circ$ Calculate :  $m(\angle DEG)$ 

[b] In the opposite figure :

ABC is a triangle in which E , M and D are the midpoints of the sides  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{CA}$  respectively  
 $\therefore BC = 8 \text{ cm}$  ,  $\therefore AC = 10 \text{ cm}$ .

Prove that : DEMC is a parallelogram.

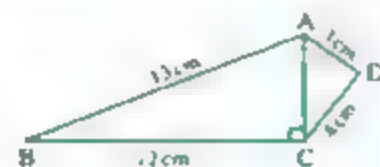


4 [a] In the opposite figure :

$m(\angle ACB) = 90^\circ$  ,  $AB = 13 \text{ cm}$  ,  $AD = 3 \text{ cm}$  ,  
 $CD = 4 \text{ cm}$  ,  $BC = 12 \text{ cm}$ .

Find : 1 The length of  $\overline{AC}$

2 The perimeter of the figure ABCD



[b] In the opposite figure :

ABC is a triangle in which D , F and E are the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{CA}$

respectively such that :  $DF = 4 \text{ cm}$  ,  $FE = 3 \text{ cm}$  ,  $DE = 6 \text{ cm}$ .

Calculate : The perimeter of  $\triangle ABC$



5 [a] Using the lattice , find the images of the points

A (-4 , 1) , B (0 , 4) and C (-2 , 2) by reflection in X-axis

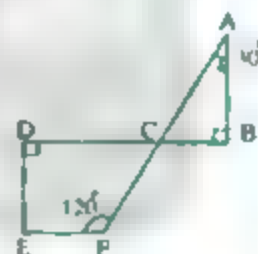
[b] In the opposite figure :

$\overline{AB} \perp \overline{BC}$  ,  $\overline{BD} \cap \overline{AF} = \{C\}$

$\therefore m(\angle A) = 30^\circ$  ,  $\therefore m(\angle F) = 120^\circ$

Find with proof : 1  $m(\angle ACB)$

2  $m(\angle E)$



Cairo Governorate

Abdeen Directorate  
Patriarchal College



Answer the following questions :

1 Choose the correct answer :

1 The sum of the measures of the interior angles of the pentagon equals

(a) 360

(b) 540

(c) 720

(d) 108

2 The two diagonals are equal in length and perpendicular in the

(a) parallelogram

(b) rectangle

(c) rhombus

(d) square



## Geometry and Measurement

- 3 The image of the point  $(4, -7)$  by reflection in  $X$ -axis is  
 (a)  $(4, 7)$  (b)  $(-4, -7)$  (c)  $(-4, 7)$  (d)  $(4, -7)$
- 4 The line segment joining the midpoints of two sides of a triangle is \_\_\_\_\_ the third side.  
 (a) parallel to (b) perpendicular to (c) bisect to (d) equal to
- 5 In  $\triangle ABC$ , if  $m(\angle B) = 90^\circ$ ,  $AB = 6$  cm and  $BC = 8$  cm, then  $AC =$  \_\_\_\_\_ cm.  
 (a) 100 (b) 8 (c) 14 (d) 10
- 6 The image of the point  $(-3, 4)$  by rotation about the origin with an angle of measure  $90^\circ$  is  
 (a)  $(-4, 3)$  (b)  $(3, -4)$  (c)  $(-4, -3)$  (d)  $(4, 3)$

## 2 Complete :

- 1 A quadrilateral in which only two opposite sides are parallel is called \_\_\_\_\_.
- 2 The measure of each angle of the regular octagon equals \_\_\_\_\_.
- 3 The image of the point  $(1, -5)$  by translation  $(-4, 6)$  is \_\_\_\_\_.
- 4 If the measure of each interior angle of a regular polygon is  $140^\circ$ , then the number of its sides is \_\_\_\_\_.
- 5 ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 100^\circ$ ,  $m(\angle B) =$  \_\_\_\_\_.

## 3 [a] In the opposite figure :

D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

,  $F \in \overline{CB}$ , where  $BF = \frac{1}{2} CB$

Prove that : DFBE is a parallelogram.



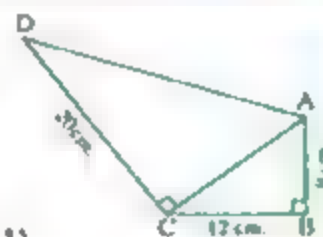
- [b] Draw the image of  $\triangle XYZ$  in which  $XY = 3$  cm,  $YZ = 5$  cm and  $ZX = 7$  cm by reflection in the straight line containing the longest side.

## 4 [a] In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$

,  $AB = 9$  cm,  $BC = 12$  cm, and  $CD = 20$  cm.

Find by proof : The length of each of  $\overline{AC}$  and  $\overline{AD}$



- [b] In the square lattice draw  $\triangle ABC$  where  $A(-4, 2)$ ,  $B(-1, 1)$  and  $C(-2, 5)$ , then draw its image by the translation :  $(x, y) \rightarrow (x + 5, y - 3)$

## Final Examinations

5 [a] In the opposite figure :

$$\overline{DC} \cap \overline{BE} = \{O\}, m(\angle A) = 85^\circ$$

$$, m(\angle D) = 100^\circ, m(\angle E) = 35^\circ \text{ and } m(\angle C) = 50^\circ$$

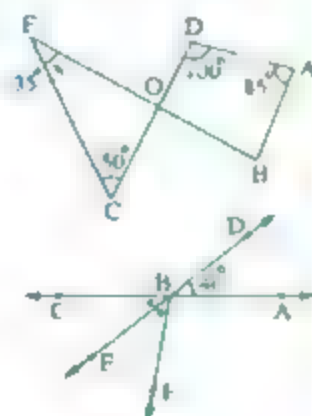
Find :  $m(\angle B)$

[b] In the opposite figure :

$$\overline{AC} \cap \overline{DE} = \{B\}, m(\angle ABD) = 40^\circ$$

,  $\overline{BE}$  bisects  $\angle CBF$

Find :  $m(\angle ABF)$



## 3 Cairo Governorate

Hedayek El Kube Educational Zone



Answer the following questions : (Calculators are Permitted)

1 Choose the correct answer :

1 The measure of the interior angle of the regular octagon equals

- (a)  $1080^\circ$  (b)  $180^\circ$  (c)  $135^\circ$  (d)  $108^\circ$

2 If ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 140^\circ$ , then  $m(\angle B) =$

- (a)  $40^\circ$  (b)  $110^\circ$  (c)  $70^\circ$  (d)  $60^\circ$

3 In  $\triangle ABC$ , if  $m(\angle B) = 90^\circ$ ,  $AB = 20$  cm,  $AC = 25$  cm, then  $BC =$  cm.

- (a) 625 (b) 12 (c) 15 (d) 225

4 If the perimeter of a square is 20 cm, then its area equals  $\text{cm}^2$

- (a) 25 (b) 100 (c) 5 (d) 16

5 The image of point  $(-3, 5)$  by rotation about origin point with an angle of measure  $90^\circ$  is

- (a)  $(-5, 3)$  (b)  $(-5, -3)$  (c)  $(5, 3)$  (d)  $(3, -5)$

6 The image of point  $(-1, 3)$  by translation  $(4, -2)$  is

- (a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$

2 Complete the following :

- 1 The line segment joining the midpoints of two sides of a triangle is parallel to
- 2 The image of the point  $(2, -4)$  by reflection in y-axis is
- 3 The number of axis of symmetry of a rectangle is

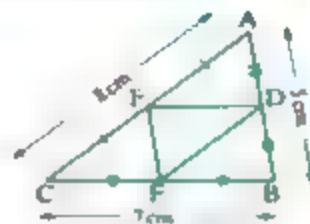
## Geometry and Measurement

- 4 The measure of exterior angle of an equilateral triangle equals .....  
 5 The volume of a cube of side length 6 cm. is .....  $\text{cm}^3$ .

## 3 [a] In the opposite figure :

ABC is a triangle , D , E and F are midpoints  
 of  $\overline{AB}$  ,  $\overline{BC}$  ,  $\overline{CA}$  respectively  
 where  $AB = 5 \text{ cm}$  ,  $BC = 7 \text{ cm}$  and  $CA = 8 \text{ cm}$ .

Calculate : The perimeter of  $\triangle DEF$



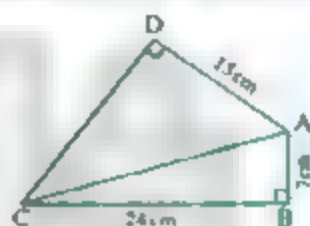
- [b] On square lattice , draw the square ABCD where  $A(1, 2)$  ,  $B(-2, 2)$  ,  $C(-2, 5)$   
 and  $D(1, 5)$  , then draw its image  
 by reflection in X-axis.

## 4 [a] In the opposite figure :

$$m(\angle B) = m(\angle D) = 90^\circ$$

$AD = 15 \text{ cm}$  ,  $AB = 7 \text{ cm}$  and  $BC = 24 \text{ cm}$ .

Find : The length of each of  $\overline{AC}$  ,  $\overline{DC}$



- [b] Draw  $\triangle ABC$  where  $A(1, 5)$  ,  $B(3, 1)$  and  $C(5, 3)$   
 , then draw its image by rotation about the origin point with an angle of measure  $180^\circ$

## 5 [a] In the opposite figure :

$$\overline{CE} \cap \overline{AD} = \{B\}$$

$$m(\angle A) = 50^\circ \text{ , } m(\angle C) = 70^\circ$$

$$m(\angle D) = 130^\circ \text{ and } m(\angle F) = 90^\circ$$

Find :  $m(\angle E)$



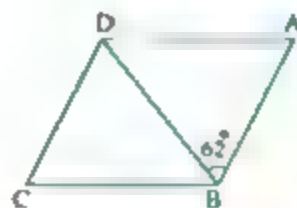
## [b] In the opposite figure :

ABCD is a rhombus

$\overline{BD}$  is a diagonal in it

$$m(\angle ABD) = 62^\circ$$

Find with proof :  $m(\angle A)$



4

Giza Governorate

El-Haram Zone

Hafiz El-Bahn Language School British Academy



Answer the following questions :

## 1 Choose the correct answer :

- The measure of each angle of the regular pentagon equals  
(a)  $100^\circ$  (b)  $120^\circ$  (c)  $108^\circ$  (d)  $110^\circ$
- The sum of measures of accumulative angles at a point equals  
(a)  $90^\circ$  (b)  $100^\circ$  (c)  $360^\circ$  (d)  $180^\circ$
- Any triangle has at least two ..... angles.  
(a) acute (b) obtuse (c) right (d) reflex
- ABCD is a parallelogram in which  $m(\angle A) = 70^\circ$ , then  $m(\angle B) =$  .....  
(a) 70 (b) 110 (c) 90 (d) 180
- $(-4, 5)$  is the image of  $(5, 4)$  by rotation in the origin point with an angle of measure  
(a)  $90^\circ$  (b)  $-90^\circ$  (c)  $180^\circ$  (d)  $360^\circ$
- The image of the point  $(-2, 2)$  by a translation of magnitude 3 units in the positive direction of y-axis is  
(a)  $(-1, 2)$  (b)  $(-2, 5)$  (c)  $(-1, 5)$  (d)  $(1, 5)$

## 2 Complete the following :

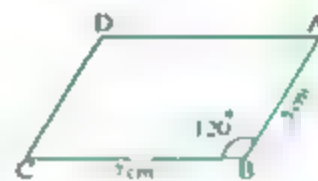
- The sum of measures of the interior angles of any triangle equals
- Each two opposite angles of a parallelogram are
- The length of the line segment joining the midpoint of two sides of a triangle is equal to ..... the length of the third side
- The image of the point  $(3, -5)$  by rotation  $360^\circ$  about the origin point is
- The image of  $(-3, 0)$  by reflection in the y-axis is

## 3 [a] A regular hexagon of side length 15 cm

Find : 1 its perimeter.

2 the measure of each angle

## [b] In the opposite figure :

ABCD is a parallelogram ,  $\overline{AB} = 5$  cm.,  $BC = 7$  cm. ,  $m(B) = 120^\circ$ Find :  $m(\angle C)$  ,  $m(\angle D)$  , the length of each of  $\overline{AD}$  and  $\overline{DC}$ 



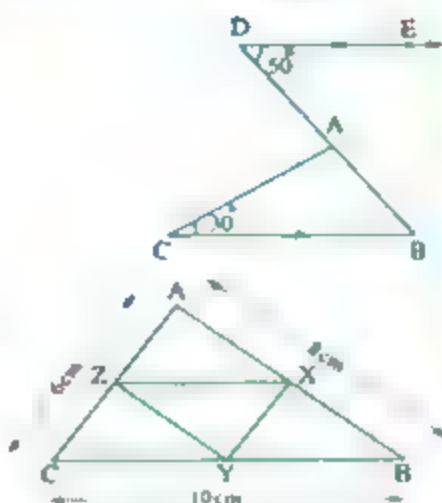
## 1. Geometry and Measurement

4 [a] In the opposite figure :

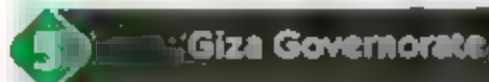
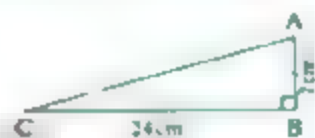
 $\overline{DE} \parallel \overline{BC}$  ,  $m(\angle D) = 50^\circ$ ,  $m(\angle C) = 30^\circ$ Find :  $m(\angle B)$  and  $m(\angle BAC)$ 

[b] In the opposite figure :

X , Y and Z

are the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{AC}$  respectively $AB = 8$  cm. ,  $AC = 6$  cm. ,  $BC = 10$  cm.Find : The perimeter of  $\triangle XYZ$ 

5 [a] In the opposite figure :

 $m(\angle B) = 90^\circ$  ,  $AB = 7$  cm. $BC = 24$  cmFind : The length of  $\overline{AC}$ [b] Draw ABC in which  $A(1, 1)$  ,  $B(5, 1)$  and  $C(3, 3)$  , then find its image by reflection in y-axis.Experimental Schools  
Math Inspector

Answer the following questions :

1 Complete the following :

- The image of the point  $(5, -3)$  under rotation of angle of measure  $90^\circ$  about the origin point is
- The ray which is drawn from the midpoint of a side in a triangle parallel to another side ..... the third side.
- In the rhombus the two diagonals are
- The sum of measures of the exterior angles of any convex polygon equals
- In  $\triangle ABC$  , if  $m(\angle A) + m(\angle B) = m(\angle C)$  , then  $\triangle ABC$  is

2 Choose the correct answer :

- If the measure of an interior angle of a regular polygon is  $135^\circ$  , then the number of its sides is
  - 6
  - 4
  - 7
  - 8

## Final Examinations

- 2 The image of the point  $(-1, 4)$  by the translation  $(x, y) \longrightarrow (x + 3, y - 2)$  followed by reflection in the  $x$ -axis is  
 (a)  $(2, 2)$  (b)  $(-2, 2)$  (c)  $(-2, -2)$  (d)  $(2, -2)$
- 3 ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 110^\circ$ , then  $m(\angle B) =$   
 (a)  $125^\circ$  (b)  $80^\circ$  (c)  $100^\circ$  (d)  $110^\circ$
- 4 The concave polygon should have ..... angle.  
 (a) an acute (b) a right (c) an obtuse (d) a reflex
- 5 The image of the point ..... is itself by reflection in  $y$  axis.  
 (a)  $(0, 3)$  (b)  $(3, 0)$  (c)  $(3, 3)$  (d)  $(-3, 3)$
- 6 The identity rotation about the origin point with an angle of measure  
 (a)  $90$  (b)  $180$  (c)  $360$  (d)  $-90$

## 2 [a] In the opposite figure :

ABCD is a quadrilateral in which

$$m(\angle B) = m(\angle ACD) = 90^\circ$$

,  $AB = 4$  cm, ,  $BC = 3$  cm, and  $AD = 13$  cm.

Find : AC and DC



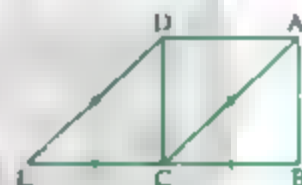
## [b] In the opposite figure :

ABCD is a square ,  $E \in \overline{BC}$

such that  $BC = CE$

1 Prove that : The figure ACED is a parallelogram

2 Find :  $m(\angle E)$



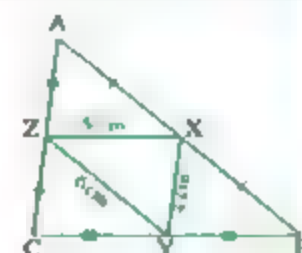
## 3 [a] In the opposite figure :

$\triangle ABC$  in which  $X$ ,  $Y$  and  $Z$  are the midpoints of

$\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CA}$  respectively

,  $XZ = 5$  cm, ,  $XY = 4$  cm, ,  $YZ = 6$  cm.

Find with proof : The perimeter of  $\triangle ABC$



## [b] In the opposite figure :

ABCD and EBCF are two parallelograms

,  $m(\angle BAD) = 60^\circ$ , ,  $m(\angle BEF) = 130^\circ$

Find with proof :  $m(\angle BXC)$



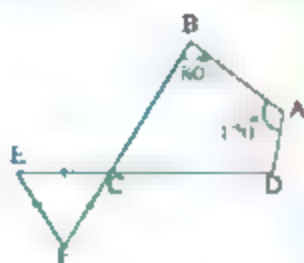
## Geometry and Measurement

5 [a] In the opposite figure :

ABCD is a quadrilateral in which  $m(\angle A) = 120^\circ$

,  $m(\angle B) = 80^\circ$ ,  $\triangle CEF$  is an equilateral triangle

Find :  $m(\angle D)$



[b] Using the square lattice draw  $\triangle ABC$  where :

$A(2, 4)$ ,  $B(4, 2)$  and  $C(3, 1)$ , then map its image by the translation

$(x, y) \rightarrow (x + 2, y + 3)$

### 6 Alexandria Governorate

East Educational Zone  
Math's Supervision (A)



Answer the following questions :

1 Complete each of the following :

- The angle whose measure is  $89^\circ$  is ..... angle.
- The length of the line segment that joins two midpoints of two sides of a triangle equals .....
- The area of one face of a cube is  $25 \text{ cm}^2$  then its volume equals .....  $\text{cm}^3$
- The image of the point  $(2, 1)$  by reflection in X-axis is .....
- In the parallelogram XYZL, if  $m(\angle X) = \frac{1}{2} m(\angle Y)$ , then  $m(\angle Y) =$  .....

2 Choose the correct answer from those given :

- The supplementary of the angle whose measure is  $70^\circ$  is an angle of measure  
(a) 30 (b) 60 (c) 120 (d) 150
- The number of the diagonals of a pentagon is  
(a) 3 (b) 5 (c) 7 (d) 9
- The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is  
(a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$
- The sum of the measures of the interior angles of the triangle equals .....  
(a) 90 (b) 360 (c) 180 (d) 540
- ABC is right-angled triangle at B,  $AB = 6 \text{ cm}$ ,  $BC = 8 \text{ cm}$ , then  $AC =$  ..... cm  
(a) 10 (b) 28 (c) 100 (d) 160
- The measure of the exterior angle of the equilateral triangle is .....  
(a) 30 (b) 45 (c) 60 (d) 120

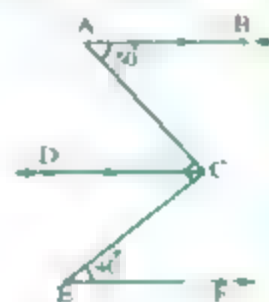
Final Examinations

3 [a] In the opposite figure :

$\overline{AB} \parallel \overline{CD}$  ,  $m(\angle A) = 50^\circ$

•  $\angle ACE$  is right angle and  $m(\angle E) = 40^\circ$ Prove that :  $\overline{AB} \parallel \overline{EF}$ 

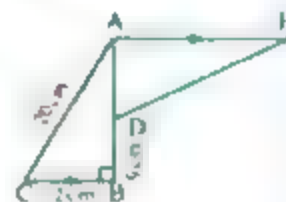
- [b] Use the translation  $(x, y) \longrightarrow (x + 2, y + 3)$  to locate the point whose image is  $(2, 3)$



4 [a] In the opposite figure :

ABC is a triangle ,  $m(\angle B) = 90^\circ$ •  $\overline{AE} \parallel \overline{BC}$  , if  $BC = 12$  cm. ,  $AC = 20$  cm.•  $D \in AB$  where  $BD = 9$  cm. and  $AE = 2 BC$ Find : The length of each of  $\overline{AD}$  ,  $\overline{ED}$ 

- [b] Using square lattice , draw  $\triangle ABC$  , where :

 $A(-2, 4)$  ,  $B(5, 0)$  ,  $C(3, -3)$ • then draw the reflected image of  $\triangle ABC$  in the origin point

5 [a] In the opposite figure :

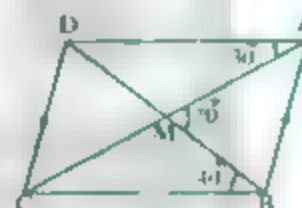
$\overline{AB} \parallel \overline{DC}$  ,  $\overline{AC} \cap \overline{BD} = \{M\}$

$m(\angle DAC) = 30^\circ$  ,  $m(\angle DBC) = 40^\circ$

and  $m(\angle AMB) = 70^\circ$

Prove that : ABCD is a parallelogram

- [b] In the opposite figure :

 $AB = 5$  cm ,  $BC = 8$  cm. ,  $AC = 7$  cm D , E and Fare the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{CA}$  respectively.Calculate : The perimeter of  $\triangle DEF$ 

7

Alexandria Governorate

Middle Educational Zone  
Math's Supervision

Answer the following questions :

1 Complete each of the following :

- The measure of the straight angle equals .....
- The image of the point  $(4, -2)$  by reflection in X-axis is



## Geometry and Measurement

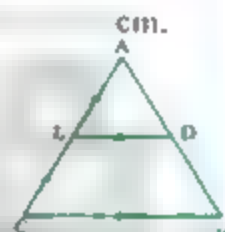
- 3 The ray drawn from the midpoint of a side parallel to another side in a triangle
- 4 The image of the point  $(-2, 1)$  by rotation with an angle of measure  $180^\circ$  about origin point is
- 5 The length of line segment joining the midpoints of two sides of a triangle equals
- 6 A rhombus is a \_\_\_\_\_ with two adjacent equal sides in length

## 2 Choose the correct answer :

- 1 The perimeter of a square with side length 1 cm equals \_\_\_\_\_ cm  
(a) 5 (b) 1 (c) 4 (d) 2
- 2 The number of diagonals in a pentagon is  
(a) 0 (b) 3 (c) 4 (d) 5
- 3 The edge length of a cube whose volume is  $27 \text{ cm}^3$  is \_\_\_\_\_ cm  
(a) 9 (b) 3 (c) 27 (d) 6
- 4 In  $\triangle ABC$ , if  $m(\angle B) = 90^\circ$ ,  $AB = 3 \text{ cm}$ ,  $BC = 4 \text{ cm}$ , then  $AC =$   
(a) 7 (b) 12 (c) 1 (d) 5

5 In the opposite figure :  $CB : ED =$  \_\_\_\_\_

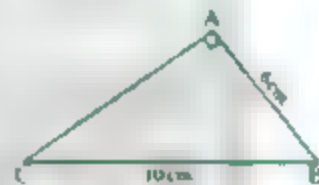
- (a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4



## 3 [a] In the opposite figure :

$\triangle ABC$  is a right-angled triangle at A  
 $BC = 10 \text{ cm}$ ,  $AB = 6 \text{ cm}$ .

Find : The length of  $\overline{AC}$



- [b] Draw the image of the rectangle XYZL where  $X(-5, 1)$ ,  $Y(-5, 4)$ ,  $Z(-1, 4)$ ,  $L(-1, 1)$  by reflection in the X-axis.

## 4 [a] In the opposite figure :

$BC = 10 \text{ cm}$ ,  $AB = 7 \text{ cm}$ ,  $AC = 8 \text{ cm}$

Calculate : The perimeter of the  $\triangle FDE$

- [b] The ratio between the measures of the angles of a quadrilateral is  $2 : 2 : 3 : 5$

Calculate : The measure of the biggest angle.

5 [a] Find the image of the point A  $(5, -5)$  by translation  $(X, y) \rightarrow (X - 2, y + 3)$ 

## [b] In the opposite figure :

$m(\angle A) = m(\angle B) = 30^\circ$

Find showing steps :  $m(\angle ACD)$



## 8 El-Kalyoubia Governorate

Directorate of Education  
Central Maths Supervision

Answer the following questions :

## 1 Choose the correct answer :

- 1 If the perimeter of a square is 20 cm, then the length of its side is ..... cm  
(a) 3 (b) 5 (c) 7 (d) 9
- 2 The number of diagonals of a pentagon is .....  
(a) 3 (b) 5 (c) 7 (d) 9
- 3 If ABCD is a parallelogram,  $m(\angle A) + m(\angle C) = 140^\circ$ , then  $m(\angle D) =$  .....  
(a)  $40^\circ$  (b)  $70^\circ$  (c)  $180^\circ$  (d)  $110^\circ$
- 4 The image of the point  $(-3, 5)$  by rotation about the origin point by an angle of measure  $90^\circ$  is .....  
(a)  $(-3, 5)$  (b)  $(5, -3)$  (c)  $(-5, -3)$  (d)  $(3, -5)$
- 5 The measure of the exterior angle of the equilateral triangle is .....  
(a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $120^\circ$
- 6 If ABC is a right-angled triangle at B,  $AB = 6$  cm and  $BC = 8$  cm, then  $AC =$  .....  
(a) 5 cm. (b) 10 cm. (c) 15 cm. (d) 20 cm.

## 2 Complete :

- 1 The number of axis of symmetry of a rectangle equals .....
- 2 The volume of a cube of edge length 0.2 m, equals .....  $\text{cm}^3$
- 3 The image of  $(3, -2)$  by reflection in the y-axis is .....
- 4 In the opposite figure :  $m(\angle BDC) = \dots\dots\dots^\circ$
- 5 The measure of each interior angle of a regular hexagon is .....



## 3 [a] In the opposite figure :

$$\overline{AD} \cap \overline{FC} = \{B\}$$

$$m(\angle A) = 50^\circ, m(\angle C) = 70^\circ$$

$$m(\angle D) = 140^\circ, \overline{EF} \perp \overline{ED}$$

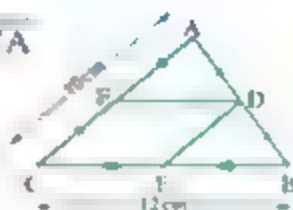
Find :  $m(\angle F)$ 

## Geometry and Measurement

[b] In the opposite figure :

ABC is a triangle , D , E , F are the midpoints of AB , BC , CA respectively , BC = 12 cm , AC = 10 cm.

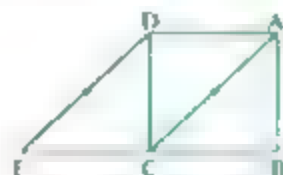
Find : The perimeter of the quadrilateral DECF



4 [a] In the opposite figure :

ABCD is a square , E ∈ BC and AC // DE

1 Prove that : ACED is a parallelogram.

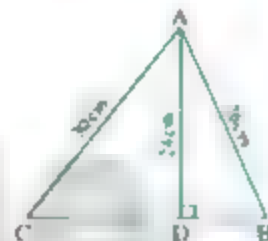
2 Find :  $m(\angle ACE)$ 

[b] In the opposite figure :

$m(\angle ADB) = 90^\circ$  , AD = 24 cm.

AB = 26 cm , AC = 30 cm.

Find : 1 The length of BC

2 The area of  $\triangle ABC$ 

5 [a] Using square lattice , draw  $\overline{AB}$  where A (4 , 3) , B ( - 1 , 1) , then find the image of  $\overline{AB}$  by translation  $(X , y) \rightarrow (X + 2 , y - 1)$

[b] Draw the image of the triangle ABC where A (1 , 1) , B (3 , 4) , C (5 , 2) by reflection in X-axis

9 El-Gharbia Governorate

Official Language Schools  
The Central Maths Supervision



Answer the following questions :

1 Choose the correct answer :

1 The measure of the exterior angle of the equilateral triangle equals

- (a) 30 (b) 45 (c) 60 (d) 120

2 If the image of the point (5 , - 3) by rotation around the origin point is itself , then the measure of the rotation angle equals .....

- (a) 90 (b) 270 (c) 180 (d) 360

3 The image of a square by rotation around the origin point with an angle of measure  $90^\circ$

- (a) rectangle (b) square (c) rhombus (d) trapezium

- 4 Any triangle has at least ..... acute angle.  
 (a) 0 (b) 1 (c) 2 (d) 3
- 5 The measure of the interior angle of the regular hexagon equals  
 (a) 60 (b) 108 (c) 120 (d) 135
- 6 The image of the point  $(-5, 0)$  by reflection on  $X$  axis is  
 (a)  $(5, 0)$  (b)  $(0, 5)$  (c)  $(-5, 0)$  (d)  $(0, -5)$

## 2 Complete :

- The line segment joining the midpoints of two sides of a triangle is
- If  $ABCD$  is a parallelogram in which  $m(\angle A) = 60^\circ$ , then  $m(\angle B) =$
- The image of the point  $(3, 2)$  by reflection in the origin point is
- Each two opposite angles in a parallelogram are
- The rectangle is a parallelogram in which one of its angles is

## 3 [a] In the opposite figure :

$$\overline{AC} \cap \overline{DE} = \{B\}, m(\angle ABD) = 40^\circ$$

and  $\overline{BE}$  bisects  $\angle CBF$

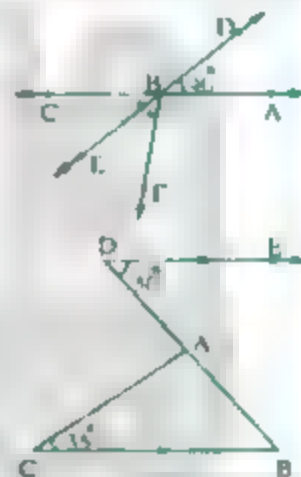
Find with proof :  $m(\angle ABE)$

## [b] In the opposite figure :

$$\overline{DE} \parallel \overline{CB}, m(\angle D) = 50^\circ, m(\angle C) = 35^\circ$$

Find with proof :

$$m(\angle B), m(\angle BAC)$$



## 4 [a] In the opposite figure :

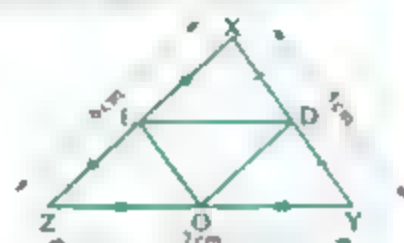
$XYZ$  is a triangle in which  $XY = 5$  cm

,  $XZ = 6$  cm.,  $YZ = 7$  cm.

Find with proof : The perimeter of  $\triangle DOE$

## [b] On the square lattice , draw the triangle whose

vertices are  $A(4, 4)$ ,  $B(4, 2)$ ,  $C(1, 2)$ , then find its image by the translation  $(-4, -2)$



## 5 [a] In the opposite figure :

$$m(\angle B) = m(\angle ACD) = 90^\circ$$

,  $AB = 12$  cm.,  $BC = 9$  cm.,  $CD = 20$  cm.

Find : The length of  $\overline{AD}$



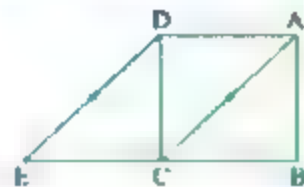


# Geometry and Measurement

[b] In the opposite figure :

ABCD is a square ,  $E \in \overline{BC}$  ,  $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



10 El-Dakahlia Governorate

Directorate of Education  
Maths Supervision



Answer the following questions :

1 Choose the correct answer :

- The sum of the measures of an exterior angles of a triangle equals  
(a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$
- The side length of a rhombus whose perimeter 36 cm. equals  
(a) 144 cm. (b) 6 cm. (c) 9 cm. (d) 18 cm.
- The measure of each angle of regular octagon equals  
(a)  $60^\circ$  (b)  $108^\circ$  (c)  $120^\circ$  (d)  $135^\circ$
- The parallelogram whose diagonals are not perpendicular but equal in length is called  
(a) rhombus (b) square (c) rectangle (d) trapezium
- The image of the point  $(-3, 5)$  by rotation about the origin point with an angle of measure  $-270^\circ$  is  
(a)  $(5, 3)$  (b)  $(-5, 3)$  (c)  $(3, 5)$  (d)  $(-5, -3)$
- The number of the acute angles in any triangle equals at least  
(a) zero (b) 1 (c) 2 (d) 3

2 Complete each of the following :

- The measure of any of the exterior angles of an equilateral triangle equals
- If  $\triangle ABC$  is right angled triangle at B ,  $m(\angle C) = 55^\circ$  , then  $m(\angle A) =$  °
- The parallelogram whose perimeter 32 cm. and the length of one of its sides is 7 cm. , then the length of its adjacent side equals ..
- If the image of the point  $(-5, 4)$  by a translation is  $(1, 4)$  , then the image of the point  $(3, -6)$  by the same translation is
- The rectangle is a parallelogram in which one of its angles is

## 3 [a] In the opposite figure :

ABC is a triangle in which  $D \in \overline{BC}$

$m(\angle ADB) = 90^\circ$ ,  $AB = 26$  cm.

$AD = 24$  cm.  $AC = 30$  cm.

Find : 1 The length of  $\overline{BC}$

2 The area of the triangle ABC



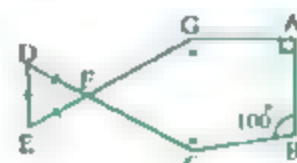
[b] Draw the image of triangle ABC where  $A(2, 2)$ ,  $B(5, 2)$ ,  $C(3, 5)$  by reflection in y-axis.

## 4 [a] In the opposite figure :

DFE is an equilateral triangle

$m(\angle A) = 90^\circ$ ,  $m(\angle B) = 100^\circ$ ,  $m(\angle C) = m(\angle G)$

Find :  $m(\angle C)$



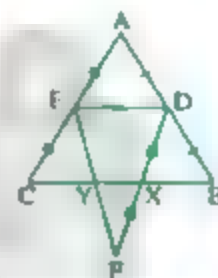
## [b] In the opposite figure :

ABC is a triangle in which D, E are the midpoints of  $\overline{AB}$

$\overline{AC}$  respectively

$CB = 12$  cm. and  $DX = XF$

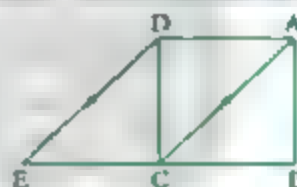
Find : The length of  $\overline{XY}$



## 5 [a] In the opposite figure :

ABCD is a square,  $E \in \overline{BC}$ ,  $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram, then find :  $m(\angle ACE)$

[b] Draw  $\triangle OBC$  on a square lattice where

$O(0, 0)$ ,  $B(4, 0)$ ,  $C(0, 3)$ , then find its image by rotation about the origin point with an angle of measure  $180^\circ$

11 Suez Governorate

Directorate of Education  
Math Supervision

Answer the following questions :

## 1 Choose the correct answer :

- 1 The two diagonals are equal in length and not perpendicular in a
  - (a) parallelogram
  - (b) rectangle
  - (c) square
  - (d) rhombus
- 2 The image of  $(1, 3)$  by translation  $(4, 2)$  is
  - (a)  $(3, 1)$
  - (b)  $(5, 5)$
  - (c)  $(5, 1)$
  - (d)  $(5, -5)$

## Geometry and Measurement

- 1 The measure of an exterior angle in an equilateral triangle is  
(a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $120^\circ$
- 4 In a parallelogram if the two adjacent sides are equal in length, then the shape is a  
(a) square (b) rhombus (c) rectangle (d) trapezium
- 5 The number of axes of symmetry of the equilateral triangle is  
(a) 0 (b) 1 (c) 2 (d) 3
- 6 The sum of measures of the interior angles in a triangle is  
(a) 90 (b) 360 (c) 180 (d) 540

## 2 Complete :

- 1 In the opposite figure :  $m(\angle ACD) = \dots$
- 2 The ray drawn from the midpoint of one side of a triangle parallel to another side
- 3 The image of the point  $(1, -2)$  by reflection in  $x$  axis is
- 4 The sum of measures of the interior angles of the pentagon is
- 5 If ABCD is a rhombus, then .....  $\perp$

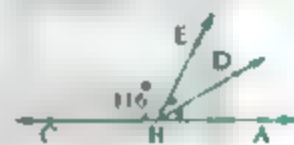


## 3 [a] In the opposite figure :

$$B \in \overline{AC}, m(\angle CBE) = 116^\circ$$

,  $\overline{BD}$  bisects  $\angle ABE$

Find :  $m(\angle ABD)$



## [b] In the opposite figure :

$$m(\angle ABC) = 110^\circ, m(\angle CBD) = 35^\circ$$

$$, m(\angle ABE) = 140^\circ$$

Find :  $m(\angle DBE)$

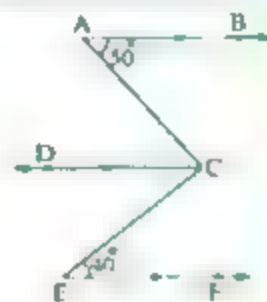


## 4 [a] In the opposite figure :

$$\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$$

$$, m(\angle A) = 50^\circ, m(\angle E) = 40^\circ$$

Find by proof :  $m(\angle ACE)$



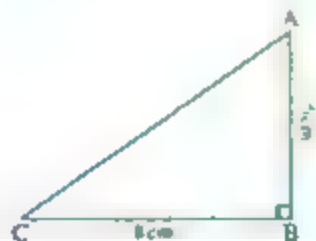
## Final Examinations

[b] In the opposite figure :

$\triangle ABC$  is right-angled at B

,  $AB = 6$  cm. ,  $BC = 8$  cm

Find : The length of  $\overline{AC}$



5 [a] Complete :

- 1 The image of  $(3, 2)$  by rotation about the origin point with an angle of measure  $90^\circ$  is
- 2 The image of  $(3, 2)$  by reflection in the origin point is
- 3 The image of  $(3, 2)$  by rotation about the origin point with an angle of measure  $180^\circ$  is .....

4 In the opposite figure :

$XY = \dots\dots\dots$  cm.

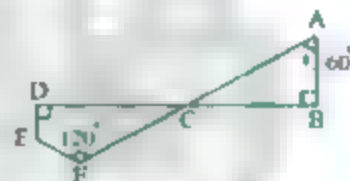


[b] In the opposite figure :

$\overline{AB}$  ,  $\overline{DE}$  are perpendicular to  $\overline{BD}$

,  $\overline{BD} \cap \overline{AF} = \{C\}$  ,  $m(\angle A) = 60^\circ$  ,  $m(\angle CFE) = 120^\circ$

Find with proof :  $m(\angle E)$



12

Port Said Governorate

East Educational Zone  
Mell's Supervision

Answer the following questions :

1 Choose the correct answer :

- 1 The image of the point  $(2, -5)$  by reflection in X-axis is  
(a)  $(2, -5)$  (b)  $(2, 5)$  (c)  $(-2, -5)$  (d)  $(5, 2)$
- 2 The measure of each interior angle of a regular hexagon equals  
(a)  $60^\circ$  (b)  $108^\circ$  (c)  $120^\circ$  (d)  $135^\circ$
- 3 The two diagonals are equal in length and not perpendicular in the  
(a) parallelogram. (b) rectangle. (c) rhombus. (d) square
- 4 The sum of the measures of the interior angles of a triangle equals  
(a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$
- 5 The image of the point  $(3, -2)$  by reflection in the y-axis is the point  
(a)  $(3, 2)$  (b)  $(-3, -2)$  (c)  $(-3, 2)$  (d)  $(-2, 3)$
- 6 The measure of the exterior angle of the equilateral triangle is  
(a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $120^\circ$



## Geometry and Measurement

## 2 Complete each of the following :

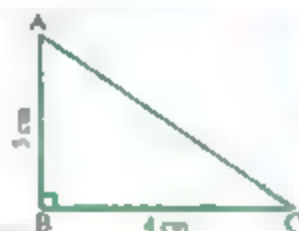
- The length of the line segment that joins two midpoints of two sides of a triangle equals ..... the length of the third side.
- The image of the point A ( - 3 , 2 ) by reflection in the origin point is the point .....
- The sum of the measures of the angles of a quadrilateral equals .....
- If ABCD is a parallelogram in which  $m(\angle A) = 60^\circ$  , then  $m(\angle B) = \dots$
- The length of the side of a rhombus whose perimeter is 24 cm equals ..... cm

## 3 [a] In the opposite figure :

ABC is a right-angled triangle at B

, AB = 3 cm. , BC = 4 cm.

Find : The length of  $\overline{AC}$



[b] On the lattice , find the image of the triangle LMN

where L ( - 4 , - 1 ) , M ( - 1 , - 3 ) , N ( 0 , - 1 )

by reflection in the X-axis

4 [a] Using the square lattice , draw  $\overline{AB}$  where A ( 4 , 3 ) , B ( - 1 , 1 ) , then find the image of  $\overline{AB}$  by reflection in the origin point.

[b] In the opposite figure :

$\overline{DC} \parallel \overline{EO}$  ,  $m(\angle D) = 90^\circ$

,  $m(\angle O) = 120^\circ$

Find :  $m(\angle C)$



## 5 [a] In the opposite figure :

ABC is a triangle in which ED = 8 cm

Find : The length of  $\overline{CB}$



[b] In the opposite figure :

$m(\angle A) = m(\angle B) = 25^\circ$

Find :  $m(\angle ACD)$



## 13 Kafr El-Sheikh Governorate

General Math Supervision



Answer the following questions :

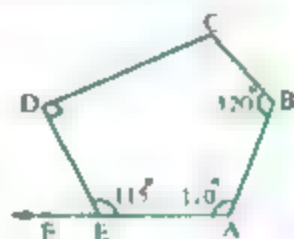
## 1 Choose the correct answer :

- The sum of measures of the interior angles of a hexagon equals
  - 180
  - 540
  - 720
  - 360
- The two diagonals are equal in length and perpendicular in the
  - trapezium
  - square
  - rectangle.
  - parallelogram.
- In  $\Delta XYZ$ , if  $m(\angle X) > m(\angle Y) + m(\angle Z)$ , then  $\angle X$  is
  - an acute
  - an obtuse
  - a right
  - a straight
- The image of  $(3, -5)$  by rotation  $R(O, 90^\circ)$  is
  - $(-3, 5)$
  - $(-3, -5)$
  - $(5, 3)$
  - $(5, -3)$
- The sum of the measures of the exterior angles of a triangle equals
  - 60
  - 120
  - 270
  - 360
- The measure of the exterior angle of the equilateral triangle is
  - 90
  - 120
  - 360
  - 60

## 2 Complete :

- The image of  $A(-2, 3)$  by translation  $(x + 3, y - 2)$  is
- The rhombus with a right angle is
- The image of  $(-1, 2)$  by reflection in the origin point is
- The length of the line segment joining two midpoints of two sides of a triangle is equal to ..... the length of the third side.
- If ABCD is a parallelogram in which  $BC = 8$  cm and  $CD = 6$  cm , then its perimeter = ..... cm

## 3 [a] In the opposite figure :

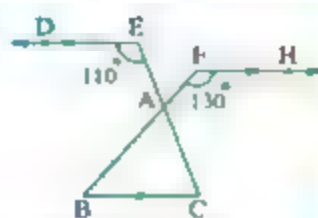
ABCDE is a pentagon in which  $F \in \overline{AE}$  $m(\angle A) = 110^\circ$ ,  $m(\angle B) = 120^\circ$ ,  $m(\angle DEA) = 115^\circ$  $m(\angle D) = 90^\circ$ Find with proof :  $m(\angle C)$ 

## Geometry and Measurement

[b] In the opposite figure :

$$\overline{ED} \parallel \overline{BC} \parallel \overline{FH}$$

$$m(\angle E) = 110^\circ, m(\angle F) = 130^\circ$$

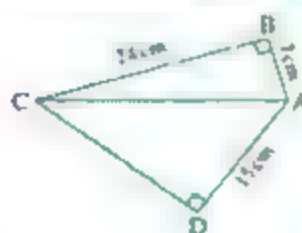
Find with proof :  $m(\angle BAC)$ 

[a] In the opposite figure :

ABCD is a quadrilateral in which

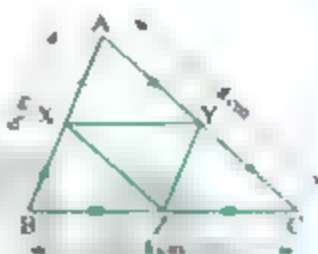
$$m(\angle B) = m(\angle D) = 90^\circ$$

$$AB = 7 \text{ cm}, BC = 24 \text{ cm}, \text{ and } AD = 15 \text{ cm}.$$

Find : The length of each of  $\overline{AC}$  and  $\overline{DC}$ 

[b] In the opposite figure :

$$AB = 6 \text{ cm}, BC = 8 \text{ cm}, AC = 8 \text{ cm}$$

 $X, Y, Z$  are the midpoints of  $\overline{AB}, \overline{AC}$  and  $\overline{BC}$  respectivelyFind with proof : The perimeter of  $\triangle XYZ$ [a] On a square lattice, draw  $\triangle ABC$ , where  $A(2, 4)$ ,  $B(2, 1)$  and  $C(6, 1)$ 

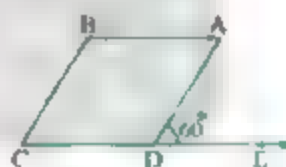
then draw its image by :

1. reflection in the y-axis.      2. rotation ( $O, 180^\circ$ )

[b] In the opposite figure :

ABCD is a parallelogram,  $E \in \overline{CD}$ 

$$m(\angle ADE) = 60^\circ$$

Find with proof :  $m(\angle B)$ 

## 14 Beni Suez Governorate

Directorate of official Language Schools  
Education Administration

Answer the following questions :

1 Choose the correct answer :

1 The measure of the interior angle of a regular polygon of  $n$  sides equals

- (a)  $\frac{(n-2) \times 90^\circ}{n}$       (b)  $\frac{(n-2) \times 180^\circ}{2}$       (c)  $\frac{(n-2) \times 180^\circ}{n}$       (d)  $180^\circ \times (n-1)$

2 In  $\triangle XYZ$ , if  $m(\angle X) = 50^\circ$ ,  $m(\angle Y) = 100^\circ$ , then  $m(\angle Z) =$ 

- (a)  $30^\circ$       (b)  $50^\circ$       (c)  $80^\circ$       (d)  $100^\circ$

## Final Examinations

- 3 The measure of the exterior angle of the equilateral triangle at any one of its vertices equals  
 (a)  $30^\circ$  (b)  $60^\circ$  (c)  $120^\circ$  (d)  $150^\circ$
- 4 If the point  $\hat{A}(-4, 5)$  is the image of the point A by translation  $(-2, 3)$ , then the point A is  
 (a)  $(-6, 8)$  (b)  $(-2, 8)$  (c)  $(-2, 2)$  (d)  $(-6, 2)$
- 5 The angle whose measure is  $179^\circ$ , its type is  
 (a) acute (b) right (c) obtuse (d) straight
- 6 If  $\triangle ABC \cong \triangle XYZ$ , then  
 (a)  $AB = YZ$  (b)  $BC = XZ$  (c)  $YX = CA$  (d)  $ZY = CB$

## 2 Complete :

- The parallelogram whose two diagonals are \_\_\_\_\_ is called a rectangle
- The ray drawn from the midpoint of a side of a triangle parallel to another side \_\_\_\_\_
- In the right-angled triangle, the area of the square drawn on the hypotenuse equals \_\_\_\_\_
- In the opposite figure :

$\triangle ABC$  is an equilateral triangle in which

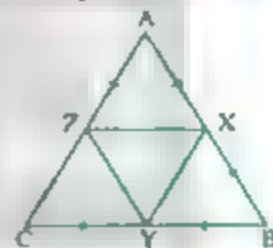
X, Y and Z are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{AC}$  respectively

• then the image of  $\triangle XBY$

by a translation of magnitude XZ in

the direction of  $\overrightarrow{XZ}$  is .

- If a straight line intersects two parallel straight lines, then every two corresponding angles are \_\_\_\_\_

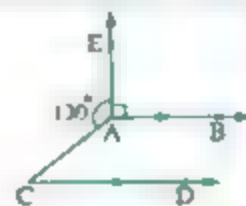


## 3 [a] In the opposite figure :

$\overline{AB} \parallel \overline{CD}$ ,  $m(\angle EAC) = 130^\circ$

and  $m(\angle EAB) = 90^\circ$

Find each of :  $m(\angle BAC)$  and  $m(\angle C)$

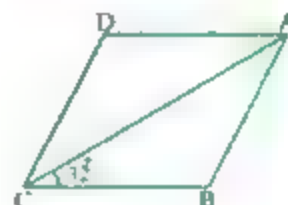


## [b] In the opposite figure :

ABCD is a rhombus

,  $\overline{AC}$  is a diagonal in it,  $m(\angle ACB) = 32^\circ$

Find :  $m(\angle D)$



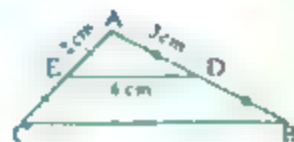


## Geometry and Measurement

## 4 [a] In the opposite figure :

D and E are the midpoints of AB and AC respectively  
 $AD = 3$  cm,  $AE = 2$  cm, and  $DE = 4$  cm

Find : The perimeter of the figure DBCE

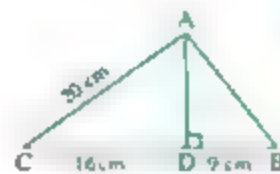


## [b] In the opposite figure :

$AD \perp BC$ ,  $BD = 9$  cm,

$DC = 16$  cm and  $AC = 20$  cm.

Find : The length of each of  $\overline{AD}$  and  $\overline{AB}$

5 On a square lattice, draw  $\triangle ABC$  where

A  $(-6, -1)$ , B  $(-2, -1)$  and C  $(-5, -6)$ , then draw :

- The image of  $\triangle ABC$  by reflection in the X axis.
- The image of  $\triangle ABC$  by rotation about the origin point with an angle of measure  $(180^\circ)$

15

Souhag Governorate

Math Supervision



Answer the following questions :

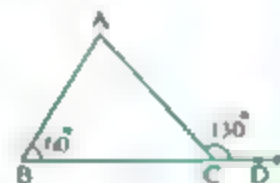
## 1 Choose the correct answer from those given :

- The sum of the measures of the interior angles of a pentagon equals  
 (a) 108 (b) 180 (c) 540 (d) 720
- In the \_\_\_\_\_ the two diagonals are perpendicular and not equal in length  
 (a) square (b) rectangle (c) rhombus (d) parallelogram
- The image of the point  $(-2, 3)$  by reflection in the y-axis is the point  
 (a)  $(3, 2)$  (b)  $(-3, 2)$  (c)  $(2, 3)$  (d)  $(-3, -2)$

## 4 In the opposite figure :

$m(\angle A) =$

- (a)  $40^\circ$  (b)  $50^\circ$   
 (c)  $60^\circ$  (d)  $70^\circ$

5 The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is \_\_\_\_\_

- (a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$

6 In the triangle ABC, if  $m(\angle A) = 50^\circ$ ,  $m(\angle B) = 100^\circ$ , then  $m(\angle C) =$ 

- (a)  $30^\circ$  (b)  $180^\circ$  (c)  $32^\circ$  (d)  $23^\circ$

## 2 Complete each of the following :

- The length of the line segment joining the midpoints of two sides of a triangle is  
and
- If  $\triangle XYZ$  is a right-angled triangle at  $X$ ,  $XY = 12$  cm and  $XZ = 9$  cm ,  
then  $YZ = \dots\dots\dots$  cm.
- The image of the point  $(-1, 2)$  by rotation about the origin point with an angle of  
measure  $90^\circ$  is
- If  $\triangle ABC \cong \triangle XYZ$  , then  $m(\angle B) = m(\angle \dots\dots\dots)$
- The ray drawn from the midpoint of a side of a triangle parallel to another side

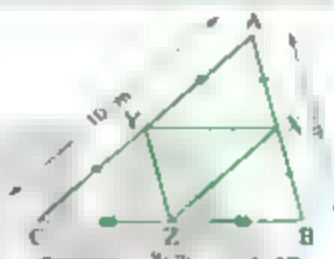
## 3 (a) In the opposite figure :

$X, Y, Z$  are the midpoints of  $\overline{AB}, \overline{AC}, \overline{BC}$

respectively ,  $AB = 7$  cm.

,  $BC = 9$  cm. ,  $AC = 10$  cm.

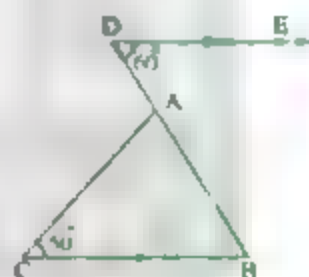
Find : The perimeter of  $\triangle XYZ$



## (b) In the opposite figure :

$\overline{DE} \parallel \overline{CB}$  ,  $m(\angle D) = 60^\circ$  ,  $m(\angle C) = 50^\circ$

Find :  $m(\angle DAC)$



## 4 (a) In the opposite figure :

$\overline{DA} \parallel \overline{BC}$  ,  $m(\angle DCB) = 60^\circ$

,  $m(\angle EAB) = 120^\circ$

Prove that : ABCD is a parallelogram.



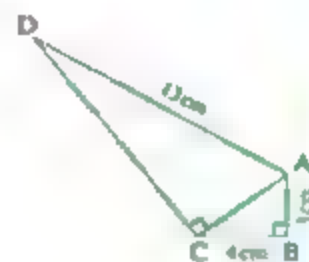
## (b) In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$

,  $AB = 3$  cm. ,  $BC = 4$  cm.

and  $AD = 13$  cm.

Find : The length of each of  $\overline{AC}, \overline{CD}$



## Geometry and Measurement

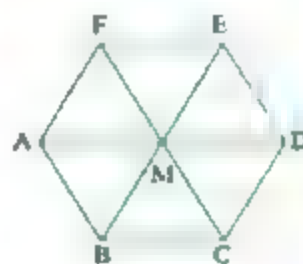
- 5 (a) On the square lattice draw  $\triangle ABC$ , where  $A(1, 2)$ ,  $B(4, 2)$ ,  $C(4, 1)$ , then find its image by rotation about the origin point with an angle of measure  $180^\circ$

(b) In the opposite figure :

ABCDEF is a regular hexagon

Find the image of  $\triangle ABM$  by :

- 1 reflection in  $\overline{EB}$
- 2 translation  $\overline{FE}$  in direction of  $\overline{FE}$
- 3 rotation ( $M, 120^\circ$ )
- 4 reflection in  $M$



# Final Examinations

on Geometry and Measurement





## Model Examinations of the School Book

on Geometry and  
Measurement

## Model

Answer the following questions :

1 Choose the correct answer from those given :

1 Circumference of a circle of radius 7 cm, = ..... cm.

$(\pi = \frac{22}{7})$

- (a) 11 (b) 22 (c) 44 (d) 88

2 The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is

- (a)
- $(3, 1)$
- (b)
- $(3, -1)$
- (c)
- $(5, 1)$
- (d)
- $(5, -5)$

3 The measure of the exterior angle of the equilateral triangle is

- (a)
- $30^\circ$
- (b)
- $45^\circ$
- (c)
- $60^\circ$
- (d)
- $120^\circ$

4 In a parallelogram if the adjacent sides are equal in the length, then the shape is

- (a) square. (b) rhombus. (c) rectangle. (d) trapezium.

5 The number of the diagonals of a pentagon is

- (a) 3 (b) 5 (c) 7 (d) 9

6 The number of axes of symmetry of an isosceles triangle =

- (a) zero (b) 1 (c) 2 (d) 3

2 Complete the following :

1 The image of the point  $(2, 1)$  by reflection in X-axis is .....

2 In the opposite figure :

$x = \dots\dots\dots^\circ$

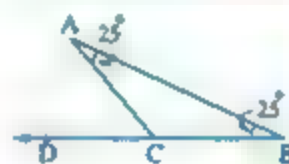
3 XYZ is a triangle in which  $m(\angle Y) = 90^\circ$ ,  $XY = 3$  cm.,  $XZ = 5$  cm, then  $YZ = \dots\dots\dots$  cm.4 ABCD is a parallelogram in which  $m(\angle A) = 100^\circ$ , then  $m(\angle B) + m(\angle D) = \dots\dots\dots^\circ$ 

5 The sum of the measures of the interior angles of a triangle = .....



3 (a) In the opposite figure :

$m(\angle A) = m(\angle B) = 25^\circ$

Find :  $m(\angle ACD)$ 

## Geometry and Measurement

- [b] Draw a triangle ABC in which  $AB = 5$  cm,  $AC = 3$  cm, and  $m(\angle A) = 40^\circ$   
 , then draw  $\hat{C}$  is the image of C under rotation  $R(A, 40^\circ)$ ,  $\hat{B}$  is the image  
 of B under rotation  $R(A, -40^\circ)$

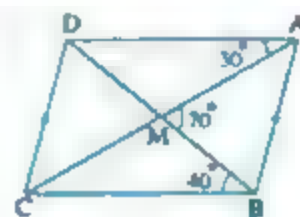
- 4 [a] In the opposite figure :

$$\overline{AB} \parallel \overline{DC}, \overline{AC} \cap \overline{BD} = \{M\},$$

$$m(\angle DAC) = 30^\circ, m(\angle DBC) = 40^\circ$$

$$\text{and } m(\angle AMB) = 70^\circ$$

Prove that : ABCD is a parallelogram.



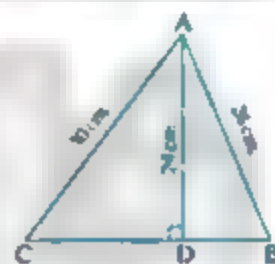
- [b] Use the translation :  $(X, y) \longrightarrow (X + 2, y + 3)$   
 to find the point whose image is  $(2, 3)$

- 5 [a] In the opposite figure :

$$\overline{AD} \perp \overline{BC}, \text{ if } AD = 24 \text{ cm, } AB = 26 \text{ cm, } AC = 30 \text{ cm.}$$

1 Find : The length of  $\overline{BC}$

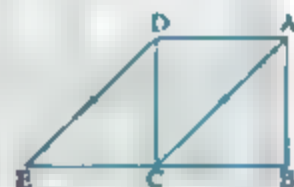
2 Find : The area of  $\triangle ABC$



- [b] In the opposite figure :

$$ABCD \text{ is a square, } E \in \overline{BC}, \overline{AC} \parallel \overline{DE}$$

Prove that : ACED is a parallelogram.



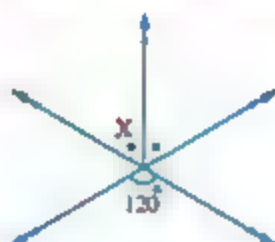
## Model 2

Answer the following questions :

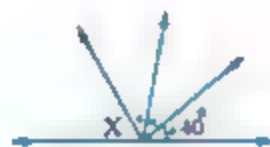
- 1 Choose the correct answer from those given :

- ABC is a right-angled triangle at B,  $AB = 6$  cm,  $BC = 8$  cm, then  $AC =$  ..... cm.  
 (a) 10 (b) 28 (c) 100 (d) 160
- The measure of each angle of regular hexagon equals .....  
 (a)  $60^\circ$  (b)  $108^\circ$  (c)  $120^\circ$  (d)  $135^\circ$
- The two diagonals are equal in length and not perpendicular in .....  
 (a) parallelogram. (b) rectangle. (c) rhombus (d) square.

- 4 In all the following shapes  $m(\angle X) = 60^\circ$  except the shape .....



(a)



(b)



(c)



(d)

- 6 In the opposite figure :

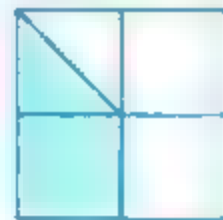
The area of the shaded part from the area of all shape equals .....

(a)  $\frac{1}{8}$

(b)  $\frac{1}{4}$

(c)  $\frac{3}{8}$

(d)  $\frac{3}{4}$



- 6 In the opposite figure :

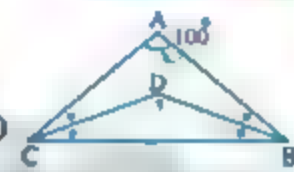
$m(\angle BDC) = \dots\dots\dots$

(a) 60

(b) 80

(c) 100

(d) 140



- 2 Complete the following :

- 1 In the opposite figure :

Semicircle of diameter 14 cm. and two semicircles the diameter of each is 7 cm.

, then the perimeter of the figure equals ..... cm.  $(\pi = \frac{22}{7})$



- 2 The image of the point (2, 3) by translation  $\overline{MN}$ , in direction  $\overline{MN}$ , where M (2, -1), N (5, 1) is

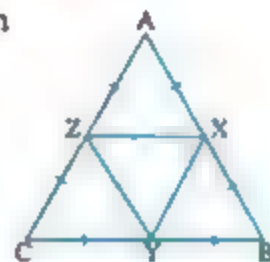
- 3 The volume of a cube of side length 1.2 m. = .....  $\text{cm}^3$

- 4 The ray drawn parallel to one side of a triangle and passing through the midpoint of another side .....

- 5 In the opposite figure :

The image of the triangle XBY

by translation  $\overline{XZ}$  in direction  $\overline{XZ}$  is .....



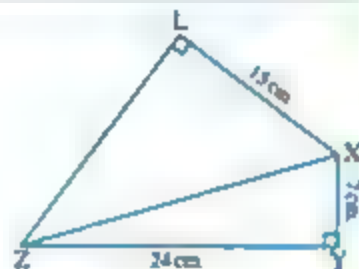
- 3 (a) In the opposite figure :

XYZL is a quadrilateral in which

$m(\angle Y) = m(\angle L) = 90^\circ$ , XY = 7 cm, ,

YZ = 24 cm, , XL = 15 cm.

Find : The length of each of  $\overline{XZ}$  and  $\overline{LZ}$



## Geometry and Measurement

- [b] Using the square lattice, draw  $\overline{AB}$  where  $A(4, 3)$ ,  $B(-1, 1)$   
then find the image of  $\overline{AB}$  by translation  $(x, y) \rightarrow (x+2, y-1)$

- 4 [a] Draw the image of triangle ABC where  $A(1, 1)$ ,  $B(3, 4)$ ,  $C(5, 2)$   
by reflection in X-axis.

- [b] In the opposite figure :

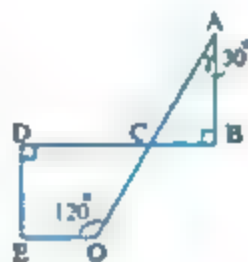
$\overline{AB}$  and  $\overline{ED}$  are perpendicular to  $\overline{BD}$

$\overline{BD} \cap \overline{AO} = \{C\}$ ,

$m(\angle A) = 30^\circ$

$m(\angle EOC) = 120^\circ$ ,

Find :  $m(\angle E)$

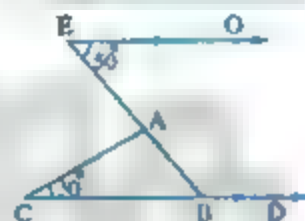


- 5 [a] In the opposite figure :

$\overline{EO} \parallel \overline{CD}$ ,  $m(\angle E) = 50^\circ$

$m(\angle C) = 30^\circ$ ,

Find the measures of angles of  $\triangle ABC$ ,  $m(\angle ABD)$



- [b] In the opposite figure :

X is the midpoint of  $\overline{AB}$

$Y \in \overline{CD}$ ,  $Z \in \overline{CE}$

$\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$ ,  $\overline{YZ} \parallel \overline{DE}$

Is  $CZ = ZE$ ? giving reason





## Model examination for the merge students

Answer the following questions :

## 1 Choose the correct answer :

- The sum of the measures of the interior angles of a triangle = .....  
(a) 90 (b) 360 (c) 180 (d) 540
- The image of the point (3 , -2) by reflection in the y-axis is the point .....  
(a) (3 , 2) (b) (-3 , -2) (c) (-3 , 2) (d) (-2 , 3)
- The diagonals are equal and perpendicular in  
(a) rhombus. (b) square. (c) rectangle. (d) parallelogram.
- In the opposite figure :

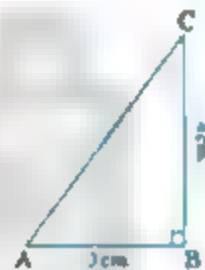
AC = ..... cm.

- (a) 5 (b) 7  
(c) 25 (d) 625

## 5 In the opposite figure :

 $m(\angle ACD) = \dots\dots\dots$ 

- (a) 40 (b) 140  
(c) 90 (d) 50



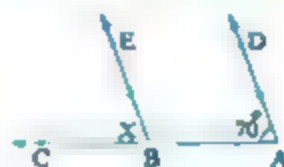
## 2 Complete each of the following :

- The length of the line segment that joins two midpoints of two sides of a triangle equals ..... the length of the third side.
- The rectangle is a parallelogram in which one of it's angles is .....
- The length of the side of a rhombus whose perimeter is 24 cm. equals ..... cm.
- The image of the point A (-3 , 2) by reflection in the origin point is the point A' ( ..... , ..... )

## Geometry and Measurement

2 In the opposite figure :

$X = \dots\dots\dots^\circ$



3 Put (✓) or (X) :

- 1 The image of the point  $(4, 3)$  by reflection in the X-axis is the point  $(3, -4)$  ( )
- 2 If ABC is a right-angled triangle at B , then  $(AB)^2 = (BC)^2 + (AC)^2$  ( )
- 3 The pentagon has 5 diagonals. ( )
- 4 ABCD is a parallelogram , in which  $m(\angle A) = 70^\circ$  , then  $m(\angle C) = 110^\circ$  ( )
- 5 Any triangle contains at least two acute angles. ( )

4 Join from the column (A) to the suitable in the column (B) :

Column (A)	Column (B)
1 The sum of the measures of the interior angles of a quadrilateral = .. ..	• $120^\circ$
2 The measure of each angle of a regular hexagon = .. ..	• $360^\circ$
3 The image of the point $(3, 2)$ by translation $(1, -2)$ is the point .. ..	• $(-1, -3)$
4 The image of the point $(1, 3)$ by rotation about the origin point , of angle of measure $180^\circ$ is the point $(\dots\dots\dots, \dots\dots\dots)$	• 45
5 The diagonal of the square divides the angle of the vertex into two angles , the measure of each = .. ..	• $(4, 0)$

5 Find the value of X :



$X = \dots\dots\dots$  cm.

Fig. (1)



$X = \dots\dots\dots$

Fig. (2)

## Schools Examinations

on Geometry and  
Measurement

1

Cairo Governorate



Answer the following questions : (Calculators are Permitted)

## 1 Choose the correct answer :

- The measure of the interior angle of the regular octagon equals .....  
(a)  $1080^\circ$  (b)  $180^\circ$  (c)  $135^\circ$  (d)  $108^\circ$
- If ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 140^\circ$ , then  $m(\angle B) =$  .....  
(a)  $40^\circ$  (b)  $110^\circ$  (c)  $70^\circ$  (d)  $60^\circ$
- In  $\triangle ABC$ , if  $m(\angle B) = 90^\circ$ ,  $AB = 20$  cm,  $AC = 25$  cm, then  $BC =$  ..... cm.  
(a) 625 (b) 12 (c) 15 (d) 225
- If the perimeter of a square is 20 cm, then its area equals .....  $\text{cm}^2$ .  
(a) 25 (b) 100 (c) 5 (d) 16
- The image of the point  $(-3, 5)$  by rotation about the origin point with an angle of measure  $90^\circ$  is ....  
(a)  $(-5, 3)$  (b)  $(-5, -3)$  (c)  $(5, 3)$  (d)  $(3, -5)$
- The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is .....  
(a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$

## 2 Complete the following :

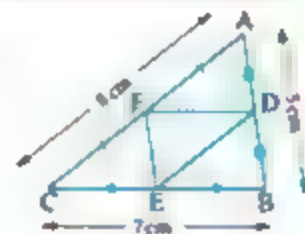
- The line segment joining the midpoints of two sides of a triangle is parallel to .....
- The image of the point  $(2, -4)$  by reflection in y-axis is .....
- The number of axes of symmetry of a rectangle is .....
- The measure of the exterior angle of an equilateral triangle equals ..... $^\circ$
- The volume of a cube of edge length 6 cm, is .....  $\text{cm}^3$

## 3 [a] In the opposite figure :

ABC is a triangle, D, E and F are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CA}$  respectively

where  $AB = 5$  cm,  $BC = 7$  cm, and  $CA = 8$  cm.

Calculate : The perimeter of  $\triangle DEF$



## Geometry and Measurement

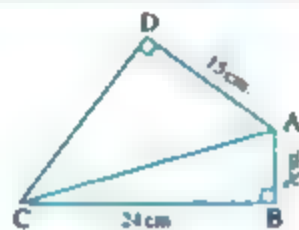
- [b] On square lattice, draw the square ABCD where A (1, 2), B (-2, 2), C (-2, 5) and D (1, 5), then draw its image by reflection in X-axis.

- 4 [a] In the opposite figure :

$$m(\angle B) = m(\angle D) = 90^\circ$$

$$AD = 15 \text{ cm}, AB = 7 \text{ cm}, \text{ and } BC = 24 \text{ cm}$$

Find : The length of each of  $\overline{AC}$ ,  $\overline{DC}$



- [b] Draw  $\triangle ABC$  where A (1, 5), B (3, 1) and C (5, 3), then draw its image by rotation about the origin point with an angle of measure  $180^\circ$

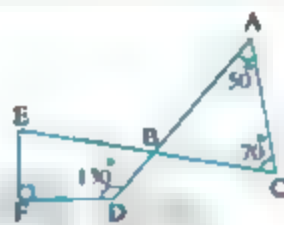
- 5 [a] In the opposite figure :

$$\overline{CE} \cap \overline{AD} = \{B\}$$

$$m(\angle A) = 50^\circ, m(\angle C) = 70^\circ$$

$$m(\angle D) = 130^\circ \text{ and } m(\angle F) = 90^\circ$$

Find :  $m(\angle E)$



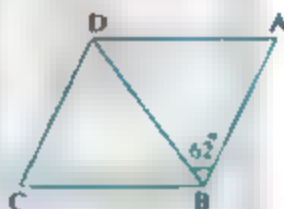
- [b] In the opposite figure :

ABCD is a rhombus

$\overline{BD}$  is a diagonal in it

$$m(\angle ABD) = 62^\circ$$

Find with proof :  $m(\angle A)$



2

Cairo Governorate

"First Star City School"  
Ministry of Education

Answer the following questions :

- 1 Complete :

- The sum of measures of the exterior angles of the heptagon is .....
- If two lines intersect, then each two vertically opposite angles are .....
- In  $\triangle ABC$ , if  $m(\angle A) + m(\angle C) = m(\angle B)$ , then  $m(\angle B) = \dots\dots\dots$
- The image of (2, 3) by translation  $(X, y) \longrightarrow (X + 4, y - 2)$  is .....
- The length of the line segment joining two midpoints of two sides of a triangle equals the length of the third side. ....



## 2 Choose the correct answer :

- 1 ABCD is a parallelogram in which  $m(\angle A) = 80^\circ$ , then  $m(\angle C) =$  .....  
 (a)  $80^\circ$  (b)  $100^\circ$  (c)  $120^\circ$  (d)  $60^\circ$
- 2 If the image of the point  $(5, -3)$  by rotation about the origin point is itself, then the measure of the rotation angle is .....  
 (a) 90 (b) 180 (c) 270 (d) 360
- 3 The line segment joining two midpoints of two sides of a triangle is ..... the third side.  
 (a) intersecting (b) parallel to (c) perpendicular to (d) congruent to
- 4 The sum of measures of the accumulative angles at a point is .....  
 (a) 90 (b) 180 (c) 270 (d) 360
- 5 If ABC is a right-angled triangle at B and  $AB = 4$  cm,  $BC = 3$  cm, then  $AC =$  ..... cm.  
 (a) 16 (b) 25 (c) 9 (d) 5
- 6 The sum of measures of the interior angles of a hexagon is .....  
 (a) 360 (b) 540 (c) 720 (d) 120

## 3 [a] In the opposite figure :

ABE is a triangle,  $\overline{AB} \parallel \overline{GH} \parallel \overline{CD}$ ,  $m(\angle CDB) = 145^\circ$

,  $m(\angle AGH) = 135^\circ$

Calculate :  $m(\angle DEG)$



## [b] In the opposite figure :

ABC is a triangle in which E, M and D are the midpoints of  $\overline{AB}$ ,  $\overline{BC}$  and  $\overline{CA}$  respectively

Prove that : DEMC is a parallelogram.



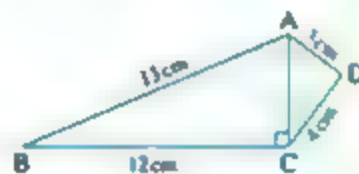
## 4 [a] In the opposite figure :

$m(\angle ACB) = 90^\circ$ ,  $AB = 13$  cm,  $AD = 3$  cm,

,  $CD = 4$  cm,  $BC = 12$  cm.

Find : 1 The length of  $\overline{AC}$

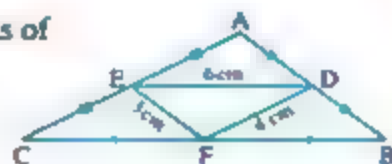
2 The perimeter of the figure ABCD



## Geometry and Measurement

[b] In the opposite figure :

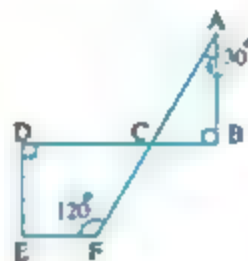
ABC is a triangle in which : D , F and E are the midpoints of  $\overline{AB}$  ,  $\overline{BC}$  and  $\overline{CA}$  respectively such that :  $DF = 4$  cm ,  
 $FE = 3$  cm ,  $DE = 6$  cm .

Calculate : The perimeter of  $\Delta ABC$ 

[a] Using the lattice , find the images of the points :

A (-4 , 1) , B (0 , 4) and C (-2 , 2) by reflection in X-axis.

[b] In the opposite figure :

 $\overline{AB} \perp \overline{BC}$  ,  $\overline{ED} \perp \overline{CD}$  ,  $\overline{BD} \cap \overline{AF} = \{C\}$  $m(\angle A) = 30^\circ$  ,  $m(\angle F) = 120^\circ$ Find with proof :  $m(\angle ACB)$  $2 m(\angle E)$ 

3

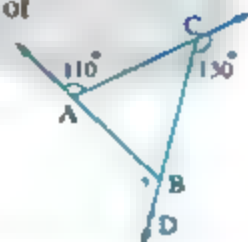
Giza Governorate

El-Dokki Governorate  
Full Language School

Answer the following questions :

1 Complete :

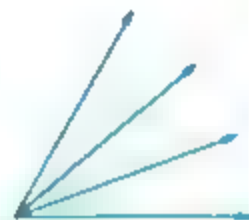
- The image of the point (2 , -1) by reflection in X-axis is
- ABCD is a parallelogram in which  $m(\angle A) = 60^\circ$  , then  $m(\angle B) =$
- The measure of each angle of the regular hexagon equals
- The length of the line segment that joins two midpoints of two sides of a triangle equals the length of the third side.
- In the opposite figure :  
 $m(\angle ABD) = \dots\dots\dots^\circ$



2 Choose the correct answer :

- The number of the diagonals of a pentagon is  
 (a) 3 (b) 5 (c) 7 (d) 9
- The image of the point (2 , 5) by rotation about the origin point with an angle of measure  $90^\circ$  is  
 (a) (-5 , 2) (b) (5 , -2) (c) (-2 , -5) (d) (-2 , 5)

- 3 ABC is a right-angled triangle at A , then  $(AC)^2 = (BC)^2 - \dots - (AB)^2$   
 (a) + (b)  $\times$  (c) + (d) -
- 4 The number of axes of symmetry of the rectangle is .....  
 (a) 1 (b) 2 (c) 3 (d) 4
- 5 If the total surface area of a cuboid =  $148 \text{ cm}^2$  and its lateral surface area =  $118 \text{ cm}^2$   
 , then its base area = .....  $\text{cm}^2$   
 (a) 30 (b) 15 (c) 45 (d) 266
- 6 Number of acute angles in the opposite figure is .....  
 (a) 3 (b) 4 (c) 5 (d) 6



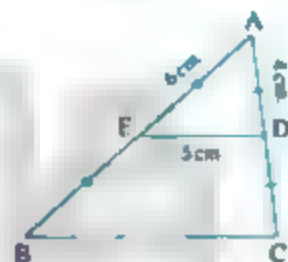
3 (a) In the opposite figure :

E and D are midpoints of  $\overline{AB}$  and  $\overline{AC}$

,  $AE = 6 \text{ cm}$ , ,  $ED = 5 \text{ cm}$ .

,  $AD = 4 \text{ cm}$ .

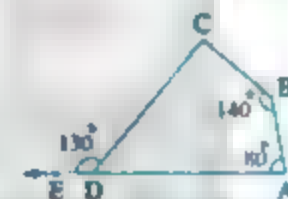
, Calculate the perimeter of  $\triangle ABC$



(b) In the opposite figure :

$D \in \overline{AE}$

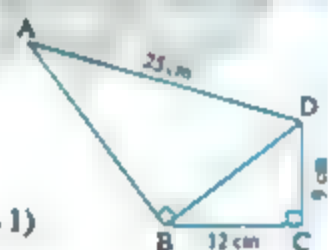
Find :  $m(\angle C)$



4 (a) In the opposite figure :

$m(\angle C) = m(\angle ABD) = 90^\circ$

Find : The length of each of  $\overline{BD}$  and  $\overline{AB}$

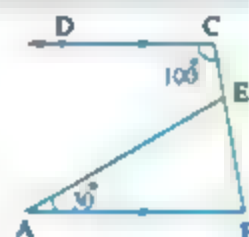


- (b) On the square lattice , draw  $\triangle ABC$  where  $A(0, 5)$  ,  $B(2, 1)$   
 ,  $C(4, 7)$  , then draw the image of  $\triangle ABC$  by reflection in y-axis.

5 (a) In the opposite figure :

$\overline{BA} \parallel \overline{CD}$  ,  $m(\angle C) = 100^\circ$  ,  $m(\angle A) = 30^\circ$

Find :  $m(\angle AEC)$



- (b) On the square lattice , draw  $\overline{AB}$  where  $A(4, 3)$  ,  $B(-1, 1)$   
 , then draw the image of  $\overline{AB}$  by translation :  $(X, y) \longrightarrow (X + 2, y - 1)$



## Geometry and Measurement

4

Giza Governorate

Answer the following questions :

## 1 Complete the following :

- The image of the point  $(5, -3)$  under rotation of an angle of measure  $90^\circ$  about the origin point is .....
- The ray which is drawn from the midpoint of a side in a triangle parallel to another side ..... the third side.
- In the rhombus the two diagonals are .....
- The sum of measures of the exterior angles of any convex polygon equals .....
- In  $\triangle ABC$ , if  $m(\angle A) + m(\angle B) = m(\angle C)$ , then  $\triangle ABC$  is .....

## 2 Choose the correct answer :

- If the measure of an interior angle of a regular polygon is  $135^\circ$ , then the number of its sides is  
(a) 6 (b) 4 (c) 7 (d) 8
- The image of the point  $(-1, 4)$  by the translation  $(X, y) \longrightarrow (X + 3, y - 2)$  followed by reflection in the X-axis is .....  
(a)  $(2, 2)$  (b)  $(-2, 2)$  (c)  $(-2, -2)$  (d)  $(2, -2)$
- ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 110^\circ$ , then  $m(\angle B) = \dots\dots\dots$   
(a) 125 (b) 80 (c) 100 (d) 110
- The concave polygon should have ..... angle.  
(a) an acute (b) a right (c) an obtuse (d) a reflex
- The image of the point ..... is itself by reflection in y-axis.  
(a)  $(0, 3)$  (b)  $(3, 0)$  (c)  $(3, 3)$  (d)  $(-3, 3)$
- The identity rotation about the origin point with an angle of measure .....  
(a) 90 (b) 180 (c) 360 (d)  $-90$

## 3 [a] In the opposite figure :

ABCD is a quadrilateral in which  
 $m(\angle B) = m(\angle ACD) = 90^\circ$   
 $AB = 4$  cm,  $BC = 3$  cm, and  $AD = 13$  cm.  
 Find : AC and DC

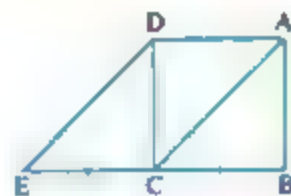




[b] In the opposite figure :

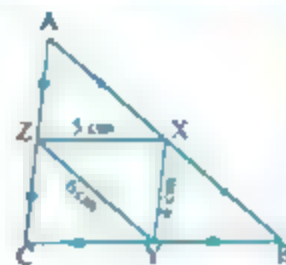
ABCD is a square ,  $E \in \overline{BC}$   
such that  $BC = CE$

- 1 Prove that : The figure ACED is a parallelogram.
- 2 Find :  $m(\angle E)$



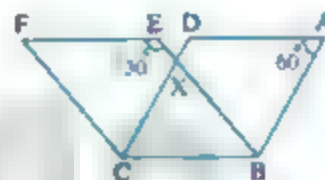
4 [a] In the opposite figure :

$\triangle ABC$  in which X , Y and Z are the midpoints of  
 $\overline{AB}$  ,  $\overline{BC}$  ,  $\overline{CA}$  respectively  
•  $XZ = 5$  cm. •  $XY = 4$  cm. •  $YZ = 6$  cm.  
Find with proof : The perimeter of  $\triangle ABC$



[b] In the opposite figure :

ABCD and EBCF are two parallelograms  
•  $m(\angle BAD) = 60^\circ$  •  $m(\angle BEF) = 130^\circ$   
Find with proof :  $m(\angle BXC)$



5 [a] In the opposite figure :

ABCD is a quadrilateral in which  $m(\angle A) = 120^\circ$   
•  $m(\angle B) = 80^\circ$  •  $\triangle CEF$  is an equilateral triangle  
Find :  $m(\angle D)$



[b] Using the square lattice draw  $\triangle ABC$  where :

A (2 , 4) , B (4 , 2) and C (3 , 1) , then map its image by the translation  
 $(x , y) \longrightarrow (x + 2 , y + 3)$

## 5 Alexandria Governorate

Final Examinations 2022  
Mathematics - Geometry

Answer the following questions :

1 Choose the correct answer :

- 1 The measure of the exterior angle of the equilateral triangle =  
(a)  $60^\circ$  (b)  $120^\circ$  (c)  $90^\circ$  (d)  $180^\circ$
- 2 The diagonals are equal in length and not perpendicular in  
(a) parallelogram. (b) rectangle. (c) rhombus. (d) square.

## Geometry and Measurement

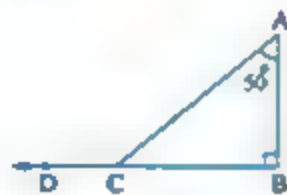
- 3 The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is .....
- (a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$

- 4 The measure of the interior angle of the regular hexagon is .....
- (a)  $60^\circ$  (b)  $108^\circ$  (c)  $120^\circ$  (d)  $135^\circ$

- 5 In the opposite figure :

$m(\angle ACD) = \dots$

- (a)  $40^\circ$  (b)  $140^\circ$   
(c)  $90^\circ$  (d)  $50^\circ$



- 6 If two adjacent sides in a parallelogram are equal in length, then it is a .....
- (a) square. (b) rhombus. (c) rectangle. (d) trapezium.

## 2 Complete :

- 1 The length of the line segment joining two midpoints of two sides of a triangle

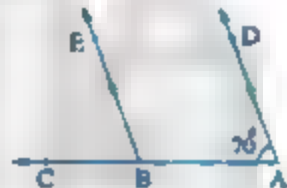
- 2 The angle of measure  $89^\circ$  is ..... angle.

- 3 In the opposite figure :

$m(\angle EBC) = \dots^\circ$

- 4 The image of  $(-3, 2)$  by reflection in origin point is

- 5 A rhombus of perimeter 24 cm. its side length equals

3 [a] Draw  $\overline{AB}$  such that  $A(2, 1)$  ,  $B(2, 4)$ 

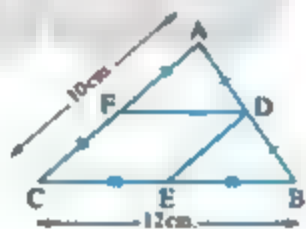
• then find its image by translation  $(x, y) \rightarrow (x + 5, y + 2)$

- [b] In the opposite figure :

D, E, F are midpoints of  $\overline{AB}$ ,  $\overline{BC}$ ,  $\overline{CA}$

,  $BC = 12$  cm. ,  $AC = 10$  cm.

Find : The perimeter of DECF

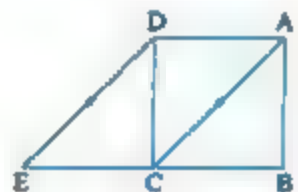
4 [a] Draw  $\triangle LMN$  such that  $L(-4, -1)$  ,  $M(-1, -3)$  ,  $N(0, -1)$ 

• then find its image by reflection in X-axis.

- [b] In the opposite figure :

ABCD is a square ,  $E \in \overline{BC}$  ,  $\overline{AC} \parallel \overline{DE}$

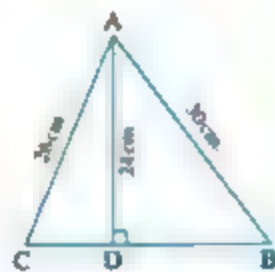
Prove that : ACED is a parallelogram.



- 5 [a] Find the number of sides of a regular polygon if the measure of its interior angle is  $140^\circ$

[b] In the opposite figure :

Find the length of  $\overline{BC}$   
and the area of  $\triangle ABC$



## 6 Alexandria Governorate

Middle Educational Zone  
Morris Supervision



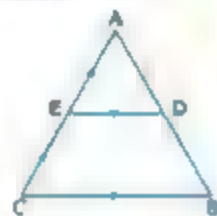
Answer the following questions :

- 1 Complete each of the following :

- The measure of the straight angle equals .....
- The image of the point  $(4, -2)$  by reflection in X-axis is .....
- The ray drawn from the midpoint of a side parallel to another side in a triangle
- The image of the point  $(-2, 1)$  by rotation with an angle of measure  $180^\circ$  about origin point is .....
- The length of line segment joining the midpoints of two sides of a triangle equals
- A rhombus is a ..... with two adjacent equal sides in length.

- 2 Choose the correct answer :

- The perimeter of a square with side length 1 cm. equals ..... cm.  
(a) 5 (b) 1 (c) 4 (d) 2
- The number of diagonals in a pentagon is .....  
(a) 0 (b) 3 (c) 4 (d) 5
- The edge length of a cube whose volume is  $27 \text{ cm}^3$  is ..... cm.  
(a) 9 (b) 3 (c) 27 (d) 6
- In  $\triangle ABC$ , if  $m(\angle B) = 90^\circ$ ,  $AB = 3 \text{ cm}$ ,  $BC = 4 \text{ cm}$ , then  $AC =$  ..... cm.  
(a) 7 (b) 12 (c) 1 (d) 5
- In the opposite figure :  $CB : ED =$  .....  
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 1 : 4





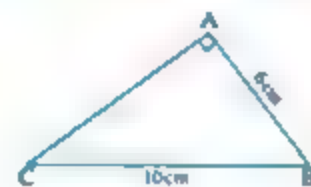
## Geometry and Measurement

## 3 [a] In the opposite figure :

ABC is a right-angled triangle at A

, BC = 10 cm. , AB = 6 cm.

Find : The length of  $\overline{AC}$



- [b] Draw the image of the rectangle XYZL where X (-5 , 1) , Y (-5 , 4) , Z (-1 , 4) , L (-1 , 1) by reflection in the X-axis.

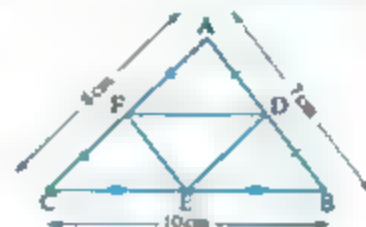
## 4 [a] In the opposite figure :

BC = 10 cm. , AB = 7 cm. , AC = 8 cm.

Calculate : The perimeter of  $\triangle FDE$

- [b] The ratio between the measures of the angles of a quadrilateral is 2 : 2 : 3 : 5

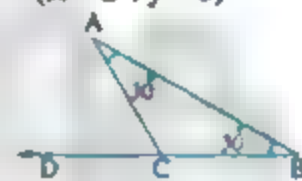
Calculate : The measure of the biggest angle.

5 [a] Find the image of the point A (5 , -5) by translation  $(X , y) \longrightarrow (X - 2 , y - 3)$ 

- [b] In the opposite figure :

$m(\angle A) = m(\angle B) = 30^\circ$

Find showing steps :  $m(\angle ACD)$



7

El-Kahyoubia Governorate

Answer the following questions :

## 1 Choose the correct answer from those given :

- The sum of the measures of the interior angles of a triangle equals ....  
 (a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$
- The image of the point (-1 , 3) by translation (4 , -2) is  
 (a) (3 , 1) (b) (3 , -1) (c) (5 , 1) (d) (5 , -5)
- The measure of the exterior angle of the equilateral triangle is -  
 (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $120^\circ$
- In a parallelogram if the adjacent sides are equal in length , then the shape is a  
 (a) square. (b) rhombus. (c) rectangle. (d) trapezium.
- The number of the diagonals of a pentagon is ....  
 (a) 3 (b) 5 (c) 7 (d) 9



## 2 Complete the following :

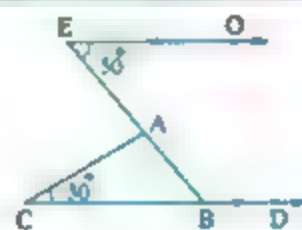
- The line segment joining the midpoints of two sides of a triangle is the third side.
- If  $\Delta XYZ$  is a right-angled triangle at  $X$ ,  $XY = 12$  cm. and  $XZ = 9$  cm.  
then  $YZ =$  ..
- If  $ABCD$  is a rhombus, then  $\angle A$  ..  $\angle C$ .
- The image of the point  $(-1, 2)$  by rotation about the origin point with an angle of measure  $90^\circ$  is ..
- The sum of the measures of the accumulative angles about a point = ..

## 3 [a] In the opposite figure :

$$\overline{EO} \parallel \overline{CD}, m(\angle E) = 50^\circ$$

$$, m(\angle C) = 30^\circ$$

Find : The measures of the angles of  $\Delta ABC$ ,  $m(\angle ABD)$



## [b] In the opposite figure :

$X$  is the midpoint of  $\overline{AB}$

$Y \in \overline{CD}, Z \in \overline{CE}$

$\overline{AD} \parallel \overline{XY} \parallel \overline{BC}, \overline{YZ} \parallel \overline{DE}$

Is  $CZ = ZE$ ? Giving reason.



- 4 [a] Using the square lattice, draw  $\Delta ABC$ , where  $A(-2, 3)$ ,  $B(2, 3)$ ,  $C(2, 6)$ , then find its image by translation  $(X, y) \rightarrow (X + 2, y - 1)$

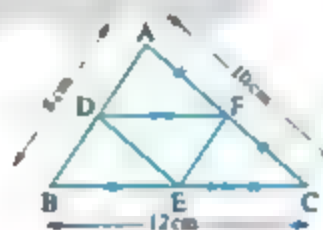
## [b] In the opposite figure :

$D, E, F$  are midpoints of

$\overline{AB}, \overline{BC}, \overline{CA}$  respectively

$AB = 8$  cm.,  $BC = 12$  cm.,  $AC = 10$  cm.

Find : The perimeter of  $\Delta DEF$

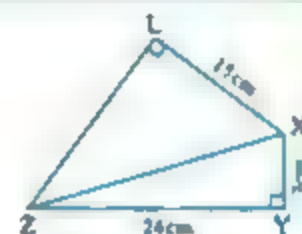


## 5 [a] In the opposite figure :

$XYZL$  is a quadrilateral in which  $m(\angle Y) = m(\angle L) = 90^\circ$

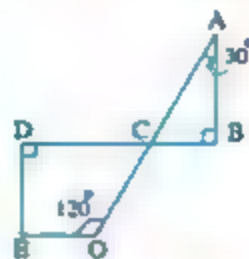
$XY = 7$  cm.,  $YZ = 24$  cm.,  $XL = 15$  cm.

Find : The length of each of  $\overline{XZ}$  and  $\overline{LZ}$



## Geometry and Measurement

[b] In the opposite figure :

 $\overline{AB}$  and  $\overline{ED}$  are perpendicular to  $\overline{BD}$  $\overline{BD} \cap \overline{AO} = \{C\}$  $m(\angle A) = 30^\circ$  ,  $m(\angle EOC) = 120^\circ$ Find :  $m(\angle E)$ 

8

El-Gharbia Governorate

Of Basic Language Subjects  
The Central Maths Supervisor

Answer the following questions :

1 Choose the correct answer :

- The measure of the exterior angle of the equilateral triangle equals .....  
(a) 30 (b) 45 (c) 60 (d) 120
- If the image of the point  $(5, -3)$  by rotation around the origin point is itself , then the measure of the rotation angle equals .....  
(a) 90 (b) 270 (c) 180 (d) 360
- The image of a square by rotation around the origin point with an angle of measure  $90^\circ$  is a ..  
(a) rectangle. (b) square. (c) rhombus. (d) trapezium.
- Any triangle has at least ..... acute angles.  
(a) 0 (b) 1 (c) 2 (d) 3
- The measure of the interior angle of the regular hexagon equals .....  
(a) 60 (b) 108 (c) 120 (d) 135
- The image of the point  $(-5, 0)$  by reflection in X-axis is .....  
(a)  $(5, 0)$  (b)  $(0, 5)$  (c)  $(-5, 0)$  (d)  $(0, -5)$

2 Complete :

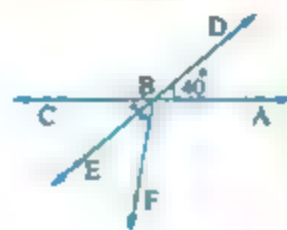
- The line segment joining the midpoints of two sides of a triangle is .....
- If ABCD is a parallelogram in which  $m(\angle A) = 60^\circ$  , then  $m(\angle B) = \dots\dots\dots$
- The image of the point  $(3, 2)$  by reflection in the origin point is .....
- Each two opposite angles in a parallelogram are .....
- The rectangle is a parallelogram in which one of its angles is .....

- 3 (a) In the opposite figure :

$$\overline{AC} \cap \overline{DE} = \{B\}, m(\angle ABD) = 40^\circ$$

and  $\overline{BE}$  bisects  $\angle CBF$

Find with proof :  $m(\angle ABF)$

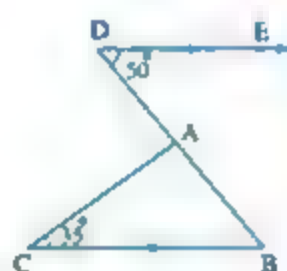


- (b) In the opposite figure :

$$\overline{DE} \parallel \overline{CB}, m(\angle D) = 50^\circ, m(\angle C) = 35^\circ$$

Find with proof :

$$m(\angle B), m(\angle BAC)$$

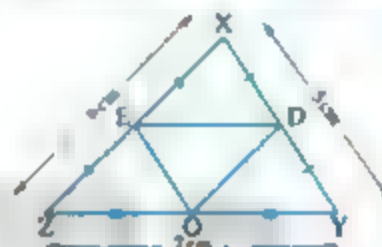


- 4 (a) In the opposite figure :

XYZ is a triangle in which  $XY = 5$  cm.

,  $XZ = 6$  cm. ,  $YZ = 7$  cm.

Find with proof : The perimeter of  $\triangle DOE$



- (b) On the square lattice , draw the triangle whose

vertices are  $A(4, 4)$  ,  $B(4, 2)$  ,  $C(1, 2)$  , then find its image by the translation  $(-4, -2)$

- 5 (a) In the opposite figure :

$$m(\angle B) = m(\angle ACD) = 90^\circ$$

,  $AB = 12$  cm. ,  $BC = 9$  cm. ,  $CD = 20$  cm.

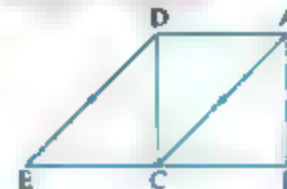
Find : The length of  $\overline{AD}$



- (b) In the opposite figure :

ABCD is a square ,  $E \in \overline{BC}$  ,  $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



9

El-Dakahlia Governorate

El-Dakahlia Directorate of Education  
Moff's Supervision

Answer the following questions :

- 1 Choose the correct answer :

1 The sum of measures of the interior angles of the hexagon equals

(a)  $270^\circ$

(b)  $180^\circ$

(c)  $720^\circ$

(d)  $360^\circ$



## Geometry and Measurement

- 2 The side length of a rhombus whose perimeter 8 cm. =  
 (a) 32 cm. (b) 2 cm. (c) 16 cm. (d) 12 cm.
- 3 The measure of each angle of a regular triangle equals  
 (a)  $60^\circ$  (b)  $108^\circ$  (c)  $120^\circ$  (d)  $135^\circ$
- 4 The parallelogram whose diagonals are perpendicular and equal in length is called .  
 (a) rhombus. (b) square. (c) rectangle. (d) trapezium.
- 5 The image of the point  $(-3, 5)$  by rotation about the origin point with an angle of measure  $270^\circ$  is ..  
 (a)  $(5, 3)$  (b)  $(-5, 3)$  (c)  $(3, 5)$  (d)  $(-5, -3)$
- 6 Any triangle has at least ..... acute angles.  
 (a) zero (b) 1 (c) 2 (d) 3

## 2 Complete each of the following :

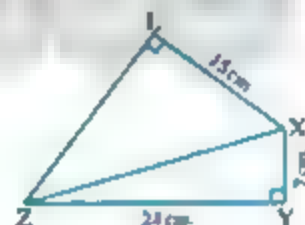
- 1 The measure of the exterior angle of an equilateral triangle equals - ..
- 2 If  $\triangle ABC$  is right-angled triangle at B ,  $m(\angle A) = 55^\circ$  , then  $m(\angle C) =$  ..
- 3 The parallelogram whose perimeter 28 cm. and the length of one of its sides is 10 cm. , then the length of its adjacent side equals ..
- 4 If the image of the point A  $(X, y)$  by a translation  $(1, -4)$  is the point  $(3, -2)$  , then point A is (.....)
- 5 The rectangle is a parallelogram in which one of its angles is - ..

## 3 [a] In the opposite figure :

XYZL is quadrilateral in which  $m(\angle Y) = m(\angle L) = 90^\circ$

,  $XY = 7$  cm. ,  $YZ = 24$  cm. ,  $XL = 15$  cm.

Find : The length of each of  $\overline{XZ}$  ,  $\overline{LZ}$



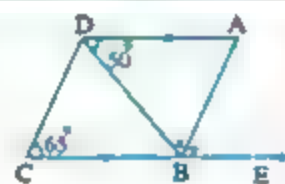
- [b] Draw on square lattice the image of triangle ABC where A  $(0, 1)$  , B  $(3, 4)$  , C  $(5, 1)$  by reflection in y-axis.

## 4 [a] In the opposite figure :

$\overline{DA} \parallel \overline{CE}$  ,  $\overline{BA}$  bisects  $\angle DBE$

,  $m(\angle ADB) = 50^\circ$  ,  $m(\angle C) = 65^\circ$

Prove that : ABCD is a parallelogram.





## Final Examinations

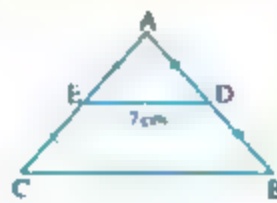
[b] In the opposite figure :

ABC is a triangle in which

D and E are the midpoints of  $\overline{AB}$  and  $\overline{AC}$  respectively

,  $\overline{DE} = 7$  cm.

Find : The length of  $\overline{BC}$



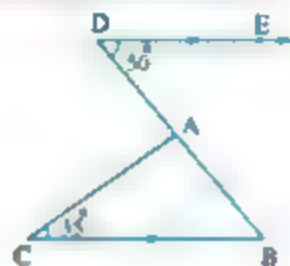
5 [a] In the opposite figure :

$\overline{DE} \parallel \overline{CB}$

,  $m(\angle D) = 50^\circ$

,  $m(\angle C) = 35^\circ$

Find :  $m(\angle B)$  and  $m(\angle BAC)$



[b] Draw  $\triangle OBC$  on a square lattice where  $O(0, 0)$  ,  $B(3, 0)$  ,  $C(0, 4)$

, then find its image by rotation about the origin point with an angle of measure  $(-180^\circ)$

10

Suez Governorate

Answer the following questions :

1 Choose the correct answer :

1 The two diagonals are equal in length and not perpendicular in a

- (a) parallelogram. (b) rectangle. (c) square. (d) rhombus.

2 The image of  $(1, 3)$  by translation  $(4, 2)$  is

- (a)  $(3, 1)$  (b)  $(5, 5)$  (c)  $(5, 1)$  (d)  $(5, -5)$

3 The measure of an exterior angle in an equilateral triangle is

- (a) 30 (b) 45 (c) 60 (d) 120

4 In a parallelogram if the two adjacent sides are equal in length , then the shape is a

- (a) square. (b) rhombus. (c) rectangle. (d) trapezium.

5 The number of axes of symmetry of the equilateral triangle is

- (a) 0 (b) 1 (c) 2 (d) 3

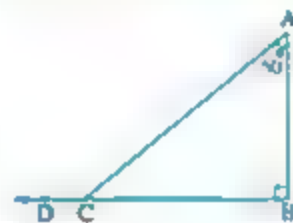
6 The sum of measures of the interior angles of a triangle is

- (a) 90 (b) 360 (c) 180 (d) 540

## Geometry and Measurement

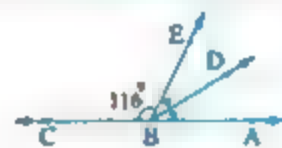
## 2 Complete :

- 1 In the opposite figure :  $m(\angle ACD) = \dots\dots\dots^\circ$
- 2 The ray drawn from the midpoint of one side of a triangle parallel to another side .....
- 3 The image of the point  $(1, -2)$  by reflection in X-axis is .....
- 4 The sum of measures of the interior angles of the pentagon is .....
- 5 If ABCD is a rhombus , then  $\angle \dots\dots\dots$



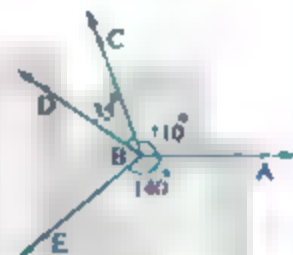
## 3 [a] In the opposite figure :

$B \in \overline{AC}$  ,  $m(\angle CBE) = 116^\circ$   
 ,  $\overline{BD}$  bisects  $\angle ABE$   
 Find :  $m(\angle ABD)$



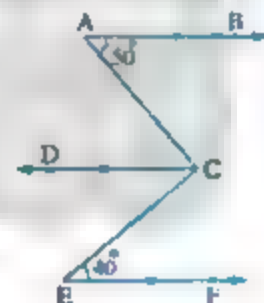
## [b] In the opposite figure :

$m(\angle ABC) = 110^\circ$  ,  $m(\angle CBD) = 35^\circ$   
 ,  $m(\angle ABE) = 140^\circ$   
 Find :  $m(\angle DBE)$



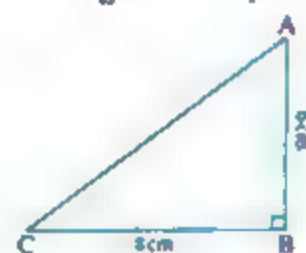
## 4 [a] In the opposite figure :

$\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$   
 ,  $m(\angle A) = 50^\circ$  ,  $m(\angle E) = 40^\circ$   
 Find with proof :  $m(\angle ACE)$



## [b] In the opposite figure :

$\triangle ABC$  is right-angled at B  
 ,  $AB = 6 \text{ cm}$  ,  $BC = 8 \text{ cm}$   
 Find : The length of  $\overline{AC}$



## 5 [a] Complete :

- 1 The image of  $(3, 2)$  by rotation about the origin point with an angle of measure  $90^\circ$  is .....
- 2 The image of  $(3, 2)$  by reflection in the origin point is .....

## Final Examinations

- 3 The image of  $(3, 2)$  by rotation about the origin point with an angle of measure  $180^\circ$  is \_\_\_\_\_

4 In the opposite figure :

$XY =$  \_\_\_\_\_ cm.



(b) In the opposite figure :

$\overline{AB}$  ,  $\overline{DE}$  are perpendicular to  $\overline{BD}$

,  $\overline{BD} \cap \overline{AF} = \{C\}$  ,  $m(\angle A) = 60^\circ$  ,  $m(\angle CFE) = 120^\circ$

Find with proof :  $m(\angle E)$



11

Damietta Governorate

Damietta Governorate of Mathematics  
of the Language School

Answer the following questions :

1 Choose the correct answer :

- 1 The measure of each angle of the regular pentagon equals \_\_\_\_\_

(a) 60 (b) 108 (c) 120 (d) 135

- 2 The image of the point  $(2, -7)$  by reflection in the origin point is \_\_\_\_\_

(a)  $(2, 7)$  (b)  $(-2, 7)$  (c)  $(-2, -7)$  (d)  $(2, -7)$

- 3 Any triangle has at least two \_\_\_\_\_ angles.

(a) reflex (b) obtuse (c) acute (d) right

- 4 ABCD is parallelogram in which :  $m(\angle A) + m(\angle C) = 100$  , then  $m(\angle B) =$  \_\_\_\_\_

(a) 50 (b) 150 (c) 130 (d) 80

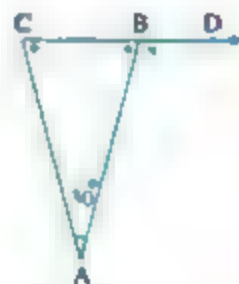
- 5 The ratio between the perimeter of an equilateral triangle and its side length = \_\_\_\_\_

(a) 3 : 1 (b) 3 : 2 (c) 1 : 3 (d) 2 : 3

6 In the opposite figure :

$m(\angle ABD) =$  \_\_\_\_\_

(a) 150 (b) 95  
(c) 105 (d) 115



## Geometry and Measurement

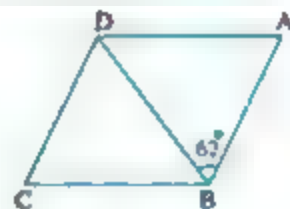
## 2 Complete each of the following :

- 1 The ray drawn from the midpoint of a side of a triangle parallel to another side . ...
- 2 The image of the point  $(3, -2)$  by rotation with an angle of measure  $90^\circ$  about the origin is
- 3 If  $m(\angle B) = 90^\circ$  , then  $m(\text{reflex } \angle B) = \dots$
- 4 If ABC is right-angled triangle at B ,  $AB = 3 \text{ cm.}$  and  $BC = 4 \text{ cm.}$  , then  $AC = \dots \text{ cm.}$

- 5 This figure  has ... line (s) of symmetry.

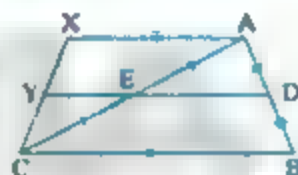
## 3 [a] In the opposite figure :

ABCD is a rhombus in which  
 $\overline{BD}$  is a diagonal and  $m(\angle ABD) = 62^\circ$   
 Find with proof :  $m(\angle A)$



## [b] In the opposite figure :

$AD = BD$  ,  $AE = EC$  ,  $\overline{AX} \parallel \overline{BC}$   
 $\overline{DE} \cap \overline{XC} = \{Y\}$   
 Prove that : Y is the midpoint of  $\overline{XC}$



## 4 [a] In the opposite figure :

$\overline{AB}$  and  $\overline{ED}$  are perpendicular to  $\overline{BD}$   
 $\overline{BD} \cap \overline{AF} = \{C\}$   
 $m(\angle A) = 30^\circ$   
 $m(\angle EFC) = 120^\circ$   
 Find :  $m(\angle E)$



## [b] In the opposite figure :

ABC is a triangle and  $\overline{AD} \perp \overline{BC}$   
 If  $AD = 24 \text{ cm.}$  ,  $AB = 26 \text{ cm.}$   
 and  $CD = 18 \text{ cm.}$   
 Find : The length of each of  $\overline{AC}$  ,  $\overline{DB}$



## 5 [a] On the square lattice find the image of the triangle LMN where :

$L(-4, -1)$  ,  $M(-1, -3)$  ,  $N(0, -1)$  by reflection in the X-axis.

- [b] Using the square lattice , draw  $\overline{AB}$  where  $A(4, 3)$  ,  $B(-1, 1)$   
 then find the image of  $\overline{AB}$  by translation  $(X, y) \longrightarrow (X + 2, y - 1)$



## 12 Kaf El-Sheikh Governorate

Answer the following questions :

## 1 Choose the correct answer :

- The sum of measures of the interior angles of a hexagon equals .....  
(a) 180 (b) 540 (c) 720 (d) 360
- The two diagonals are equal in length and perpendicular in the .....  
(a) trapezium, (b) square, (c) rectangle, (d) parallelogram.
- In  $\Delta XYZ$ , if  $m(\angle X) > m(\angle Y) + m(\angle Z)$ , then  $\angle X$  is ..... angle.  
(a) an acute (b) an obtuse (c) a right (d) a straight
- The image of  $(3, -5)$  by rotation  $R(O, 90^\circ)$  is  
(a)  $(-3, 5)$  (b)  $(-3, -5)$  (c)  $(5, 3)$  (d)  $(5, -3)$
- The sum of the measures of the exterior angles of a triangle equals .....  
(a) 60 (b) 120 (c) 270 (d) 360
- The measure of the exterior angle of the equilateral triangle is .....  
(a) 90 (b) 120 (c) 360 (d) 60

## 2 Complete :

- The image of  $A(2, 3)$  by translation  $(X + 3, Y - 2)$  is .....
- The rhombus with a right angle is .....
- The image of  $(-1, 2)$  by reflection in the origin point is .....
- The length of the line segment joining two midpoints of two sides of a triangle is equal to ..... the length of the third side.
- If ABCD is a parallelogram in which  $BC = 8$  cm. and  $CD = 6$  cm., then its perimeter = ..... cm.

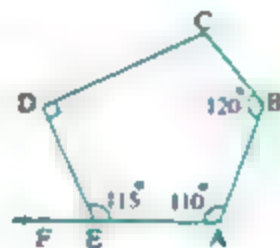
## 3 (a) In the opposite figure :

ABCDE is a pentagon in which  $F \in \overline{AE}$

$m(\angle A) = 110^\circ$ ,  $m(\angle B) = 120^\circ$ ,  $m(\angle DEA) = 115^\circ$

$m(\angle D) = 90^\circ$

Find with proof :  $m(\angle C)$

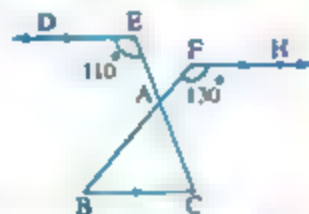


## Geometry and Measurement

[b] In the opposite figure :

$\overline{ED} \parallel \overline{BC} \parallel \overline{FH}$

$m(\angle E) = 110^\circ, m(\angle F) = 130^\circ$

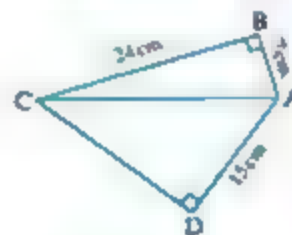
Find with proof :  $m(\angle BAC)$ 

4 [a] In the opposite figure :

ABCD is a quadrilateral in which

$m(\angle B) = m(\angle D) = 90^\circ$

$AB = 7 \text{ cm}, BC = 24 \text{ cm}, \text{ and } AD = 15 \text{ cm}.$

Find : The length of each of  $\overline{AC}$  and  $\overline{DC}$ 

[b] In the opposite figure :

$AB = 6 \text{ cm}, BC = 8 \text{ cm}, AC = 8 \text{ cm}.$

 $X, Y, Z$  are the midpoints of  $\overline{AB}, \overline{AC}$  and  $\overline{BC}$  respectively.Find with proof : The perimeter of  $\triangle XYZ$ 5 [a] On a square lattice, draw  $\triangle ABC$ , where  $A(2, 4), B(2, 1)$  and  $C(6, 1)$ 

, then draw its image by :

- 1 reflection in the y-axis.
- 2 rotation  $(O, 180^\circ)$

[b] In the opposite figure :

ABCD is a parallelogram,  $E \in \overline{CD}$ 

$m(\angle ADE) = 60^\circ$

Find with proof :  $m(\angle B)$ 

13

El-Beheira Governorate

Beheira Educational Development  
Kand El-Beheira G1 C

Answer the following questions :

1 Complete :

- 1 In the right-angled triangle, area of the square drawn on the hypotenuse equals
- 2 The reflection in a line reserves .....
- 3 The measure of the exterior angle of the triangle is equal to the sum of
- 4  $(-3, 2)$  is the image of the point  $(3, 2)$  by reflection in ..... axis.
- 5 The line segment joining the midpoints of two sides of the triangle is ..... the third side.

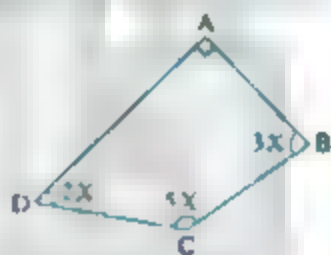
## 2 Choose the correct answer :

- The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is ...  
 (a)  $(3, 1)$  (b)  $(3, -1)$  (c)  $(5, 1)$  (d)  $(5, -5)$
- The measure of the exterior angle of the equilateral triangle equals ...  
 (a)  $60^\circ$  (b)  $100^\circ$  (c)  $120^\circ$  (d)  $150^\circ$
- The image of the point  $(-3, 5)$  by rotation about the origin point by an angle of measure  $90^\circ$  is .....  
 (a)  $(5, 3)$  (b)  $(5, -3)$  (c)  $(-5, -3)$  (d)  $(-5, 3)$
- The sum of measures of the exterior angles of any polygon = ..  
 (a)  $180^\circ$  (b)  $360^\circ$  (c)  $120^\circ$  (d)  $540^\circ$
- The number of lines of symmetry of the square =  
 (a) 1 (b) 2 (c) 3 (d) 4
- The parallelogram whose diagonals are ..... is a rectangle.  
 (a) parallel (b) perpendicular (c) equal in length (d) bisect each other

## 3 (a) In the opposite figure :

$$m(\angle A) = 90^\circ$$

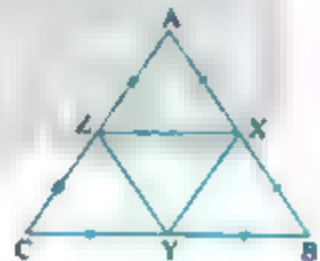
Find the value of : X



## (b) In the opposite figure :

$$AC = AB = 7 \text{ cm.}$$

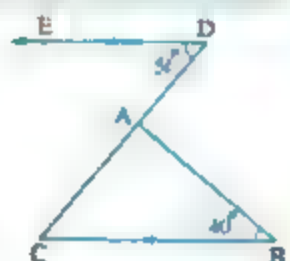
$$BC = 8 \text{ cm.}$$

Find : The perimeter of  $\triangle XYZ$ 

## 4 (a) In the opposite figure :

$$\overline{DE} \parallel \overline{BC}, m(\angle D) = 50^\circ$$

$$m(\angle B) = 40^\circ$$

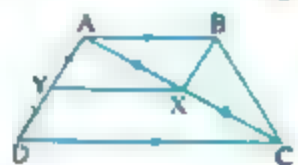
Find :  $m(\angle BAC)$ 

## (b) In the opposite figure :

$$\overline{AB} \parallel \overline{DC}, AB = \frac{1}{2} DC$$

X is the midpoint of  $\overline{AC}$ , Y is the midpoint of  $\overline{AD}$ 

Prove that : ABXY is a parallelogram.





## Geometry and Measurement

5 [a] In the opposite figure :

$$m(\angle B) = m(\angle ACD) = 90^\circ$$

$$AB = 3 \text{ cm.}, BC = 4 \text{ cm.}, AD = 13 \text{ cm.}$$

Find : The length of each of  $\overline{AC}$  and  $\overline{DC}$



[b] On the square lattice, draw  $\triangle ABC$  where  $A(1, 1)$ ,  $B(4, 1)$ ,  $C(4, 4)$ , then find the image of  $\triangle ABC$  by reflection in  $X$  axis.

14

Souhag Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 The sum of the measures of the interior angles of a pentagon equals

(a) 108

(b) 180

(c) 540

(d) 720

2 In the \_\_\_\_\_ the two diagonals are perpendicular and not equal in length

(a) square

(b) rectangle

(c) rhombus

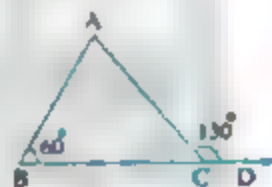
(d) parallelogram

3 The image of the point  $(-2, 3)$  by reflection in the  $y$ -axis is the point

(a)  $(3, 2)$ (b)  $(-3, 2)$ (c)  $(2, 3)$ (d)  $(-3, -2)$ 

4 In the opposite figure :

$$m(\angle A) = \dots\dots\dots$$

(a)  $40^\circ$ (b)  $50^\circ$ (c)  $60^\circ$ (d)  $70^\circ$ 

5 The image of the point  $(-1, 3)$  by translation  $(4, -2)$  is ...

(a)  $(3, 1)$ (b)  $(3, -1)$ (c)  $(5, 1)$ (d)  $(5, -5)$ 

6 In the triangle ABC, if  $m(\angle A) = 50^\circ$ ,  $m(\angle B) = 100^\circ$ , then  $m(\angle C) =$

(a)  $30^\circ$ (b)  $180^\circ$ (c)  $32^\circ$ (d)  $23^\circ$ 

2 Complete each of the following :

1 The length of the line segment joining the midpoints of two sides of a triangle is \_\_\_\_\_

2 If  $\triangle XYZ$  is a right-angled triangle at  $X$ ,  $XY = 12 \text{ cm.}$  and  $XZ = 9 \text{ cm.}$ , then  $YZ = \dots\dots\dots \text{ cm.}$

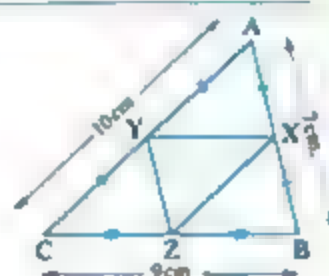
3 The image of the point  $(-1, 2)$  by rotation about the origin point with an angle of measure  $90^\circ$  is \_\_\_\_\_



- 4 If  $\triangle ABC \cong \triangle XYZ$ , then  $m(\angle B) = m(\angle \quad)$
- 5 The ray drawn from the midpoint of a side of a triangle parallel to another side

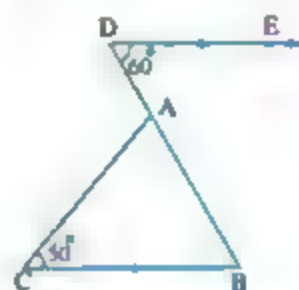
3 [a] In the opposite figure :

X, Y, Z are the midpoints of  $\overline{AB}$ ,  $\overline{AC}$ ,  $\overline{BC}$  respectively,  $AB = 7$  cm,  
 $BC = 9$  cm,  $AC = 10$  cm.  
 Find : The perimeter of  $\triangle XYZ$



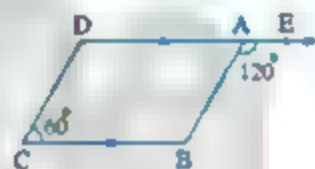
[b] In the opposite figure :

$\overline{DE} \parallel \overline{CB}$ ,  $m(\angle D) = 60^\circ$ ,  $m(\angle C) = 50^\circ$   
 Find :  $m(\angle DAC)$



4 [a] In the opposite figure :

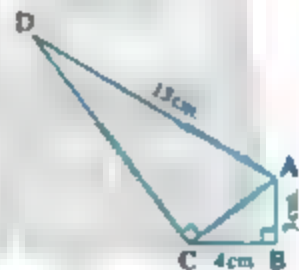
$\overline{DA} \parallel \overline{BC}$ ,  $m(\angle DCB) = 60^\circ$   
 $m(\angle EAB) = 120^\circ$   
 Prove that : ABCD is a parallelogram.



[b] In the opposite figure :

$m(\angle B) = m(\angle ACD) = 90^\circ$   
 $AB = 3$  cm,  $BC = 4$  cm,  
 and  $AD = 13$  cm.

Find : The length of each of  $\overline{AC}$ ,  $\overline{CD}$



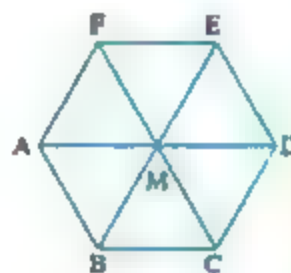
5 [a] On the square lattice draw  $\triangle ABC$ , where  $A(1, 2)$ ,  $B(4, 2)$ ,  $C(4, -1)$ , then find its image by rotation about the origin point with an angle of measure  $180^\circ$

[b] In the opposite figure :

ABCDEF is a regular hexagon

Find the image of  $\triangle ABM$  by :

- 1 reflection in  $\overline{EB}$
- 2 translation FE in direction of  $\overline{FE}$
- 3 rotation (M,  $120^\circ$ )
- 4 reflection in M



## Geometry and Measurement

15

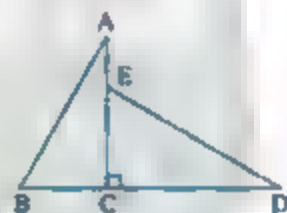
Qena Governorate

Qena Directorate of Education  
Math's supervision

Answer the following questions :

## 1 Choose the correct answer :

- The image of the point  $(-3, 4)$  by reflection in y-axis is  
(a)  $(2, -4)$  (b)  $(-3, -4)$  (c)  $(3, -4)$  (d)  $(3, 4)$
- ABCD is a parallelogram,  $m(\angle A) = 60^\circ$ , then  $m(\angle B) = \dots$   
(a) 120 (b) 60 (c) 90 (d) 100
- ABCD is a parallelogram  $m(\angle A) = m(\angle B)$ , then ABCD is a  
(a) rectangle. (b) square. (c) rhombus. (d) trapezium.
- Measure of each angle in the regular pentagon =  $\dots$   
(a)  $108^\circ$  (b)  $100^\circ$  (c)  $90^\circ$  (d)  $60^\circ$
- The image of the point  $(-1, -2)$  by translation  $(3, 1)$  is  $\dots$   
(a)  $(1, 2)$  (b)  $(2, -1)$  (c)  $(0, 2)$  (d)  $(3, -2)$
- In the opposite figure :  
 $\Delta ABC$  is the image of  $\Delta DEC$   
which is right-angled at C by rotation  
about C with an angle of measure  $\dots$   
(a)  $90^\circ$  (b)  $-90^\circ$   
(c)  $180^\circ$  (d)  $360^\circ$



## 2 Complete :

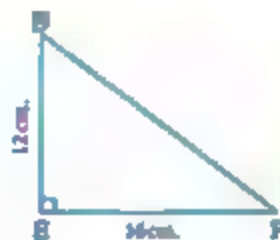
- The ray drawn parallel to one side of a triangle and passing through the midpoint of another side
- $(-5, 1)$  is the image of the point  $(5, 1)$  by reflection in  $\dots$  axis.
- If two adjacent sides are equal in length in a parallelogram, then it becomes  $\dots$
- Sum of measures of the exterior angles of any convex polygon =  $\dots$
- In  $\Delta LMN$  if  $m(\angle M) = 90^\circ$ , then  $(LM)^2 = \dots$

- 3 [a] In the opposite figure :

EDF is a right-angled triangle at E ,  $DE = 12$  cm.

,  $EF = 16$  cm.

Find : DF

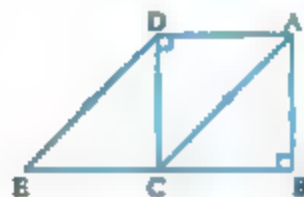


- [b] In the opposite figure :

ABCD is a square ,  $E \in \overline{BC}$

,  $\overline{AC} \parallel \overline{DE}$

Prove that : ACED is a parallelogram.



- 4 [a] Using the square lattice , draw  $\triangle ABC$  where  $A(-4, 2)$  ,  $B(0, 1)$  and  $C(-1, -2)$   
 , then find the image of  $\triangle ABC$  by rotation  $R(O, 180^\circ)$

- [b] Find the number of sides of the regular polygon if the measure of its angle =  $144^\circ$

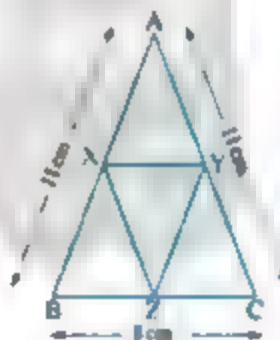
- 5 [a] Using the square lattice , draw  $\triangle ABC$  where  $A(-1, 3)$  ,  $B(2, 1)$   
 and  $C(4, 5)$  , then find the image of  $\triangle ABC$  by reflection in X-axis.

- [b] In the opposite figure :

$AB = 11$  cm. ,  $BC = 8$  cm. ,  $CA = 11$  cm.

X , Z , Y are midpoints of  $\overline{AB}$  ,  $\overline{BC}$  ,  $\overline{CA}$

Find perimeter of :  $\triangle XYZ$





ELMOASSER

# In Mathematics

## Guide Answers

1<sup>st</sup> Prep  
Second Term



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Guide  
Answers

## of Algebra and Statistics Exercises



هذا العمل خاص بموقع ذاكرولي التعليمي ولا يسمح بتداوله على مواقع أخرى

## Answers of Unit One

## Answers of Exercise 1

1.  $(\frac{1}{2})^3 = \frac{1^3}{2^3} = \frac{1}{8}$       2.  $(\frac{1}{3})^4 = \frac{1^4}{3^4} = \frac{1}{81}$   
 3.  $(\frac{3}{5})^2 = \frac{3^2}{5^2} = \frac{9}{25}$       4.  $(-\frac{1}{7})^3 = -\frac{1^3}{7^3} = -\frac{1}{343}$   
 5.  $(-\frac{3}{4})^4 = \frac{3^4}{4^4} = \frac{81}{256}$       6.  $(\frac{5}{9})^0 = 1$   
 7.  $(1\frac{1}{3})^2 = (\frac{4}{3})^2 = \frac{16}{9}$   
 8.  $(-2\frac{1}{2})^3 = (-\frac{5}{2})^3 = -\frac{125}{8}$   
 9.  $(0.04)^2 = (\frac{4}{100})^2 = (\frac{1}{25})^2 = \frac{1}{625}$   
 10.  $(1.5)^3 = (\frac{15}{10})^3 = (\frac{3}{2})^3 = \frac{27}{8}$   
 11.  $(-3.2)^2 = (-\frac{16}{5})^2 = \frac{256}{25}$   
 12.  $(1 - 1\frac{2}{3})^2 = (-\frac{1}{3})^2 = \frac{1}{9}$

2

1.  $8 \times \frac{1}{8} = 1$       2.  $\frac{2}{16} \times \frac{8}{27} = \frac{1}{6}$   
 3.  $-\frac{27}{125} \times (-\frac{25}{27}) = \frac{1}{5}$   
 4.  $\frac{9}{25} \div (-\frac{9}{125}) = \frac{9}{25} \times (-\frac{125}{9}) = -5$   
 5.  $\frac{16}{9} \times \frac{27}{8} = 6$   
 6.  $\frac{35}{36} \div \frac{1}{4} = \frac{35}{36} \times \frac{4}{1} = \frac{5}{9}$   
 7.  $(\frac{5}{2})^2 \times \frac{4}{25} = \frac{25}{4} \times \frac{4}{25} = 1$   
 8.  $\frac{25}{9} \div (-\frac{5}{3})^2 = \frac{25}{9} \div \frac{25}{9} = 1$

3

1.  $\frac{16}{25} \times \frac{5}{16} \times 1 = \frac{1}{5}$   
 2.  $\frac{3}{4} \times (-\frac{8}{27}) \times \frac{9}{4} = -\frac{1}{2}$   
 3.  $\frac{625}{81} \times (-\frac{27}{125}) \times (-1) = \frac{5}{3}$   
 4.  $-\frac{8}{27} \times \frac{1}{27} \div \frac{4}{81} = -\frac{8}{27} \times \frac{1}{27} \times \frac{81}{4} = -\frac{2}{9}$

$$5. (\frac{125}{8} \div \frac{81}{16}) \times \frac{27}{125} = \frac{125}{8} \times \frac{16}{81} \times \frac{27}{125} = \frac{2}{3}$$

$$6. -\frac{1}{8} \div (-3) = -\frac{1}{8} \times \frac{1}{3} = -\frac{1}{24}$$

4

1. (c)      2. (d)      3. (a)      4. (b)      5. (c)  
 6. (b)      7. (c)      8. (c)      9. (b)      10. (b)

5

1. 3      2. 2      3. 3      4. 2  
 5. 3      6. 2      7.  $-\frac{8}{125}$       8.  $\frac{9}{25}$   
 9.  $\frac{1}{125}$       10.  $\frac{1}{9}$       11.  $-\frac{3}{8}$       12. 3  
 13.  $\frac{81}{256} \div \frac{243}{1024}$       14.  $(\frac{1}{4})^2$

6

$$x^2 \cdot y^3 = (-\frac{2}{3})^2 \cdot (-\frac{1}{3})^3 = \frac{4}{9} \cdot -\frac{1}{27} = -\frac{17}{27} \quad \frac{1}{27} = \frac{11}{27}$$

$$7. a^3 \div b^3 = (\frac{2}{3})^3 \div (-\frac{4}{3})^3 = \frac{8}{27} \div (-\frac{64}{27}) = -\frac{8}{27} \div -\frac{64}{27} = -\frac{1}{8}$$

$$|a^3 \div b^3| = |- \frac{1}{8}| = \frac{1}{8}$$

$$8. 9xy^2 - z^3 = 9 \times \frac{1}{2} \times (\frac{2}{3})^2 - (-3)^3 = 9 \times \frac{1}{2} \times \frac{4}{9} + 27 = 2 + 27 = 29$$

9

$$a^3b^2 \div b^2c - 8abc = (-\frac{1}{2})^3 \times 2^2 \div 2^2 = \frac{3}{4} - 8 \times (-\frac{1}{2}) \times 2 \times \frac{3}{4} = -\frac{1}{8} \times 4 + 4 \times \frac{3}{4} + 6 = -\frac{1}{2} + 3 + 6 = 8\frac{1}{2}$$

10

$$1. x^2y^2z^2 = (\frac{3}{2})^2 \times (\frac{1}{2})^2 \times (-\frac{4}{3})^2 = \frac{9}{4} \times \frac{1}{4} \times \frac{16}{9} = 1$$

$$2. x^2 \div z^2 = (-\frac{3}{2})^2 \div (-\frac{4}{3})^2 = \frac{9}{4} \div \frac{16}{9} = \frac{9}{4} \times \frac{9}{16} = \frac{81}{64}$$

$$3. x^2 - yz^2 = (-\frac{3}{2})^2 - \frac{1}{2} \times (-\frac{4}{3})^2 = \frac{9}{4} - \frac{1}{2} \times \frac{16}{9} = \frac{9}{4} - \frac{16}{18} = \frac{81}{36} - \frac{32}{36} = \frac{49}{36}$$

## Algebra and Statistics

$$x^2 y^2 z^2 = 1 \text{ from (1)}$$

$$x + y = \frac{3}{2} + \frac{1}{2} = -1$$

$$\text{i.e. } \frac{x^2 y^2 z^2}{x + y} = \frac{1}{-1} = -1$$

$$11 \text{ The volume of the cube } = \left(1\frac{1}{2}\right)^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8} \text{ cm}^3$$

$$12 \text{ (a)}$$

$$(b)$$

$$13 \text{ The order is } \left(-\frac{2}{3}\right)^3, \left(-\frac{1}{3}\right)^3, \left(-\frac{1}{3}\right)^2, \left(\frac{2}{3}\right)^2$$

## Answers to Exercise 2

1

$$(1) \left(\frac{2}{3}\right)^{3+1} = \left(\frac{2}{3}\right)^4 = \frac{32}{243}$$

$$(2) -\left(\frac{2}{3}\right)^{3+1} = -\left(\frac{2}{3}\right)^4 = -\frac{32}{243}$$

$$(3) \left(\frac{1}{3}\right)^{1+4} = \left(\frac{1}{3}\right)^5 = \frac{1}{3125}$$

$$(4) \left(\frac{1}{6}\right)^{8-8} = \frac{1}{6} \quad (5) \left(\frac{2}{3}\right)^{3-3} = \left(\frac{2}{3}\right)^0 = \frac{4}{27}$$

$$(6) -\left(\frac{2}{3}\right)^{7-3} = -\left(\frac{2}{3}\right)^4 = -\frac{2}{25}$$

$$(7) \left(\frac{2}{3}\right)^2 + \frac{2}{3} = \left(\frac{2}{3}\right)^{2+1} = \frac{2}{3}$$

$$(8) \left(\frac{1}{2}\right)^{2+1+3} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$(9) \left(\frac{4}{5}\right)^{8+8+1} = \left(\frac{4}{5}\right)^{17} = \frac{64}{125}$$

2

$$(1) 3^{7+3-6} = 3^4 = 81 \quad (2) 2^{8+1-3-4} = 2^0 = 1$$

$$(3) 5^{4+2-3} = 5^3 = 125 \quad (4) 2^{3+4-3-2} = 2^2 = 16$$

$$(5) x^{2+3+4-7-1} = x$$

$$(6) (-3)^{2-3} \times (-2)^{3-5} = (-3)^2 \times (-2)^2 = ((-3) \times (-2))^2 = 6^2 = 36$$

$$(7) x^{4+3-6} \times y^{3-2} = x^1 y \quad (8) \frac{x^3 y^3}{x^2 y^2} = \frac{(xy)^3}{(xy)^2} = xy$$

$$(9) \frac{\left(-\frac{1}{2}\right)^5 \times \left(-\frac{2}{3}\right)^8}{\left(-\frac{1}{2}\right)^4 \times \left(-\frac{2}{3}\right)^6} = \left(-\frac{1}{2}\right)^{5-4} \times \left(-\frac{2}{3}\right)^{8-6} = \left(-\frac{1}{2}\right) \times \left(\frac{2}{3}\right)^2 = \left(-\frac{1}{2}\right) \times \frac{4}{9} = -\frac{2}{9}$$

$$(1) \frac{a^2 b^3}{c^3}$$

$$(2) \frac{5^2 x^2}{3^2 y^3} = \frac{25 x^2}{9 y^3}$$

$$(3) \frac{2^4 a^4 b^4}{3^4 c^4} = \frac{16 a^4 b^4}{81 c^4} \quad (4) \frac{x^{2+2}}{y^{3+2}} = \frac{x^4}{y^5}$$

$$(5) \frac{a^{3+3} \times b^{2+3}}{c^{3+3}} = \frac{a^6 b^5}{c^6} \quad (6) -\frac{c^{2+3}}{d^3} = -\frac{c^5}{d^3}$$

$$(7) \frac{x^3+3}{y^3+2} = \frac{x^6}{y^4}$$

$$(8) \left(\frac{4x^3 y^2}{2x^2 y}\right)^3 = (2xy)^3 = 2^3 x^3 y^3 = 128 x^3 y^3$$

$$(9) \frac{2^3 a^3 \times 2^4 a^4}{2^6 a^6 \times a} = 2^{3+4-6-1} \times a^{3+4-6-1} = 2 \times a^0 = 2 \times 1 = 2$$

$$(1) \left(\frac{1}{2}\right)^{3+3} = \left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$(2) \left(\frac{2}{3}\right)^{2+3} = \left(\frac{2}{3}\right)^5 = \frac{59049}{1024}$$

$$(3) \left[\left(\frac{5}{2}\right)^3\right]^2 = \left(\frac{5}{2}\right)^{3 \times 2} = \left(\frac{5}{2}\right)^6 = \frac{15625}{64}$$

$$(4) \left[(-\frac{4}{3})^2\right]^3 = \left(\frac{4}{3}\right)^{2 \times 3} = \left(\frac{4}{3}\right)^6 = \frac{4096}{729}$$

$$(5) \left(\frac{2}{3} \times \frac{3}{2}\right)^{10} = 1^{10} = 1$$

$$(6) \left(\frac{2}{3}\right)^6 \times \left(\frac{2}{3}\right)^6 = \left(\frac{2}{3} \times \frac{2}{3}\right)^6 = 1^6 = 1$$

$$(7) \left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3} \times \frac{2}{3}\right)^3 = 1^3 = 1$$

$$(8) \left(\frac{x^3}{y^2} + \frac{x^2}{y^2}\right)^{16} = \left(\frac{x^3}{y^2} \times \frac{y^2}{x^2}\right)^{16} = x^{16}$$

$$(1) (c) \quad (2) (a) \quad (3) (h) \quad (4) (i)$$

$$(5) (f) \quad (6) (c) \quad (7) (b) \quad (8) (d)$$

$$(9) (a) \quad (10) (d) \quad (11) (c) \quad (12) (d)$$

$$(13) (c) \quad (14) (a) \quad (15) (d) \quad (16) (c)$$

$$(17) (d) \quad (18) (c) \quad (19) (c) \quad (20) (d)$$

$$(1) \frac{2^4 y^4 \times 3^2 y^2}{12 y^3} = \frac{16 y^4 \times 9 y^2}{12 y^3} = \frac{144 y^6}{12 y^3} = 12 y$$

$$\text{At } y = -\frac{1}{6}$$

$$\text{The result} = 12 \times \left(-\frac{1}{6}\right) = -2$$

$$(2) (1) (c^2 b)^3 = \left(\left(-\frac{2}{3}\right)^2 \times \frac{3}{4}\right)^3 = \left(\frac{4}{9} \times \frac{3}{4}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$\begin{aligned} \text{[2]} (4a^3c)^2 &= 16a^6c^2 = 16 \times \left(\frac{1}{2}\right)^6 \times \left(-\frac{2}{3}\right)^2 \\ &= 16 \times \frac{1}{64} \times \frac{4}{9} = \frac{1}{9} \end{aligned}$$

$$\begin{aligned} \text{[3]} (a^2b^3c^2)^2 &= a^4b^6c^4 = \left(\frac{1}{2}\right)^4 \times \left(\frac{3}{4}\right)^6 \times \left(-\frac{2}{3}\right)^4 \\ &= \frac{1}{16} \times \frac{9}{16} \times \frac{16}{81} = \frac{1}{144} \end{aligned}$$

8

$$\begin{aligned} \text{[1]} \frac{(a^2c^2)^2}{b} &= \frac{a^4c^4}{b} = \left(\frac{5}{3}\right)^4 \times \left(\frac{2}{3}\right)^4 \times \left(-\frac{3}{2}\right) \\ &= \left(\frac{5}{3} \times \frac{2}{3}\right)^4 \times \left(-\frac{3}{2}\right) \\ &= \left(\frac{10}{3}\right)^4 \times \left(-\frac{3}{2}\right) = -\left(\frac{10}{3}\right)^3 = -\frac{32}{243} \end{aligned}$$

$$\text{[2]} \left(\frac{2ab}{5c}\right)^3 = \left(\frac{2 \times \frac{3}{2} \times \left(-\frac{1}{2}\right)}{5 \times \frac{2}{3}}\right)^3 = \left(-\frac{3}{5}\right)^3 = -\frac{125}{8}$$

$$\text{[10]} \text{[1]} x^3y^2 = \left(-\frac{1}{2}\right)^3 \times \left(\frac{3}{4}\right)^2 = -\frac{1}{8} \times \frac{9}{16} = -\frac{9}{128}$$

$$\text{[2]} y^3x^2 = \left(\frac{3}{4}\right)^3 \times \left(-\frac{1}{2}\right)^2 = \frac{27}{64} \times \frac{1}{4} = \frac{27}{256}$$

$$\begin{aligned} \text{[3]} \frac{x^3}{y^2z^2} &= \frac{\left(-\frac{1}{2}\right)^3}{\left(\frac{1}{3}\right)^2 \times \left(-\frac{1}{2}\right)^2} = \frac{-\frac{1}{8}}{\frac{1}{16} \times \frac{1}{4}} \\ &= \frac{-\frac{1}{8}}{\frac{1}{64}} = -\frac{1}{8} \times \frac{64}{1} = -8 \end{aligned}$$

11

[1] 24

[2]  $\frac{2}{16}$ [3]  $((-3)^2)^4$ [4]  $2x^2y$ 

[5] 4

[6]  $2x$ 

12

$$\text{The area of the square} = \left(\frac{2x}{5}\right)^2 = \frac{2^2 \times x^2}{5^2} = \frac{4x^2}{25} \text{ cm}^2$$

13

$$\begin{aligned} \text{The volume of the cube} &= \left(\frac{3a^2}{7}\right)^3 = \frac{3^3 \times (a^2)^3}{7^3} \\ &= \frac{27a^6}{343} \text{ cm}^3 \end{aligned}$$

14

The area of the shaded part

= the area of the great square - the area of the small square

$$= \left(\frac{3x}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = \frac{9x^2}{4} - \frac{x^2}{4} = \frac{8x^2}{4} = 2x^2 \text{ cm}^2$$

$$\begin{aligned} \text{[15]} \text{The number of bytes in one terabyte} \\ &= 2^{10} \times 2^{10} \times 2^{10} \times 2^{10} = 2^{40} \text{ bytes} \end{aligned}$$

$$\text{[16]} \text{The number is } 4^3 \div 4 = 4^2$$

$$\text{Then } \frac{3}{4} \text{ the number} = \frac{3}{4} \times 4^2 = 12$$

17

$$x = \left(\frac{2}{3}\right)^{10}$$

$$y = \left(\frac{3}{2}\right)^9 \times \left(3 - \frac{3}{2}\right) = \left(\frac{3}{2}\right)^9 \times \left(\frac{6}{2} - \frac{3}{2}\right)$$

$$= \left(\frac{3}{2}\right)^9 \times \frac{3}{2} = \left(\frac{3}{2}\right)^{10}$$

$$xy = \left(\frac{2}{3}\right)^{10} \times \left(\frac{3}{2}\right)^{10} = \left(\frac{2}{3} \times \frac{3}{2}\right)^{10} = 1^{10} = 1$$

= the multiplicative neutral element.

i.e.  $x$  is the multiplicative inverse of  $y$ 

$$\text{[18]} x^{15}y^{14} = x \times x^{14} \times y^{14} = x(xy)^{14}$$

$$= \frac{1}{3} \left(\frac{1}{3} \times 5\right)^{14} = \frac{1}{3} \times (1)^{14} = \frac{1}{3}$$

19

$$\text{[1]} \text{L.H.S.} = 5^x(5^2 - 5) = 5^x(25 - 5) = 20 \times 5^x = \text{R.H.S.}$$

$$\text{[2]} 3^{13} \div 3^{14} = 3^{14} (3 + 1) = 4 \times 3^{14}$$

i.e.  $3^{13} \div 3^{14}$  is divisible by 4

## Answers of Exercise 3

$$\text{[1]} 4^{-1} = \frac{1}{4}$$

$$\text{[2]} 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$\text{[3]} \left(\frac{1}{2}\right)^{-1} = 2$$

$$\text{[4]} \left(-\frac{3}{2}\right)^{-2} = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\text{[5]} (0.2)^{-2} = \left(\frac{2}{10}\right)^{-2} = \left(\frac{10}{2}\right)^2 = 25$$

$$\text{[6]} (1.2)^{-1} = \left(\frac{6}{5}\right)^{-1} = \frac{5}{6}$$

$$\text{[7]} \text{[1]} 3^7 \times 3^{-3} = 3^{7+(-3)} = 3^4 = 81$$

$$\text{[2]} 2^{-2} \times 2^{-3} = 2^{-2+(-3)} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

$$\text{[3]} \frac{3^{-2}}{3^{-1}} = 3^{1+(-2)} = 3^{-1} = \frac{1}{3}$$

$$\text{[4]} \frac{6^{-2}}{6^{-3}} = 6^{-2+(-(-3))} = 6^1 = 6$$

$$\text{[8]} \text{[1]} (5^{-1})^{-3} = 5^3 = 125 \quad \text{[2]} (3^{-2})^2 = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$$

$$\text{[3]} (0.25)^{-2} = \left(\frac{1}{4}\right)^{-2} = 4^2 = 16$$



## Algebra and Statistics

$$[4] (2^{-1} \times 2^{-2})^3 = (2^{-3})^3 = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}$$

$$[6] \left(\frac{2^{-1}}{3}\right)^3 = \frac{2^{-3}}{3^3} = 2^{-3-3} = 2^{-6} = \frac{1}{2^6} = \frac{1}{64}$$

$$[8] \left(\frac{2^4}{2^{-4}}\right)^0 = 1$$

[4]

$$[1] 8^{1-2-(-3)} = 8^2 = 64 \quad [2] 7^{-2+3-3} = 7^0 = 1$$

$$[3] 2^{3-2-(-4)-3} = 2^4 = 16$$

$$[4] \frac{2^3 \times 2^{-3}}{2^4} = 2^{3-3-4} = 2^{-4} = \frac{1}{2^4} = \frac{1}{16}$$

$$[6] \frac{3^{-4}}{3^{-2} \times 3^{-4}} = \frac{1}{3^{-2}} = 3^2 = 9$$

$$[8] (9^3 \div 1 \div 9)^{-3} = (9^{-1})^{-3} = 9^3 = 729$$

$$[7] (4^{-2} \div (-2) \times 3)^{-3} = (4 \times 3)^{-3} = \frac{1}{12^3} = \frac{1}{1728}$$

$$[9] (2^{3-3} \times 3^{2-4})^{-1} = (2^0 \times 3^{-2})^{-1} = 2^{-0} \times 3^2 = \frac{9}{4}$$

$$[10] 1 \times 2^4 = 1 \times 16 = 16$$

$$[10] \left(1 - \frac{1}{2}\right)^{-2} = \left(1 - \frac{1}{4}\right)^{-2} = \left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

$$[11] \frac{(10)^2 \times (10^{-2})^3}{(10)^{-3}} = \frac{(10)^2 \times (10)^{-6}}{(10)^{-3}} = (10)^{2-6+3} = (10)^{-1} = \frac{1}{10}$$

[5]

$$[1] \frac{x}{x} \quad [8] \frac{x^2}{x} \quad [12] \frac{1}{a^2 b^3}$$

$$[4] x^{3-3} = x^{-3} = \frac{1}{x^3} \quad [6] x^{3-2-1} = x^0 = 1$$

$$[8] c^{-3-2} = c^{-5} = \frac{1}{c^5} \quad [7] x^{1-(-3)} = x^{1+3} = x^4$$

$$[9] a^{-6} = \frac{1}{a^6} \quad [9] b^3$$

$$[10] (a^{2-3})^2 = (a^{-1})^2 = a^{-2} = \frac{1}{a^2}$$

$$[11] x^{-6} \times x^6 = x^0 = 1$$

$$[12] (y^{3-(-2)})^{-3} = (y^5)^{-3} = y^{-15} = \frac{1}{y^{15}}$$

$$[13] x^{2-3-(-4)-1} = x^2$$

$$[14] \frac{x^{-6} \times x^{-2}}{x^{-3} \times x^{-8}} = x^{-6-2-(-3)-(-8)} = x^{-1} = \frac{1}{x}$$

$$[15] (x^{-2-1} \times y^{1-(-3)})^{-1} = (x^{-3} y^4)^{-1} = \frac{x^3}{y^4}$$

$$[16] \frac{a^{-1}}{b^2} \times \frac{a^2}{2^{-2} b^{-4}} = \frac{2^2 \times a^{-1+2}}{b^{2-4}} = \frac{4a}{b^{-2}} = 4ab^2$$

$$[17] x^2 + 2xx^{-1} + (x^{-1})^2 = x^2 + 2 + x^{-2} = x^2 + \frac{1}{x^2} + 2$$

$$[6] [1] \frac{1}{3}$$

$$[8] 3$$

$$[3] x^3$$

$$[4] -2, x^2$$

$$[6] \frac{x^4}{9}$$

$$[8] 3x^2$$

$$[7] x^2 y^3$$

$$[9] \frac{y}{x}$$

$$[9] 1$$

$$[10] -2$$

$$[11] 0$$

$$[12] 1, x^3$$

$$[13] 4$$

[7]

$$[1] (b)$$

$$[2] (c)$$

$$[3] (c)$$

$$[4] (c)$$

$$[5] (b)$$

$$[6] (b)$$

$$[7] (d)$$

$$[8] (d)$$

$$[9] (d)$$

$$[10] (d)$$

$$[11] (b)$$

$$[12] (a)$$

[8]

$$[1] >$$

$$[2] <$$

$$[3] <$$

$$[4] >$$

$$[5] >$$

$$[6] =$$

[9]

$$b^{-1} = \frac{1}{b}, \text{ substituting } b = 0, \text{ then } \frac{1}{0} = \frac{1}{0} \text{ (undefined).}$$

[10]

$$[1] \text{ At } x = -2, y = 2$$

$$\text{Then: } \left(-\frac{2}{3}\right)^{-2} \times \left(\frac{2}{3}\right)^2 = \left(\frac{3}{2}\right)^2 \times \left(\frac{2}{3}\right)^2 = \left(\frac{3}{2} \times \frac{2}{3}\right)^2 = 1^2 = 1$$

$$[2] \text{ At } x = -1, y = 2$$

$$\text{Then } \left(-\frac{1}{3}\right)^{-1} \times \left(\frac{2}{3}\right)^2 = -\frac{3}{1} \times \frac{4}{9} = -\frac{4}{3}$$

[11]

$$\left(\frac{2}{x}\right)^{-2} = \left(\frac{2}{3} + \left(-\frac{1}{3}\right)^2\right)^{-2} = \left(\frac{2}{3} + \frac{1}{9}\right)^{-2}$$

$$= \left(\frac{2}{3} \times \frac{9}{7}\right)^{-2} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$$

[12]

$$\begin{aligned} \text{The height of the jump} &= 2^3 + 2^{-4} \\ &= 2^3 - (-4) = 2^7 \\ &= 128 \text{ times of its length} \end{aligned}$$

[13]

$$[1] \text{ In 2 years the population will}$$

$$\text{be } 2(1.03)^2 = 2.1218 \text{ millions.}$$

$$[2] \text{ The population now } = 2(1.03)^0 = 2 \text{ millions.}$$

$$[3] \text{ The population last year was}$$

$$2(1.03)^{-1} = \frac{2}{1.03} = 1.94 \text{ millions}$$

[14]

$$\begin{aligned} \frac{2^{10} \times 3^4}{(2 \times 2 \times 3)^5} &= \frac{2^{10} \times 3^4}{(2^2 \times 3)^5} = \frac{2^{10} \times 3^4}{2^{10} \times 3^5} \\ &= 2^{10-10} \times 3^{4-5} = 3^{-1} = \frac{1}{3} \end{aligned}$$

## Answers of Unit 1

15

$$\frac{(2 \times 3)^{2n+1} \times (2^2)^{-n}}{2^n \times 3^{2n+1}} = \frac{2^{2n+1} \times 3^{2n+1} \times 2^{-2n}}{2^n \times 3^{2n+1}}$$

$$= 2^{2n+1-2n} \times 3^{2n+1-2n-1}$$

$$= 2^{1-n} \times 3^0 = 2^{1-n}$$

When  $n = 3$ , then  $2^{1-n} = 2^{1-3} = 2^{-2} = \frac{1}{4}$ 

16

1)  $2^{n+1} = 2^n \times 2 = 3 \times 2 = 6$

2)  $4^n = (2 \times 2)^n = 2^n \times 2^n = 3 \times 3 = 9$

3)  $4^{-n} = (2 \times 2)^{-n} = 2^{-n} \times 2^{-n} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

4)  $2^{n-1} = 2^n \times 2^{-1} = 3 \times \frac{1}{2} = \frac{3}{2}$

17

$$a^{31} b^{30} = a \times a^{30} b^{30} = a (ab)^{30}$$

$$= 5 \left(5 \times \frac{1}{5}\right)^{30} = 5 \times 1 = 5$$

18

The order is  $(-5)^{15} \cdot (-2)^{15} \cdot (-2)^{-15} \cdot 2^{-30}$   
 $\cdot (-2)^{30} \cdot (-5)^{20}$ 

## Answers of Exercise 4

1

The numbers which are in the standard form are (1), (4), (6) and (8)

2

1)  $6 \times 10^3$       2)  $-2 \times 10^6$       3)  $7 \times 10^6$   
 4)  $1.9 \times 10^3$       5)  $4.687 \times 10^3$       6)  $5.8 \times 10$

3

1)  $6 \times 10^{-4}$       2)  $5.3 \times 10^{-5}$       3)  $8.64 \times 10^{-4}$   
 4)  $4.21 \times 10^{-1}$       5)  $2.90003 \times 10$       6)  $-3.00901 \times 10^2$

4

1)  $(5.1 \times 10^8) \text{ km}^2$

5

1)  $(1.67 \times 10^{-24}) \text{ gm}$

6

The velocity of light =  $300\,000 \times 1000$   
 $= 300\,000\,000$   
 $= 3 \times 10^8 \text{ m/sec}$

7

1)  $1 \times 10^{-15} \text{ seconds}$

8

18 zeroes

9

1)  $6.8 \times 10^6$       2)  $6.8 \times 10^{-4}$   
 3)  $7.2 \times 10^6$       4)  $7.5 \times 10^{-3}$

1)  $3.24 \times 10^3$

2)  $4 \times 10^{-11}$

3)  $3.6 \times 10^{-7}$

4)  $-7.025 \times 10^{-6}$

5)  $5 \times 10^{11}$

6)  $2.0205 \times 10^9$

10

1)  $>$       2)  $<$       3)  $<$       4)  $<$   
 5)  $>$       6)  $>$       7)  $>$       8)  $>$

11

The order is  
 $8.35 \times 10^{-3}, 1 \times 10^{-2}, 3.6 \times 10^{-3}, 5.2 \times 10^{-3}, 6.08 \times 10^{-4}$

12

1) (d)      2) (b)      3) (c)      4) (b)  
 5) (c)      6) (b)      7) (a)      8) (b)  
 9) (c)      10) (c)      11) (d)

13

1)  $9.6 \times 10^{13}$       2)  $17.22 \times 10^3 = 1.722 \times 10^4$   
 3)  $0.502 \times 10^{-7} = 5.02 \times 10^{-8}$   
 4)  $(4.4 \times 10^3) \times (32 \times 10^3) = 140.8 \times 10^6 = 1.408 \times 10^{10}$   
 5)  $2 \times 10^3$       6)  $25.1 \times 10^{-7} = 2.51 \times 10^{-6}$   
 7)  $1 \times 10^3$   
 8)  $(625 \times 10^4) \div (2.5 \times 10^{-3}) = 250 \times 10^7 = 2.5 \times 10^9$

14

1)  $10^4 (3.8 \times 10 + 4.6) = 10^4 (38 + 4.6)$   
 $= 42.6 \times 10^4 = 4.26 \times 10^5$   
 2)  $10^3 (4.54 \times 10 + 3.76) = 10^3 \times 49.16 = 4.916 \times 10^4$   
 3)  $10^3 (5.3 \times 10 - 0.8) = 10^3 \times 52.2 = 5.22 \times 10^4$   
 4)  $10^{-3} (2.65 \times 10 - 6.34) = 10^{-3} \times 20.16 = 2.016 \times 10^{-2}$

15

1)  $5 \times 10^3 \times 3 \times 10^3 = 15 \times 10^6 = 1.5 \times 10^7$   
 2)  $4 \times 10^2 \times 7 \times 10^{-3} = 28 \times 10^{-1} = 2.8 \times 10^{-2}$   
 3)  $8 \times 10^3 \div 4 \times 10^{-3} = 2 \times 10^6$   
 4)  $3.3 \times 10^{-3} \div 5 \times 10^2 = 0.66 \times 10^{-7} = 6.6 \times 10^{-8}$   
 5)  $(2 \times 10^4)^3 = 8 \times 10^{12}$   
 6)  $(2 \times 10^{-3})^2 = 4 \times 10^{-6}$   
 7)  $(1 \times 10^{-1})^{-8} = 10^8$

16

1)  $800\,000 = 8 \times 10^5$        $\therefore n = 5$   
 2)  $0.00000006 = 6 \times 10^{-8}$        $\therefore n = -8$   
 3)  $0.00052 = 5.2 \times 10^{-4}$        $\therefore n = -4$   
 4)  $0.000357 = 3.57 \times 10^{-4}$        $\therefore n = -4$



7  $2\left(-\frac{5}{4} \times \frac{3}{4} + \frac{3}{4}\right) = 4$

8  $(2+5)^2 = 49$

2  $(5-2)^3 = 27$

3  $\left(\frac{5}{2}\right)^3 = \frac{125}{8}$

4  $\frac{6^2}{5-1} = 9$

5  $\frac{5 \cdot 2}{5^3} = \frac{3}{125}$

6  $\frac{12}{4 \cdot 25} = \frac{3}{25}$

9  $10 \times 9 - (4 \times 6) + 3 \times 6 \times 9 = 168$

10  $x = 38, y = 9$

$2x + 4y = 2 \times 38 + 4 \times 9 = 112$

11  $x = 32, y = 8$

$x - 4y = 32 - 4 \times 8 = \text{zero}$

12  $x = 15, y = 30$

$\left(\frac{1}{x}\right)^{-3} = \left(\frac{30}{15}\right)^{-3} = \frac{1}{8}$

13 1  $T = 6x^2 = 6 \times 3^2 = 54 \text{ m}^2$

2  $T = 6x^2 = 6 \times (0.8)^2 = 3.84 \text{ cm}^2$

14 1  $T = 2(xy + yz + zx)$

$= 2(2 \times 3 + 3 \times 5 + 5 \times 2) = 62 \text{ cm}^2$

2  $T = 2(xy + yz + zx)$

$= 2\left(\frac{1}{3} \times 0.4 + 0.4 \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}\right) = \frac{22}{225} \text{ m}^2$

15 1  $A = \frac{1}{2} h(a+b) = \frac{1}{2} \times 2 \left(\frac{1}{2} + \frac{1}{4}\right) = 1 \text{ m}^2$

2  $A = \frac{1}{2} h(a+b) = \frac{1}{2} \times 4 \left(\frac{1}{2} + \frac{1}{2}\right) = 2 \text{ m}^2$

16 3  $3 + 96 - (12 \times 4) = 5$

2  $3 + (96 + 12) \times 4 = 35$

3  $(3 + 96) + 12 \times 4 = 13$

### Answers of Exercise 6

1 1 4 2 -5 3  $\pm 50$

4  $\pm 200$  5  $\frac{1}{2}$  6  $-\frac{1}{2}$

7  $\sqrt{\frac{81}{100}} = \frac{9}{10}$  8  $\pm \sqrt{\frac{144}{100}} = \pm \frac{12}{10} = \pm \frac{6}{5}$

9  $\sqrt{\frac{25}{4}} = \frac{5}{2}$  10  $-\sqrt{\frac{36}{25}} = -\frac{6}{5}$  11 -4

12  $\pm 8$  13  $\frac{81}{100}$  14  $|- \frac{3}{4}| = \frac{3}{4}$

15  $\pm \frac{24}{35}$  16  $-\sqrt{\frac{25}{400}} = -\sqrt{\frac{1}{16}} = -\frac{1}{4}$

17  $|- \frac{7a^2}{5b^3}|$  18  $\pm \sqrt{\frac{4b^4}{11h}}$  19  $|7a^2b|$

20  $|\frac{5xy}{6}|$

2 1  $\pm 8$

2  $\pm 12$

3  $\pm \frac{3}{2}$

4  $\pm \sqrt{\frac{25}{4}} = \pm \frac{5}{2}$

5  $\pm 0.5$

6  $\pm 0.07$

3 1  $3 + 4 = 7$

2  $\sqrt{100} = 10$

3  $\sqrt{16} = 4$

4  $-\sqrt{144} = -12$

5  $\sqrt{9+16} = \sqrt{25} = 5$

6  $-\sqrt{100-64} = -\sqrt{36} = -6$

7  $\sqrt{\frac{9}{16} + \frac{16}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

8  $-\sqrt{\frac{1}{4}(\frac{4}{3} - \frac{1}{3})} = -\sqrt{\frac{1}{4} \times \frac{1}{3}} = -\frac{1}{4}$

9  $\sqrt{5^{4+2-3}} = \sqrt{5^3} = 5$

10  $\sqrt{(\frac{1}{2} - \frac{1}{2})^2} = \sqrt{(\frac{1}{2} \times 5)^2} = \sqrt{(\frac{5}{2})^2} = \frac{5}{2}$

11  $\sqrt{(\frac{1}{2} \times \frac{1}{3})^4} = \sqrt{(\frac{1}{6})^4} = (\frac{1}{6})^2 = \frac{1}{36}$

12  $\sqrt{(\frac{1}{4})^3} = \sqrt{((\frac{1}{2})^2)^3} = \sqrt{(\frac{1}{2})^6} = (\frac{1}{2})^3 = \frac{1}{8}$

4 1 1

2  $\frac{1}{2}$

3 1

4 100

5 80

6  $\frac{1}{2}$

7 10

8  $\frac{1}{2}$

9  $\frac{1}{4}$

10  $(\frac{5}{2})^2$  or  $(-\frac{5}{2})^2$

11  $\sqrt{(\frac{1}{2})^2} = \frac{1}{2}$

12 1

13  $a^2b^4$

14 1

15  $\frac{1}{4}$

16 3

17 2

18 2222

5 1 (c)

2 (b)

3 (b)

4 (d)

5 (a)

6 (c)

7 (b)

8 (b)

9 (d)

10 (b)

6 1  $\frac{1}{3} \times \frac{8}{3} = -\frac{1}{3} \times \frac{8}{3} = -\frac{8}{9}$

2  $\frac{1}{2} \times 1 \times \frac{2}{3} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$

3  $\frac{2}{3} \times \frac{1}{4} \times (-\frac{1}{2}) = \frac{2}{3} \times \frac{1}{4} \times (-\frac{8}{4}) = -\frac{12}{12}$

4  $\frac{1}{3} + \frac{8}{9} - 1 = \frac{1}{9} + \frac{8}{9} - \frac{9}{9} = \text{zero}$

5  $\frac{1}{4} \times (\frac{2}{3}) = \frac{9}{4} = \frac{1}{4} \times \frac{8}{27} \times \frac{9}{4} = \frac{1}{2}$

6  $\frac{25}{4} \times \frac{4}{25} = 1$

Another solution .

$\frac{25}{4} \times (\frac{2}{3})^2 = (\frac{5}{2})^2 \times (\frac{2}{3})^2$

$= (\frac{5}{2} \times \frac{2}{3})^2 = 1^2 = 1$



## Algebra and Statistics

7  $4 + 5 = 9 \quad \therefore \sqrt{4+5} = \sqrt{9} = 3$   
 $\therefore \sqrt{(4+5)^2} = \sqrt{(9)^2} = 9$

8  $\sqrt{\frac{4}{9}} = \frac{2}{3}$   
 $\therefore$  The L.C.M. of 3 and 4 is 12  
 $\therefore \frac{2}{3} = \frac{8}{12}, \frac{1}{4} = \frac{3}{12} \therefore \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$   
 $\therefore$  The two numbers are  $\frac{23}{36}$  and  $\frac{26}{36}$

9  $\sqrt{\frac{4}{9}} = \frac{2}{3}$   
 $\therefore$  The L.C.M. of 3 and 5 is 15  
 $\therefore \frac{2}{3} = \frac{10}{15}, \frac{1}{5} = \frac{3}{15} \therefore \sqrt{\frac{4}{9}} > \frac{1}{5}$   
 $\therefore$  The difference =  $\frac{10}{15} - \frac{3}{15} = \frac{7}{15}$

10  $\sqrt{2\frac{1}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2} \therefore (-\frac{3}{2})^2 = \frac{9}{4}$   
 $\therefore$  L.C.M. of 2 and 9 is 18  
 $\therefore \frac{3}{2} = \frac{27}{18}, \frac{1}{9} = \frac{2}{18} \therefore \sqrt{\frac{9}{4}} > (-\frac{1}{9})^2$   
 $\therefore$  The difference =  $\frac{27}{18} - \frac{2}{18} = \frac{25}{18}$

11  $\therefore \sqrt{25-10+1} = \sqrt{16} = 4$   
 Another solution:  $\sqrt{(5-1)^2} = \sqrt{4^2} = 4$   
 $\therefore \frac{1}{16} - \frac{1}{2} + 1 = \sqrt{\frac{1}{16} - \frac{8}{16} + \frac{16}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$   
 Another solution:  
 $\therefore (\frac{1}{4} - 1)^2 = \sqrt{(-\frac{3}{4})^2} = 1 - \frac{3}{4} = \frac{1}{4}$

12  $\sqrt{9} = 3 \quad \sqrt{16} = 4$   
 $\sqrt{121} = 11 \quad \sqrt{6+3 \times 10-11} = \sqrt{25} = 5$

- 13  $\therefore (XY)^2 = 25 \quad \therefore XY = \sqrt{25} = 5 \text{ cm.}$   
 $\therefore$  E is the midpoint of XY  
 $\therefore XE = 2.5 \text{ cm}$   
 $\therefore (AB)^2 = 144 \quad \therefore AB = \sqrt{144} = 12 \text{ cm}$   
 $\therefore (BC)^2 = 625 \quad \therefore BC = \sqrt{625} = 25 \text{ cm.}$   
 $\therefore AC = 37 \text{ cm.}$   
 14 The side length of the square =  $\sqrt{0.49} = 0.7 \text{ cm}$   
 $\therefore$  Its perimeter = the side length  $\times 4$   
 $= 0.7 \times 4 = 2.8 \text{ cm.}$   
 15 The area of the square = the area of the triangle  
 $= \frac{1}{2} \times 9 \times 8 = 36 \text{ cm}^2$   
 $\therefore$  The side length of the square =  $\sqrt{36} = 6 \text{ cm}$

10

6  $\therefore$  The area of the circle =  $\pi r^2$   
 $\therefore 154 = \frac{22}{7} \times r^2 \quad \therefore r^2 = 154 \div \frac{22}{7} = 49$   
 $\therefore r = \sqrt{49} = 7 \text{ cm.}$

8  $\therefore$  The area of the circle =  $\pi r^2$   
 $78.5 = 3.14 \times r^2 \quad \therefore r^2 = 78.5 \div 3.14 = 25$   
 $\therefore r = \sqrt{25} = 5 \text{ cm}$   
 $\therefore$  Its diameter length =  $2 \times 5 = 10 \text{ cm.}$

7  $\therefore$  The area of the circle =  $\pi r^2$   
 $\therefore 616 = \frac{22}{7} \times r^2 \quad \therefore r^2 = 616 \div \frac{22}{7} = 196$   
 $\therefore r = \sqrt{196} = 14 \text{ cm}$   
 $\therefore$  Its circumference =  $2 \times \pi \times r = 2 \times \frac{22}{7} \times 14 = 88 \text{ cm.}$

9  $\therefore$  The area of the square =  $1\frac{1}{4} \times \frac{4}{3} = \frac{25}{12} \text{ m}^2$   
 The side length =  $\sqrt{\frac{25}{12}} = \frac{5}{2} = 1\frac{1}{2} \text{ m}$

10  $2 \times$  the width of the rectangle  $\times$  the width of the rectangle = 24.5

$\therefore$  The width of the rectangle  $\times$  the width of the rectangle = 12.25

$\therefore$  The width of the rectangle =  $\sqrt{12.25} = 3.5 \text{ cm.}$

$\therefore$  The length of the rectangle =  $2 \times 3.5 = 7 \text{ cm.}$

13  $\frac{m^2}{n^2} = \frac{16}{100} \quad \therefore \frac{m}{n} = \pm \sqrt{\frac{16}{100}} = \pm \frac{4}{10}$   
 $\therefore (\frac{m}{n})^3 = (\pm \frac{4}{10})^3 = \pm \frac{64}{1000} = \pm 0.064$

## Answers of Exercise 7

- 1  $X - 7 = 3 \quad \therefore X - 7 + 7 = 3 + 7$   
 $\therefore X = 10 \quad \therefore$  The S.S. = {10}
- 2  $X + 17 = 13 \quad \therefore X + 17 - 17 = 13 - 17$   
 $\therefore (13 - 17) \in R \quad \therefore$  The S.S. =  $\emptyset$
- 3  $5X = 20 \quad \therefore 5X \times \frac{1}{5} = 20 \times \frac{1}{5}$   
 $\therefore X = 4 \quad \therefore$  The S.S. = {4}
- 4  $\frac{2}{3}X = \frac{1}{2} \quad \therefore \frac{2}{3}X \times \frac{3}{2} = \frac{1}{2} \times \frac{3}{2}$   
 $\therefore X = \frac{1}{2} \quad \therefore$  The S.S. =  $\{\frac{1}{2}\}$
- 5  $4 + y + 4 = 13 + 4$   
 $\therefore y = 17 \quad \therefore$  The S.S. = {17}
- 6  $m - (-3) = 1 \quad \therefore m + 3 - 3 = 1 - 3$   
 $\therefore m = -2 \quad \therefore$  The S.S. = {-2}

$$7 \quad \therefore x - 7 = 0 \quad \therefore x - 7 + 7 = 0 + 7$$

$$\therefore x = 7 \quad \therefore \text{The S.S.} = \{7\}$$

$$8 \quad \therefore y - (-5) = -3 \quad \therefore y + 5 - 5 = -3 - 5$$

$$\therefore y = -8 \quad \therefore \text{The S.S.} = \{-8\}$$

$$9 \quad \therefore x - 6\frac{1}{4} = 12\frac{1}{2}$$

$$\therefore x - 6\frac{1}{4} + 6\frac{1}{4} = 12\frac{1}{2} + 6\frac{1}{4}$$

$$\therefore x = 18\frac{3}{4} \quad \therefore \text{The S.S.} = \{18\frac{3}{4}\}$$

$$10 \quad 891 \div x = 11.09$$

$$\therefore 891 \div x - 891 \div x = 11.09 - 891$$

$$\therefore x = 2.18 \quad \therefore \text{The S.S.} = \{2.18\}$$

11

$$1 \quad \therefore 2x - 1 = 5$$

$$2x - 1 + 1 = 5 + 1$$

$$2x = 6$$

$$\therefore 2x \times \frac{1}{2} = 6 \times \frac{1}{2}$$

$$\therefore x = 3$$

$$2 \quad \therefore 8x + 4 = 12$$

$$\therefore 8x + 4 - 4 = 12 - 4$$

$$\therefore 8x = 8$$

$$\therefore 8x \times \frac{1}{8} = 8 \times \frac{1}{8}$$

$$\therefore x = 1$$

$$3 \quad \therefore 3x - 13 \div 13 = 26 \div 13$$

$$3x = 39$$

$$\therefore \frac{3x}{3} = \frac{39}{3}$$

$$x = 13$$

$$4 \quad \therefore 2x + 14 - 14 = 14 - 14$$

$$\therefore 2x = \text{zero}$$

$$\therefore \frac{2x}{2} = \frac{0}{2}$$

$$\therefore x = \text{zero}$$

$$5 \quad \therefore 8 + 2x - 8 = 14 - 8$$

$$\therefore 2x = 6$$

$$\therefore \frac{2x}{2} = \frac{6}{2}$$

$$\therefore x = 3$$

$$6 \quad \therefore \frac{5}{6}x - 4 \div 4 = 11 \div 4$$

$$\therefore \frac{5}{6}x = 15$$

$$\therefore \frac{5}{6}x \times \frac{6}{5} = 15 \times \frac{6}{5}$$

$$\therefore x = 18$$

$$7 \quad \therefore 8 - 2x - 8 = -2 - 8$$

$$\therefore -2x = -10$$

$$\therefore \frac{-2x}{2} = \frac{-10}{2}$$

$$\therefore x = 5$$

$$8 \quad \therefore 2 - 5x - 2 = 0 - 2$$

$$\therefore -5x = -2$$

$$\therefore -5x \times -\frac{1}{5} = -2 \times -\frac{1}{5}$$

$$\therefore x = \frac{2}{5}$$

$$9 \quad \therefore 2x + 3x + 25 = 5$$

$$\therefore 5x + 25 - 25 = 5 - 25$$

$$\therefore 5x = -20$$

$$\therefore \frac{5x}{5} = \frac{-20}{5}$$

$$\therefore x = -4$$

$$10 \quad \therefore 6x - 2x + 7 = 4$$

$$\therefore 4x + 7 - 7 = 4 - 7$$

$$\therefore 4x = -3$$

$$\therefore \frac{4x}{4} = \frac{-3}{4}$$

where  $-\frac{3}{4}$  is impossible in  $\mathbb{Z}$

$\therefore$  The equation has no solution in  $\mathbb{Z}$

12

$$1 \quad 2x - 6 = 4$$

$$\therefore 2x = 10$$

$$x = 5$$

$$2 \quad \therefore 3x + 10x - 6 = 7$$

$$\therefore 13x = 13 \quad \therefore x = 1$$

$$3 \quad \therefore 7x - 14 - 3x - 3 = 3$$

$$\therefore 4x - 17 = 3$$

$$\therefore 4x = 20$$

$$\therefore x = 5$$

$$4 \quad \therefore 3x + 6 + 7x - 7 = 12$$

$$\therefore 10x - 1 = 12$$

$$10x = 13$$

$$\therefore x = 1.3$$

$$5 \quad 4x - 4 - x - 3 = 0$$

$$\therefore 3x - 7 = 0$$

$$\therefore 3x = 7$$

$$\therefore x = \frac{7}{3}$$

$$6 \quad \therefore 5x - 10 + 2x + 8 = -16$$

$$\therefore 7x - 2 = -16$$

$$\therefore 7x = -14$$

$$\therefore x = -2$$

$$7 \quad \therefore 2x - 6 + 3x - 6 - 4x = -3$$

$$\therefore x - 12 = -3$$

$$\therefore x = 9$$

$$8 \quad \therefore 3y + 6y + 18 - 8y + 16 = 60$$

$$\therefore y + 34 = 60$$

$$\therefore y = 26$$

13

$$1 \quad \therefore 2x - x = 9 - 5$$

$$\therefore x = 4$$

$$\therefore \text{The S.S.} = \{4\}$$

$$2 \quad 5x - 2x = 11 \div 4$$

$$\therefore 3x = 15$$

$$\therefore x = 5$$

$$\therefore \text{The S.S.} = \{5\}$$

$$3 \quad \therefore x + 3x = 18 - 3$$

$$\therefore 4x = 15$$

$$\therefore x = \frac{15}{4}$$

$$\therefore \text{The S.S.} = \{\frac{15}{4}\}$$

$$4 \quad 3x + 5x = 30 - 6$$

$$\therefore 8x = 24$$

$$\therefore x = 3$$

$$\therefore \text{The S.S.} = \{3\}$$

$$5 \quad 4x + 4 = 2x - 2$$

$$\therefore 4x - 2x = -2 - 4$$

$$\therefore 2x = -6$$

$$\therefore x = -3$$

$$\therefore \text{The S.S.} = \{-3\}$$

$$6 \quad 3x - 6 = 5x - 10$$

$$\therefore -6 + 10 = 5x - 3x$$

$$\therefore 4 = 2x$$

$$\therefore x = 2$$

$$\therefore \text{The S.S.} = \{2\}$$

$$7 \quad 6a - 2 = 6 - 2a$$

$$\therefore 6a + 2a = 6 + 2$$

$$\therefore 8a = 8$$

$$\therefore a = 1$$

$$\therefore \text{The S.S.} = \{1\}$$

$$8 \quad 6x - 24 - 2x - 2 = x - 3$$

$$4x - 26 = x - 3$$

$$\therefore 4x - x = -3 + 26$$

$$\therefore 3x = 23$$

$$\therefore x = \frac{23}{3}$$

$$\therefore \text{The S.S.} = \{\frac{23}{3}\}$$

$$9 \quad 4(x + 1) = 3(x - 1)$$

$$\therefore 4x + 4 = 3x - 3$$

$$\therefore x = -7$$

$$\therefore \text{The S.S.} = \{-7\}$$

## Algebra and Statistics

10  $5(1 - 2x) = 3(4 + 4x)$

$5 - 10x = 12 + 12x$

$\therefore 5 - 12 = 12x + 10x \quad -7 = 22x$

$\therefore x = -\frac{7}{22}$  The S.S. =  $\{-\frac{7}{22}\}$

5	1 2	8 12	3 10	4 14
	8 2	8 9	7 $1\frac{1}{3}$	(8) $(x-5)$ years
	8 $(y+4)$ years		10 $(x-5)$ years	
	11 $(x-6)$ years		18 $\frac{1}{3}x$	(13) $6x$
	14 $5-x$		15 $x+2$	

11	1) (a)	2 (d)	3 (c)	4 (c)
	(5) (a)	6 (a)	7 (c)	8 (b)

7 Since the sum of measures of the angles of the triangle =  $180^\circ$

1  $x + 2x + 2x + 5^\circ = 180^\circ$

$5x + 5^\circ = 180^\circ \quad 5x = 175^\circ$

$x = 35^\circ$

$\therefore$  The measures of the angles of the triangle are  $35^\circ, 70^\circ, 75^\circ$

2  $y + 3y + 8y = 180^\circ \quad \therefore 12y = 180^\circ$

$\therefore y = 15^\circ \quad \therefore 3y = 45^\circ, 8y = 120^\circ$

$\therefore$  The measures of the angles of the triangle are  $15^\circ, 45^\circ, 120^\circ$

3  $a + \frac{1}{2}a + 90^\circ = 180^\circ \quad \frac{3}{2}a + \frac{1}{2}a = 90^\circ$

$\therefore \frac{3}{2}a = 90^\circ$

$a = 90^\circ \times \frac{2}{3} = 60^\circ, \frac{1}{2}a = 30^\circ$

$\therefore$  The measures of the angles of the triangle are  $75^\circ, 15^\circ, 90^\circ$

8  $\because C \in \overline{AB}$  then  $x + 4x + 3x + 20^\circ = 180^\circ$

$\therefore 8x + 20^\circ = 180^\circ \quad \therefore 8x = 160^\circ \quad x = 20^\circ$

$\therefore m(\angle DCE) = 4x = 4 \times 20^\circ = 80^\circ$

9 1  $\overline{DE} \parallel \overline{BC}, \overline{DB}$  is a transversal to them

$\therefore m(\angle DBC) + m(\angle D) = 180^\circ$

(Two interior angles in the same side of the transversal)

$\therefore m(\angle DBC) = 180^\circ - 110^\circ = 70^\circ$

$\therefore \overline{AG} \parallel \overline{BD}, \overline{AB}$  is the transversal

$\therefore m(\angle A) = m(\angle DBC)$  (corresponding angles)

$3x + 10^\circ = 70^\circ \quad \therefore 3x = 60^\circ$

$x = \frac{60^\circ}{3} = 20^\circ$

2  $\therefore \overline{DE} \parallel \overline{AB}, \overline{DB}$  is a transversal to them

$m(\angle D) + m(\angle DBA) = 180^\circ$

By the same way,  $m(\angle A) + m(\angle DBA) = 180^\circ$

$\therefore m(\angle A) = m(\angle D) \quad \therefore x + 30^\circ = 2x - 30^\circ$

$\therefore 2x - x = 30^\circ + 30^\circ \quad \therefore x = 60^\circ$

10 Let the length of the rectangle =  $x$  metres

the width of the rectangle =  $(x - 4)$  metres

$\therefore$  the perimeter =  $(\text{length} + \text{width}) \times 2$

$\therefore 68 = (x + x - 4) \times 2 \quad \therefore 34 = 2x - 4$

$2x = 38 \quad \therefore x = 38 \div 2 = 19$  metres

the dimensions of the rectangle are 19 metres + 15 metres

11 Let the width of the rectangle =  $x$  cm

$\therefore$  the length of the rectangle =  $(2x - 4)$  cm

$\therefore$  the perimeter of the rectangle = the perimeter of the square

$\therefore (\text{length} + \text{width}) \times 2 = \text{side length} \times 4$

$(x + 2x - 4) \times 2 = 7 \times 4$

$(3x - 4) \times 2 = 28 \quad \therefore 3x - 4 = 14$

$\therefore 3x = 18 \quad \therefore x = 18 \div 3 = 6$  cm

$\therefore$  the width of the rectangle = 6 cm

$\therefore$  the length of the rectangle =  $2 \times 6 - 4 = 8$  cm

12 Let the width of the rectangle =  $x$  cm.

$\therefore$  the length of the rectangle =  $2x$  cm

$2x - 5 = x + 6 \quad 2x - x = 5 + 6$

$\therefore x = 11$

$\therefore$  the width of the rectangle = 11 cm

and the length of the rectangle = 22 cm.

the area of the rectangle =  $11 \times 22 = 242 \text{ cm}^2$

13  $7x - 2x = 25 \quad 5x = 25$

$x = \frac{25}{5} = 5$

$\therefore$  The two numbers are : 10 , 35



- 14 Let one of the two numbers be  $x$   
the other number =  $2x$   
 $\therefore x + 2x = 108 \quad \therefore 3x = 108$   
 $x = \frac{108}{3} = 36$   
 $\therefore$  the two numbers are  $36 + 72$

- 15 Let the great number be  $x$   
 $\therefore$  the small number =  $x - 5$   
 $x + x - 5 = 21 \quad 2x - 5 = 21$   
 $2x = 26 \quad x = \frac{26}{2} = 13$   
The two numbers are  $13 + 8$

- 16 Let the number be  $x$  its triple =  $3x$   
 $3x + x = 32 \quad \therefore 4x = 32$   
 $x = \frac{32}{4} = 8$  The number is 8

- 17 Let the number be  $x$  its triple =  $3x$   
 $3x - 9 = 6 \quad 3x = 15$   
 $\therefore x = \frac{15}{3} = 5$  The number is 5

- 18 Let the small number be  $x$   
the middle number =  $x + 1$   
the great number =  $x + 2$   
 $x + x + 1 + x + 2 = 213 \quad \therefore 3x + 3 = 213$   
 $\therefore 3x = 210 \quad \therefore x = \frac{210}{3} = 70$   
 $\therefore$  the numbers are  $70 + 71 + 72$

- 19 Let the three numbers be  $x, x + 2, x + 4$   
 $x + x + 2 + x + 4 = 966 \quad 3x + 6 = 966$   
 $\therefore 3x = 960 \quad \therefore x = 320$   
 $\therefore$  the numbers are  $320 + 322 + 324$

- 20 Let the three numbers be  $x, x + 2, x + 4$   
 $x + x + 2 + x + 4 = 357 \quad 3x + 6 = 357$   
 $\therefore 3x = 351 \quad \therefore x = \frac{351}{3} = 117$   
 $\therefore$  the numbers are  $117 + 119 + 121$

- 21 Let the age of the son be  $x$  years  
 $\therefore$  the age of the father =  $3x$   
after 2 years the age of the son =  $(x + 2)$  years  
the age of the father =  $(3x + 2)$  years  
 $x + 2 + 3x + 2 = 52 \quad 4x + 4 = 52$

- $4x = 48 \quad x = \frac{48}{4} = 12$   
the age of the son = 12 years  
the age of the father = 36 years

- 22 Let the age of Bassim be  $x$  years  
Amgad's age =  $x + 2$  years  
Ayman's age =  $x - 6$  years  
 $x + x + 2 + x - 6 = 89 \quad 3x - 4 = 89$   
 $\therefore 3x = 93 \quad \therefore x = \frac{93}{3} = 31$  years  
 $\therefore$  the age of Bassim = 31 years  
the age of Amgad =  $31 + 2 = 33$  years  
the age of Ayman =  $31 - 6 = 25$  years

- 23 Let the price of one metre of silk =  $x$  pounds  
 $\therefore$  the price of one metre of wool =  $(x + 2)$  pounds  
 $3(x + 2) + 4x = 671 \quad 3x + 6 + 4x = 671$   
 $\therefore 7x = 665 \quad \therefore x = 95$   
 $\therefore$  the price of one metre of silk is 95 pounds and  
the price of one metre of wool is 97 pounds

- 24 1  $5 + 1 = 6 \quad 6 = 6$   
 $x = 1$  The S.S. =  $\{1\}$   
2  $\frac{x}{10} + \frac{x}{5} = -\frac{1}{3} + \frac{3}{4} \quad \frac{x}{10} + \frac{2x}{10} = \frac{2}{3}$   
 $\frac{3x}{10} = \frac{2}{3} \quad 3x = \frac{2 \times 10}{3}$   
 $3x = 4 \quad x = \frac{4}{3}$   
The S.S. =  $\{\frac{4}{3}\}$

- 25 1  $\therefore x^2 + 6x + 9 - (x^2 - 4x + 4) = 15$   
 $x^2 + 6x + 9 - x^2 + 4x - 4 = 15$   
 $\therefore 10x + 5 = 15 \quad \therefore 10x = 10$   
 $\therefore x = 1 \quad \therefore$  The S.S. =  $\{1\}$   
2  $\therefore 4x^2 + 4x - 3 - (4x^2 - 4x + 1) = 14$   
 $\therefore 4x^2 + 4x - 3 - 4x^2 + 4x - 1 = 14$   
 $\therefore 8x - 4 = 14 \quad 8x = 18$   
 $x = \frac{18}{8} = \frac{9}{4} \quad \therefore$  The S.S. =  $\{\frac{9}{4}\}$

- 26  $\therefore 12x + 3 = 39 \quad \therefore 12x = 36 \quad \therefore x = 3$   
 $\therefore$  The S.S. =  $\{3\}$   
 $\therefore x = 3$  satisfies the equation  $a - x = 12$   
 $3 - 12 = a \quad \therefore 3 - a = 12 \quad \therefore 2a = 12$   
 $a = 6$


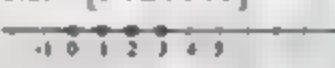


## Algebra and Statistics

27.  $a + 1$  is a solution for the equation  
 $\therefore X = a + 1$  satisfies the equation.  
 $\therefore (a + 1 + a)(a + 1 - a) = (a + 1)^2 - a(a + 1) + 3$   
 $\therefore 2a + 1 = a^2 + 2a + 1 - a^2 - a + 3$   
 $\therefore 2a + 1 = a + 4 \quad \therefore 2a - a = 4 - 1 \quad \therefore a = 3$

28. Let the age of the boy who was born in 1980 be  $X$   
 $\therefore$  the ages of the two other boys are  $X - 4$  &  $X - 6$   
 $\therefore X + X - 4 + X - 6 = 41$   
 $3X - 10 = 41 \quad \therefore 3X = 51 \quad \therefore X = \frac{51}{3} = 17$   
the year in which the sum of their ages became  
41 years =  $1980 + 17 = 1997$

## Answers of Exercise 8

1. 5      2. 4      3. 7      4. -9  
5. 1.5      6. -0.6      7.  $2\frac{1}{2}$       8.  $-\frac{1}{3}$
2.  $X + 3 - 3 \leq 6 - 3 \quad \therefore X \leq 3$   
1) The S.S. =  $\{3, 2, 1, 0, \dots\}$   
  
2) The S.S. =  $\{3, 2, 1, 0\}$   

3. 1.  $X + 2 - 2 > 5 - 2 \quad \therefore X > 3$   
 $\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X > 3\}$   
2.  $X + 4 - 4 > 1 - 4 \quad \therefore X > -3$   
 $\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X > -3\}$   
3.  $y - 5 + 5 > 7 + 5 \quad \therefore y > 12$   
 $\therefore$  The S.S. =  $\{y : y \in \mathbb{Q}, y > 12\}$   
4.  $19 - 14 < y + 14 - 14 \quad \therefore 5 < y$   
 $\therefore$  The S.S. =  $\{y : y \in \mathbb{Q}, y > 5\}$   
5.  $-1 + 3 \geq X - 3 + 3 \quad \therefore 2 \geq X$   
 $\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X \leq 2\}$   
6.  $-5\frac{1}{2} - 1\frac{1}{4} > a + 1\frac{1}{4} - 1\frac{1}{4} \quad \therefore -6\frac{3}{4} > a$   
 $\therefore$  The S.S. =  $\{a : a \in \mathbb{Q}, a < -6\frac{3}{4}\}$   
7.  $-2X - \frac{1}{2} > 12X - \frac{1}{2} \quad \therefore X > -6$   
 $\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X > -6\}$

8.  $\frac{2}{3}X \times \frac{3}{2} \geq 1 \times \frac{3}{2} \quad \therefore X \geq \frac{3}{2}$   
 $\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X \geq \frac{3}{2}\}$   
9.  $-\frac{1}{4}X \times -4 \geq \frac{1}{4} \times -4 \quad \therefore X \geq -1$   
 $\therefore$  The S.S. =  $\{X : X \in \mathbb{Q}, X \geq -1\}$

1.  $3X - 2 + 2 < 1 + 2 \quad \therefore 3X < 3$   
 $\therefore 3X \times \frac{1}{3} < 3 \times \frac{1}{3} \quad \therefore X < 1$   
2.  $2X + 3 - 3 < 9 - 3 \quad \therefore 2X < 6$   
 $\therefore 2X \times \frac{1}{2} < 6 \times \frac{1}{2} \quad \therefore X < 3$   
3.  $4X + 2 - 2 \geq -10 - 2 \quad \therefore 4X \geq -12$   
 $\therefore 4X \times \frac{1}{4} \geq -12 \times \frac{1}{4} \quad \therefore X \geq -3$   
4.  $3X - 2 + 2 \geq 5 + 2 \quad \therefore 3X \geq 7$   
 $\therefore 3X \times \frac{1}{3} \geq 7 \times \frac{1}{3} \quad \therefore X \geq \frac{7}{3}$   
5.  $3X - 9 + 9 < 0 + 9 \quad \therefore 3X < 9$   
 $\therefore 3X \times \frac{1}{3} < 9 \times \frac{1}{3} \quad \therefore X < 3$   
6.  $1 + 2X - 1 \leq -3 - 1 \quad \therefore 2X \leq -4$   
 $2X \times \frac{1}{2} \leq -4 \times \frac{1}{2} \quad \therefore X \leq -2$   
7.  $9 - 6X - 9 < 15 - 9 \quad \therefore -6X < 6$   
 $-6X \times -\frac{1}{6} > 6 \times -\frac{1}{6} \quad \therefore X > -1$   
8.  $2 - 3X - 2 \leq 4 - 2 \quad \therefore -3X \leq 2$   
 $\therefore -3X \times -\frac{1}{3} \geq 2 \times -\frac{1}{3} \quad \therefore X \geq -\frac{2}{3}$   
9.  $\frac{3X - 2}{5} \times 5 \geq \frac{1}{2} \times 5 \quad \therefore 3X - 2 \geq \frac{5}{2}$   
 $\therefore 3X - 2 + 2 \geq \frac{5}{2} + 2 \quad \therefore 3X \geq \frac{9}{2}$   
 $\therefore 3X \times \frac{1}{3} \geq \frac{9}{2} \times \frac{1}{3} \quad \therefore X \geq \frac{3}{2}$   
10.  $5X + 1 - 1 \leq 29 - 1 \quad \therefore 5X \leq 28$   
 $\therefore 5X \times \frac{1}{5} \leq 28 \times \frac{1}{5} \quad \therefore X \leq \frac{28}{5}$   
11.  $4n - 2n + 2 \geq 0 \quad \therefore 2n + 2 - 2 \geq 0 - 2$   
 $\therefore 2n \geq -2 \quad \therefore 2n \times \frac{1}{2} \geq -2 \times \frac{1}{2}$   
 $\therefore n \geq -1$   
12.  $-3m + 6m - 24 > 9 \quad \therefore 3m - 24 + 24 > 9 + 24$   
 $\therefore 3m \times \frac{1}{3} > 33 \times \frac{1}{3} \quad \therefore m > 11$

5

1  $6d - 5d \leq -3 - 1$   $\therefore d \leq -4$

2  $6X - 5X \geq 14 - 2$   $\therefore X \geq 12$

3  $-2 + 8 < 5X - 3X$   $6 < 2X$

$3 < X$   $\therefore X > 3$

4  $8 \leq 5X + 2X$   $\therefore 8 \leq 7X$

$X \geq \frac{8}{7}$

5  $5X + 1 \geq 2X + 4$   $\therefore 5X - 2X \geq 4 - 1$

$3X \geq 3$   $\therefore X \geq 1$

6  $3X + 6 < -X + 4$   $\therefore 3X + X < 4 - 6$

$4X < -2$   $\therefore X < -\frac{1}{2}$

7  $3X + 6 \geq -2X - 2$   $\therefore 3X + 2X \geq -2 - 6$

$5X \geq -8$   $\therefore X \geq -\frac{8}{5}$

8  $2 - 3X + 15 \geq X + 7$   $\therefore 17 - 7 \geq X + 3X$

$10 \geq 4X$   $\therefore X \leq \frac{10}{4}$

$X \leq \frac{5}{2}$

9  $21y - 1 \leq 20y - 1$   $\therefore 21y - 20y \leq -1 + 1$   $\therefore y \leq 0$

10  $3 - 1 \leq 2X - \frac{X}{2}$   $2 \leq \frac{3}{2}X$

$X \geq \frac{4}{3}$

6

1  $9 - 1 \leq 4X + 1 - 1 \leq 17 - 1$   $\therefore 8 \leq 4X \leq 16$

$2 \leq X \leq 4$

$\therefore \text{The S.S.} = \{2, 3, 4\}$

2  $2 \leq 3X + 2 - 2 < 12 - 2$

$7 \times \frac{1}{3} \leq 3X \times \frac{1}{3} < 10 \times \frac{1}{3}$   $\frac{7}{3} \leq X < \frac{10}{3}$

$\therefore \text{The S.S.} = \{X \in \mathbb{Q} : \frac{7}{3} \leq X < \frac{10}{3}\}$

3  $9 - 6 > X + 6 - 6 > 2 - 6$

$3 > X > -4$   $\therefore \text{The S.S.} = \{0, 1, 2\}$

4  $1 >$   $2 <$   $3 \geq$   $4 <$   $5 >$

5  $a > -5$   $7 <$   $8 >$   $9 >$

6 1 (b) 2 (d) 3 (d) 4 (d) 5 (b)

7 (d) 8 (b) 9 (c) 10 (c)

8

When  $a = 4$  and  $b = 3$ , then  $a > b$  $c = 8$  and  $d = 2$ , then  $c > d$  then  $a - c = 4 - 8 = -4$ and  $b - d = 3 - 2 = 1$  $\therefore a - c < b - d$ 

So we see that

If  $a > b$  and  $c > d$  it is not always correct that  $a - c > b - d$ 

10

1 (✓)

2 (x)

For example

When  $X = -3$  and  $y = -5$ , then  $-3 > -5$  but  $-3 < 0$ 

3 (✓)

4 (x) For example

When  $y = \frac{1}{2}$ , then  $y^2 = \frac{1}{4}$  but  $\frac{1}{4} < \frac{1}{2}$ 

5 (x) For example:

When  $X = 4$ ,  $y = -2$ , then  $4 \times -2 < 0$   $-8 < 0$ 

6 (x) For example:

When  $X = -4$ ,  $y = -9$ , then $X + y = -4 + (-9) = -13$   $-13 < -9$ 

7 (x) For example:

When  $y = -1$ ,  $X = 5$ , then  $y^2 = 1$  but  $1 < 5$ 

8 (x) For example

When  $X = 4$ ,  $y = -2$ , then  $y^2 = 4$ ,  $Xy = -8$ but  $4 > -8$ 

9 (x) For example:

When  $X = -2$ ,  $y = -4$ , then  $Xy = -2 \times -4 = 8$ , $X^2 = 4$ but  $8 > 4$ 

10 (x) For example

When  $X = 2$ ,  $y = 1$ , then  $X^3 = 8$ ,  $y^2 = 1$ but  $8 > 1$ 

11

Let the number of shirts be  $X$ 

1  $40X + 70 \leq 200$   $\therefore 40X \leq 200 - 70$

2  $40X \leq 130$   $\therefore X \leq 3.25$

 $\therefore$  The greatest number of shirts can be bought is 3 shirts

## Algebra and Statistics

12

$$a \leq 3X - 5 \leq b \quad a + 5 \leq 3X \leq b + 5$$

$$\frac{a+5}{3} \leq X \leq \frac{b+5}{3} \quad 2 \leq X \leq 5$$

$$\frac{a+5}{3} = 2 \quad a + 5 = 6 \quad a = 1$$

$$\frac{b+5}{3} = 5 \quad b + 5 = 15 \quad b = 10$$

13

- 1 The greatest possible value of the expression  $X + y$  is when  $X = 5, y = 7 \rightarrow$  the value  $= 5 + 7 = 12$
- 2 The greatest possible value of the expression  $y - X$  is when  $y = 7, X = -4$   
The value  $= 7 - (-4) = 11$
- 3 The smallest possible value of the expression  $X \cdot y$  is when  $X = -4, y = 7$   
The value  $= -4 \times 7 = -28$
- 4 The smallest possible value of the expression  $X^2 + y^2$  is when  $X = 0, y = 0$   
The value  $= 0^2 + 0^2 = 0$

## Answers of exams on the second part of unit one

## Model - 1

- 1 1 (b) 4 (c) 3 (d)  
4 (b) 5 (a) 6 (d)

$$1 \quad 1. X - 9 \quad 2. \frac{7}{3} |ab| \quad 3. 143$$

$$4. \{X : X \in \mathbb{Q}, X > 0\} \quad 5. 14$$

- 3 (a) The S.S.  $= \{X : X \in \mathbb{Q}, X \leq 1\}$   
(b)  $\frac{2}{3}$

- 4 (a) 1) 23 2) 40  
(b) 4 cm, 8 cm

- 5 (a) The S.S.  $= \{\frac{15}{4}\}$   
(b) The S.S.  $= \{X : X \in \mathbb{Q}, \frac{7}{3} \leq X < \frac{10}{3}\}$

## Model - 2

- 1 (c) 2 (b) 3 (a)  
4 (a) 5 (b) 6 (a)

- 2  
1 zero 2 0.03 3 3 4 -5 5  $3 \cdot x$

- 3 (a) 22  
(b)  $\frac{2}{3}$

- 4 (a) The S.S.  $= \{\frac{5}{3}\}$   
(b) The S.S.  $= \{X : X \in \mathbb{Q}, X \leq 5\}$

- 5 (a) The S.S.  $= \{X : X \in \mathbb{Q}, X \geq 6\}$   
(b) The number is 7

## Answers of Unit Two

## Answers of Exercise 9

Answer by yourself.

## Answers of Exercise 10

First : Problems on experimental probability

Answer by yourself.

Second : Problems on theoretical probability

1

- 1  $\frac{2}{3}$  2  $\frac{1}{3}$  3  $\frac{1}{2}$  4  $\frac{1}{6}$  5 zero  
6 1 7  $\frac{1}{2}$  8  $\frac{1}{6}$  9  $\frac{1}{6}$  10  $\frac{1}{3}$

2

- 1 zero + 1 2  $\frac{1}{2}$  3  $\frac{1}{2}$  4  $\frac{1}{3}$  5 zero  
6  $\frac{2}{3}$  7  $\frac{1}{2} + \frac{1}{2}$  8  $\frac{3}{8}$  9  $\frac{1}{3}$  10 600

3

- 1 (d) 2 (b) 3 (a) 4 (b) 5 (b)  
6 (c) 7 (b) 8 (d) 9 (b) 10 (c)

4

- 1  $\therefore$  The numbers from 1 to 25 and divisible by 5 are  
5, 10, 15, 20, 25 and their number = 5  
 $\therefore$  The probability =  $\frac{5}{25} = \frac{1}{5}$   
2  $\therefore$  The numbers from 1 to 25 and more than or  
equal to 20 are 20, 21, 22, 23, 24, 25 its  
number is 6  
 $\therefore$  The probability =  $\frac{6}{25}$   
3  $\therefore$  The numbers from 1 to 25 and each of them is  
a perfect square are 1, 4, 9, 16, 25 and their  
number is 5  
 $\therefore$  The probability =  $\frac{5}{25} = \frac{1}{5}$

5

S = {1, 2, 3, 4, 5, 6, 7, 8}

- 1 The probability of getting an even number  
=  $\frac{4}{8} = \frac{1}{2}$   
2 The probability of getting an odd number  
=  $\frac{4}{8} = \frac{1}{2}$

- 3 The probability of getting a number greater than  
or equal to 6 =  $\frac{3}{8}$

- 4 The probability of getting a number divisible by 3  
=  $\frac{2}{8} = \frac{1}{4}$

6

- 1 The probability that the letter S =  $\frac{1}{3}$   
2 The probability that the letter E =  $\frac{1}{3}$   
3 The probability that the letter R = zero

7

The total number of balls =  $5 + 3 + 2 = 10$ 

- 1 The probability that the ball is yellow =  $\frac{3}{10} = 0.3$   
2 The probability that the ball is yellow or red =  $\frac{3+5}{10}$   
=  $\frac{8}{10} = 0.8$   
3 The probability that the ball is not yellow =  $\frac{5+2}{10}$   
=  $\frac{7}{10} = 0.7$

8

- 1 The probability that it shows an odd number  
=  $\frac{5}{10} = \frac{1}{2}$   
2 The probability that it shows a prime number  
=  $\frac{4}{10} = \frac{2}{5}$   
3 The probability that it shows an even number  
=  $\frac{5}{10} = \frac{1}{2}$   
4 The probability that it shows an odd number  
greater than 3 =  $\frac{3}{10}$

9

- 1 The probability of appearing an even number  
less than or equal to 4 =  $\frac{2}{6} = \frac{1}{3}$   
2 The probability of appearing a number between  
0 and 10 =  $\frac{6}{6} = 1$   
3 The probability of appearing a number divisible  
by 7 =  $\frac{0}{6} = \text{zero}$   
4 The probability of appearing a number not divisible  
by 2 =  $\frac{3}{6} = \frac{1}{2}$

10

S = {1, 2, 3, 4, 5, 6}

- 1 The probability of getting a number greater than 6  
=  $\frac{0}{6} = \text{zero}$ .



## Algebra and Statistics

- 11 The probability of getting a number satisfying the inequality  $1 \leq X \leq 6 = \frac{6}{6} = 1$   
 12 The probability of getting a number satisfying the inequality  $2 < X < 4 = \frac{1}{6}$

11

- 1 The probability that the card carries a number whose tens digit is even =  $\frac{2}{4} = \frac{1}{2}$   
 2 The probability that the card carries a number whose units digit is odd =  $\frac{2}{4} = \frac{1}{2}$   
 3 The probability that the card carries a number multiple of 4 =  $\frac{2}{4} = \frac{1}{2}$

12

1.  $S = \{1, 2, 3\}$   
 2 The probability that the appeared number on the upper face is 2 =  $\frac{1}{3}$   
 3 The probability that the appeared number is odd =  $\frac{2}{3}$

- 13 The number of red marbles =  $\frac{2}{3} \times 30 = 12$  marbles

14

- The probability of drawing a red ball =  $\frac{1}{4}$   
 The probability of drawing a blue ball =  $\frac{3}{4}$   
 The number of blue balls =  $\frac{3}{4} \times 80 = 60$  balls

15

- $S = \{22, 32, 52, 33, 23, 33, 55, 25, 35\}$   
 1 The probability that the tens digit is odd =  $\frac{6}{9} = \frac{2}{3}$   
 2 The probability that the units digit is odd =  $\frac{6}{9} = \frac{2}{3}$   
 3 The probability that the sum of the two digits 7 =  $\frac{2}{9}$   
 4 The probability that the product of the two digits 15 =  $\frac{2}{9}$

16

- The number of red marbles =  $22 - 12 = 10$  and after drawing two red marbles the rest marbles will be 20 and the red marbles is 8  
 The probability that the drawn marble is black =  $\frac{12}{20} = \frac{3}{5}$

17

- The number of girls = 20 , The number of boys = 30  
 The probability that the student is a boy =  $\frac{30}{50} = \frac{3}{5}$

18

- 19 (1) (c) (2) (c) (3) (b) (4) (d)  
 (5) (c) (6) (b) (7) (b)

19

- (1) (a)  $\frac{1}{8}$  (b)  $\frac{3}{8}$  (c)  $\frac{4}{8} = \frac{1}{2}$   
 (2) The probability that the pointer does not stop at the red colour = The probability that the pointer stops at green or yellow =  $\frac{7}{8}$

20

- The probability that the pointer stops at yellow colour =  $1 - (\frac{1}{8} + \frac{1}{4} + \frac{3}{8}) = 1 - \frac{6}{8} = \frac{2}{8}$   
 $\therefore$  The probability that the pointer stops at the yellow or red colour =  $\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

21

- (1) The probability that the student succeeded in math =  $\frac{30}{40} = \frac{3}{4}$   
 (2) The probability that the student succeeded in science =  $\frac{24}{40} = \frac{3}{5}$   
 (3) The probability that the student failed in science =  $1 - \frac{3}{5} = \frac{2}{5}$   
 (4)  $\therefore$  The number of succeeded students in both math and science is 20 students.  
 $\therefore$  The number of students who succeeded in math only =  $30 - 20 = 10$  students  
 $\therefore$  The number of students who succeeded in science only =  $24 - 20 = 4$  students  
 $\therefore$  The number of students who failed in both math and science =  $40 - (20 + 10 + 4) = 6$  students  
 $\therefore$  The probability that the student failed in both math and science =  $\frac{6}{40} = \frac{3}{20}$

22

- $\therefore$  The probability that the first player scores a goal =  $\frac{18}{21} = 0.86$   
 The probability that the second player scores a goal =  $\frac{25}{32} = 0.78$   
 $0.86 > 0.78$   
 $\therefore$  The best is choosing the first player because his probability is the greater

23

- 1 No because the product of an odd number  $\times$  an even number = an even number  
i.e. The probability of getting an even number is the greater
- Souad because she wins when the result is an even number and its probability is the greater

24

The probability of shooting the shaded part

$$= \frac{\text{The area of the shaded rectangle}}{\text{The area of the external rectangle}} = \frac{5 \times 10}{20 \times 10} = \frac{1}{4}$$

25

1.  $\therefore$  The area of the part E =  $\frac{1}{2} \times \frac{1}{4}$  the area of the rectangle =  $\frac{1}{8}$  the area of the rectangle  
 $\therefore$  The probability of shooting the part E =  $\frac{1}{8}$
- $\therefore$  The area of one of the parts A + B + C =  $\frac{1}{4} \times \frac{1}{2}$  the area of the rectangle =  $\frac{1}{8}$  the area of the rectangle  
 $\therefore$  The area of the part formed from A + B + C together =  $\frac{3}{8}$  the area of the rectangle.  
 $\therefore$  The probability of shooting the part formed from A + B + C together =  $\frac{3}{8}$

26

- $\therefore$  The probability of drawing a red ball =  $\frac{2}{3}$   
 $\therefore$  The probability of drawing a white ball =  $1 - \frac{2}{3} = \frac{1}{3}$   
 $\therefore$  The total number of balls =  $3 \times 5 = 15$  balls

27

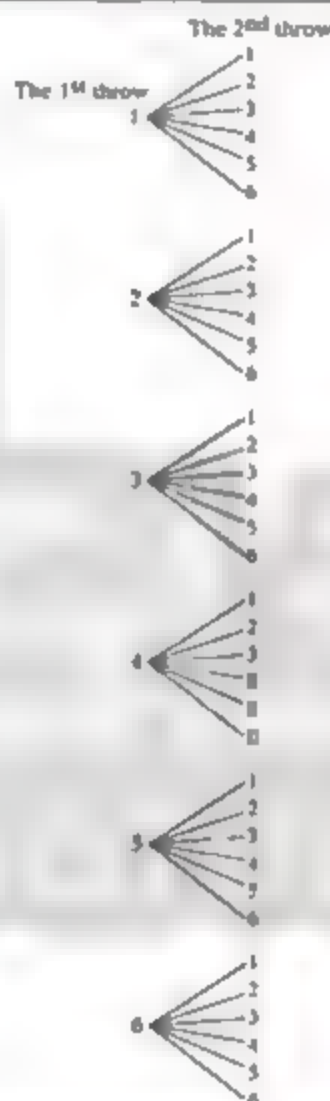
The probability that the drawn card carries a number less than or equal to 8

$$= 1 - \frac{1}{3} = \frac{2}{3}$$

$$\therefore \text{The number of cards} = 8 \times \frac{3}{2} = 12 \text{ cards}$$

$$\therefore n = 12$$

28



- $\therefore$  The probability of appearance of the number 3 in the two times =  $\frac{1}{36}$

## Algebra and Statistics

## Answer of exam on unit two

1 1 (a) 2 (d) 3 (d)

4 (b) 5 (c) 6 (c)

2 1 zero 2  $\frac{1}{2}$  3  $\frac{3}{7}$ 4  $\frac{1}{3}$  5  $\frac{1}{6}$ 3 1  $\frac{1}{3}$  2  $\frac{8}{15}$  3 1 4  $\frac{2}{3}$ 4 1  $\frac{1}{3}$  2  $\frac{1}{6}$  3  $\frac{5}{6}$ 

4 zero 5 1

5 1  $\frac{2}{3}$  2  $\frac{1}{3}$  3  $\frac{2}{3}$ 

## Answers of accumulative basic skills

1 1 (c) 2 (b) 3 (a) 4 (c)

5 (b) 6 (a) 7 (b) 8 (a)

9 (c) 10 (c) 11 (d) 12 (d)

13 (c) 14 (a) 15 (c)

2 1 30 398 2 -2 3  $\frac{1}{9}$  4 115  $\frac{9x}{8}$  6 90 7 50 8  $\frac{5}{6}$ 

9 10 10 40 11 25 12 15

13 1 14 3 15 15

Guide  
Answers

of Geometry and  
Measurement Exercises





## Geometry and Measurement

## Answers of Unit Three

## Answers of Exercise 1

- 1 ① Given ②  $A \in \overline{CE}$  ③ Given  
④ Corresponding angles, ⑤ Alternate angles

2

Given :

$$m(\angle AMB) = 50^\circ, m(\angle EMD) = 80^\circ,$$

$$m(\angle CMD) = 65^\circ, \overline{MC} \text{ bisects } \angle BMD$$

R.T.F. :  $m(\angle AME)$ 

Proof :

$$\because \overline{MC} \text{ bisects } \angle BMD \text{ (given)}$$

$$\therefore m(\angle BMC) = m(\angle CMD) = 65^\circ$$

$$\therefore m(\angle AMB) + m(\angle BMC) + m(\angle CMD)$$

$$+ m(\angle DME) + m(\angle AME) = 360^\circ$$

$$\therefore m(\angle AME) = 360^\circ - 260^\circ = 100^\circ \quad (\text{The req.})$$

3

$$\text{Given : } \overline{AC} \cap \overline{BD} = \{M\}, m(\angle BMC) = 120^\circ,$$

 $\overline{ME}$  bisects  $\angle AMD$ R.T.F. :  $m(\angle EMC)$ 

Proof :

$$\because \overline{AC} \cap \overline{BD} = \{M\}$$

$$\therefore m(\angle BMC) = m(\angle AMD) \text{ (V.O.A.)}$$

$$\therefore m(\angle AMD) = 120^\circ,$$

 $\therefore \overline{ME}$  bisects  $\angle AMD$ 

$$\therefore m(\angle AME) = m(\angle EMD)$$

$$\therefore m(\angle EMD) = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore M \in \overline{BD}$$

$$\therefore m(\angle BMC) + m(\angle CMD) = 180^\circ$$

$$\therefore m(\angle DMC) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle EMC) = m(\angle EMD) + m(\angle DMC)$$

$$\therefore m(\angle EMC) = 60^\circ + 60^\circ = 120^\circ \quad (\text{The req.})$$

4

Given :

$$AB = AC, BD = CD$$

R.T.F. :  $\overline{AD}$  bisects  $\angle BAC$ Proof :  $\because$  In  $\triangle ADB, \triangle ADC$  :

$$AB = AC \text{ (given)}$$

$$BD = CD \text{ (given)}$$

 $\overline{AD}$  is a common side $\therefore \triangle ADB \cong \triangle ADC$  ; then we deduce that :

$$m(\angle BAD) = m(\angle CAD)$$

 $\therefore \overline{AD}$  bisects  $\angle BAC$ 

(Q.E.D)

5

$$\because m(\angle EBD) + m(\angle CBD) + m(\angle ABC)$$

$$+ m(\angle ABE) = 360^\circ$$

$$\therefore m(\angle EBD) + 35^\circ + 110^\circ + 140^\circ = 360^\circ$$

$$\therefore m(\angle EBD) = 360^\circ - 285^\circ = 75^\circ \quad (\text{The req.})$$

6

$$\therefore B \in \overline{FD}$$

$$\therefore m(\angle DBC) = 180^\circ - (30^\circ + 40^\circ) = 110^\circ \quad (\text{The req.})$$

7

$$\therefore B \in \overline{AC}$$

$$\therefore m(\angle ABE) + m(\angle CBE) = 180^\circ$$

$$\therefore m(\angle ABE) = 180^\circ - 116^\circ = 64^\circ$$

 $\therefore \overline{BD}$  bisects  $\angle ABE$ 

$$\therefore m(\angle ABD) = m(\angle DBE) = 64^\circ \div 2 = 32^\circ \quad (\text{The req.})$$

8

$$\therefore \overline{AC} \cap \overline{DE} = \{B\}$$

$$\therefore m(\angle CBE) = m(\angle ABD) = 40^\circ \text{ (V.O.A.)}$$

 $\overline{BE}$  bisects  $\angle CBF$ 

$$\therefore m(\angle FBE) + m(\angle EBC) = 80^\circ$$

$$\therefore m(\angle FBC) = 80^\circ$$

$$B \in \overline{AC}$$

$$m(\angle ABF) = 180^\circ - 80^\circ = 100^\circ \quad (\text{The req.})$$

9

$$\therefore m(\angle BEC) + m(\angle AED) + m(\angle AEB)$$

$$+ m(\angle CED) = 360^\circ$$

$$\therefore m(\angle BEC) + m(\angle AED) + 65^\circ + 85^\circ = 360^\circ$$

$$\therefore m(\angle BEC) + m(\angle AED)$$

$$= 360^\circ - (65^\circ + 85^\circ) = 210^\circ$$

$$\therefore m(\angle BEC) = m(\angle AED)$$

$$\therefore m(\angle BEC) = \frac{210^\circ}{2} = 105^\circ \quad (\text{First req.})$$

$$\therefore m(\angle AEB) + m(\angle BEC) = 65^\circ + 105^\circ$$

$$= 170^\circ \neq 180^\circ$$

 $\therefore A, E$  and  $C$  are not on the same straight line

(Second req.)

10

$$\therefore m(\angle ABC) = 180^\circ - 72^\circ = 108^\circ$$

(Two interior angles in the same side of the transversal)

$$2 \quad m(\angle ABC) = m(\angle BAD) = 57^\circ \text{ (Alternate angles)}$$

$$3 \quad m(\angle ABC) = m(\angle EAD) = 63^\circ$$

(Corresponding angles)

11

$$\therefore m(\angle EAC) + m(\angle BAC) + m(\angle EAB) = 360^\circ$$

$$\therefore m(\angle BAC) = 360^\circ - (130^\circ + 90^\circ) = 140^\circ \text{ (First req.)}$$

$\overline{AB} \parallel \overline{CD}$ ,  $\overline{AC}$  is a transversal to them

$$m(\angle C) + m(\angle CAB) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle C) = 180^\circ - 140^\circ = 40^\circ \text{ (Second req.)}$$

12

$\overline{AB} \parallel \overline{CD}$ ,  $\overline{AC}$  is a transversal to them

$$\therefore m(\angle ACD) + m(\angle A) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle ACD) = 180^\circ - 60^\circ = 120^\circ \quad (1)$$

$$\therefore \overline{AB} \parallel \overline{CD}, \overline{AB} \parallel \overline{EF}$$

$\therefore \overline{CD} \parallel \overline{EF}$ ,  $\overline{CE}$  is a transversal to them.

$$\therefore m(\angle DCE) + m(\angle E) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle DCE) = 180^\circ - 35^\circ = 145^\circ \quad (2)$$

$$\therefore m(\angle ACD) + m(\angle DCE) + m(\angle ACE) = 360^\circ$$

From (1) and (2)

$$\therefore m(\angle ACE) = 360^\circ - (120^\circ + 145^\circ) = 95^\circ \text{ (The req.)}$$

13

$\therefore \overline{DE} \parallel \overline{BC}$ ,  $\overline{AB}$  is a transversal to them

$$\therefore m(\angle B) = m(\angle DAB) = 80^\circ \text{ (Alternate angles)}$$

$\therefore \overline{DE} \parallel \overline{BC}$ ,  $\overline{AC}$  is a transversal to them

$$\therefore m(\angle C) = m(\angle EAC) = 50^\circ \text{ (Alternate angles)}$$

$$\therefore A \in \overline{DE}$$

$$m(\angle BAC) = 180^\circ - (50^\circ + 80^\circ) = 50^\circ \text{ (The req.)}$$

14

$\therefore \overline{OA} \parallel \overline{CE}$ ,  $\overline{DC}$  is a transversal to them

$$\therefore m(\angle DOA) = m(\angle C) = 60^\circ$$

(Corresponding angles)

$\therefore \overline{OD} \parallel \overline{AB}$ ,  $\overline{AO}$  is a transversal to them.

$$\therefore m(\angle BAX) = m(\angle DOA) = 60^\circ$$

(Corresponding angles)

$\therefore \overline{OA} \parallel \overline{CE}$ ,  $\overline{AE}$  is a transversal to them

$$\therefore m(\angle XAE) = m(\angle E) = 70^\circ \text{ (Alternate angles)}$$

$$\therefore m(\angle BAE) = 60^\circ + 70^\circ = 130^\circ \text{ (The req.)}$$

15

$$1 \quad \therefore E \in \overline{AB} \quad \therefore m(\angle BEF) = 180^\circ - 122^\circ = 58^\circ$$

$$\therefore m(\angle BEF) = m(\angle EFC) \text{ (Alternate angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.})$$

$$2 \quad \therefore E \in \overline{AB}$$

$$\therefore m(\angle BEF) = 180^\circ - 100^\circ = 80^\circ$$

$$\therefore m(\angle BEF) = m(\angle DFN) \text{ (Corresponding angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.})$$

$$3 \quad \therefore m(\angle BEF) = m(\angle AEM) = 132^\circ \quad (\text{V.O.A.})$$

$$\therefore m(\angle BEF) = m(\angle DFN) \text{ (Corresponding angles)}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D.})$$

16

$\therefore \overline{AB} \parallel \overline{CD}$ ,  $\overline{AC}$  is a transversal to them

$$m(\angle ACD) = m(\angle A) = 50^\circ \text{ (Alternate angles)}$$

$$\therefore m(\angle ACE) = 90^\circ$$

$$\therefore m(\angle DCE) = 90^\circ - 50^\circ = 40^\circ$$

$$m(\angle DCE) = m(\angle E) \text{ (But they are alternate angles)}$$

$$\overline{CD} \parallel \overline{EF}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad \therefore \overline{AB} \parallel \overline{EF} \quad (\text{Q.F.D.})$$

17

$\therefore \overline{EF} \parallel \overline{CD}$ ,  $\overline{EC}$  is a transversal to them

$$m(\angle ECD) = m(\angle CEF) = 95^\circ \text{ (Alternate angles)}$$

$$m(\angle ACD) = 95^\circ - 30^\circ = 65^\circ$$

$$m(\angle ACD) + m(\angle A) = 65^\circ + 115^\circ = 180^\circ$$

(But they are interior angles in the same side of the transversal)

$$\overline{AB} \parallel \overline{CD}$$

$$\therefore \overline{AB} \parallel \overline{EF}$$

$$\therefore \overline{CD} \parallel \overline{EF} \quad (\text{Q.E.D.})$$

18

$$\therefore \triangle ABC \cong \triangle DEF$$

$$\therefore m(\angle ACB) = m(\angle DFE)$$

(But they are corresponding angles)

$$\therefore \overline{BC} \parallel \overline{EF} \quad (\text{Q.E.D.})$$

19

$$\therefore \overline{AD} \cap \overline{BC} = \{M\}$$

$$\therefore m(\angle AMB) = m(\angle DMC) \text{ (V.O.A.)}$$

$$\therefore \text{In } \triangle \triangle AMB \text{ and } DMC$$

$$\begin{cases} AM = DM \\ BM = CM \end{cases}$$

$$m(\angle AMB) = m(\angle DMC)$$

$$\therefore \text{The two triangles are congruent and we deduce that } AB = CD \quad (\text{Q.E.D. 1})$$

$$m(\angle A) = m(\angle D)$$

(But they are alternate angles)

$$\therefore \overline{AB} \parallel \overline{CD} \quad (\text{Q.E.D. 2})$$

## Geometry and Measurement

20

 $\therefore R \in \overline{DX}$ 

$$\therefore m(\angle FRC) = 180^\circ - 110^\circ = 70^\circ$$

$$\therefore m(\angle FCR) = m(\angle FRC) = 70^\circ$$

$$\therefore m(\angle FCR) = m(\angle A) \text{ (But they are alternate angles)}$$

$$\therefore \overline{CD} \parallel \overline{AB} \quad \text{(First req.)}$$

$$\therefore C \in \overline{DR} \quad \therefore m(\angle DCF) = 180^\circ - 70^\circ = 110^\circ$$

$$\therefore m(\angle AFE) = m(\angle DCF) = 110^\circ$$

(Corresponding angles)

$$\therefore \overline{CD} \parallel \overline{AB}, \overline{CD} \parallel \overline{EF} \quad \therefore \overline{AB} \parallel \overline{EF}$$

 $\therefore \overline{AF}$  a transversal to them

$$\therefore m(\angle AFE) = 180^\circ - m(\angle A) = 180^\circ - 70^\circ = 110^\circ$$

$$\therefore m(\angle DCF) = 110^\circ$$

$$\therefore m(\angle EFC) = 180^\circ - 110^\circ = 70^\circ$$

(interior angles in the same side of the transversal)

 $\therefore F \in \overline{AC}$ 

$$\therefore m(\angle AFE) = 180^\circ - 70^\circ = 110^\circ \quad \text{(Second req.)}$$

21

1) Let  $\overline{AB} \parallel \overline{CD}$ ,  $\overline{EF} \perp \overline{AB}$  $\therefore \overline{AB} \parallel \overline{CD}$  and  $\overline{EF}$ 

is a transversal to them.

$$\therefore m(\angle XYZ) = m(\angle EXA)$$

(Corresponding angles)

$$\therefore m(\angle XYZ) = 90^\circ$$

$$\therefore \overline{EF} \perp \overline{CD} \quad \text{(Q.E.D.)}$$

2) Let the straight line  $L \parallel$  the straight line  $M$  $\therefore$  The straight line  $L \parallel$  the straight line  $N$  $\therefore$  the straight line  $K$ 

is a transversal to all of them.

 $\therefore$  The straight line $L \parallel$  the straight line  $M$ ,and the straight line  $K$ 

is a transversal to them.

$$\therefore m(\angle ABC) = m(\angle DEB)$$

(corresponding angles)

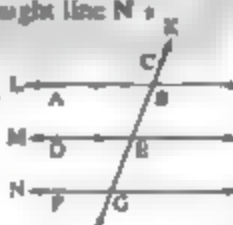
 $\therefore$  The straight line  $L \parallel$  the straight line  $N$ ,and the straight line  $K$  is a transversal to them

$$\therefore m(\angle ABC) = m(\angle FGE)$$

(Corresponding angles)

$$\therefore m(\angle DEB) = m(\angle FGE)$$

(But they are corresponding angles)

 $\therefore$  The straight line  $M \parallel$  the straight line  $N$  (Q.E.D.)22)  $\Delta \triangle ABD \cong \triangle CBD$  in them :

$$\begin{cases} AB = CB \\ AD = CD \\ \overline{BD} \text{ is a common side} \end{cases}$$

$$\therefore \Delta \triangle ABD \cong \Delta \triangle CBD$$

$$\therefore \Delta \triangle ABD \cong \Delta \triangle CBD$$

 $\therefore$  then we deduce that :

$$m(\angle ADB) = m(\angle CDB)$$

$$\therefore \overline{DB} \text{ bisects } \angle ADC$$

(Q.E.D. 1)

 $\therefore \Delta \triangle ADF \cong \triangle CDF$  in them

$$\begin{cases} AD = CD \\ \overline{DF} \text{ is a common side} \\ m(\angle ADF) = m(\angle CDF) \end{cases}$$

$$\therefore \Delta \triangle ADF \cong \Delta \triangle CDF$$

$$\therefore \Delta \triangle ADF \cong \Delta \triangle CDF$$

 $\therefore$  Then we deduce that ,

$$m(\angle AFD) = m(\angle CFD)$$

$$\therefore F \in \overline{AC}$$

$$\therefore m(\angle AFD) + m(\angle CFD) = 180^\circ$$

$$m(\angle AFD) = m(\angle CFD) = \frac{180^\circ}{2} = 90^\circ$$

$$\therefore \overline{AC} \perp \overline{DB} \quad \text{(Q.E.D. 2)}$$

23

$$\therefore \overline{AE} \cap \overline{DC} = \{F\}$$

$$\therefore m(\angle AFD) = m(\angle EFC) \quad \text{(V.O.A.)}$$

$$\therefore \Delta \triangle AFD \cong \Delta \triangle EFC$$

$$\therefore \Delta \triangle AFD \cong \Delta \triangle EFC$$

$$\begin{cases} m(\angle D) = m(\angle FCE) \\ m(\angle AFD) = m(\angle EFC) \\ FD = FC \end{cases}$$

$$\therefore \Delta \triangle AFD \cong \Delta \triangle EFC$$

$$\therefore \Delta \triangle AFD \cong \Delta \triangle EFC$$

$$\therefore \Delta \triangle AFD \cong \Delta \triangle EFC$$

Then we deduce that  $CE = AD$  $\therefore$  ABCD is a square

$$\therefore BC = AD$$

$$\therefore CE = CB \quad \text{(Q.E.D.)}$$

24

 $\Delta \triangle ABD \cong \triangle CBD$  in them :

$$\begin{cases} AD = CB \\ \overline{BD} \text{ is a common side} \\ m(\angle ADB) = m(\angle CBD) \end{cases}$$

 $\therefore \Delta \triangle ABD \cong \Delta \triangle CBD$  then we deduce that :

$$AB = CD$$

(Q.E.D. 1)

$$m(\angle ABD) = m(\angle CBD)$$

$$m(\angle ABD) = m(\angle CBD)$$

(And they are two alternate angles)

$$\therefore \overline{AB} \parallel \overline{CD} \quad \text{(Q.E.D. 2)}$$

24



25

 $\Delta \Delta DXB, EYC$  in them :

$$\begin{cases} DB = EC \\ DX = EY \\ m(\angle DXB) = m(\angle EYC) = 90^\circ \end{cases}$$

 $\therefore \Delta DXB = \Delta EYC$  . Then we deduce that :

$$m(\angle B) = m(\angle C) \quad (1)$$

 $\overline{BC} \parallel \overline{DE}$  and  $\overline{AB}$  is a transversal to them

$$\therefore m(\angle ADE) = m(\angle B) \quad (2)$$

(Corresponding angles)

$$\text{Similarly : } m(\angle AED) = m(\angle C) \quad (3)$$

(Corresponding angles)

From (1) and (2) and (3) :

$$\therefore m(\angle ADE) = m(\angle AED) \quad (\text{Q.E.D.})$$

26

 $\Delta \Delta ABD, ACE$  in them :

$$\begin{cases} AD = AE \\ \angle A \text{ is a common angle} \\ m(\angle ADB) = m(\angle AEC) = 90^\circ \end{cases}$$

 $\therefore \Delta ABD = \Delta ACE$ 

$$\therefore BD = CE \quad (\text{Q.E.D.})$$

27

 $\Delta \Delta ABE, ACD$  in them

$$\begin{cases} AE = AD \\ m(\angle AEB) = m(\angle ADC) \\ \angle A \text{ is a common angle} \end{cases}$$

 $\therefore \Delta ABE = \Delta ACD$ Then we deduce that ,  $BE = CD$  (Q.E.D. 1)

$$\therefore AB = AC \quad \therefore AD + DB = AE + EC$$

$$\therefore AD = AE$$

$$\therefore DB = EC \quad (\text{Q.E.D. 2})$$

28

$$1 \therefore 3x - 5^\circ = 70^\circ \text{ (V.O.A.)}$$

$$\therefore 3x = 70^\circ + 5^\circ = 75^\circ$$

$$\therefore x = \frac{75^\circ}{3} = 25^\circ$$

$$2 \therefore x^2 = 36^\circ + 64^\circ = 100^\circ \text{ (V.O.A.)}$$

$$\therefore x = \pm 10^\circ$$

$$3 \therefore (3x - 8^\circ) + x = 180^\circ \quad \therefore 4x - 8^\circ = 180^\circ$$

$$\therefore 4x = 180^\circ + 8^\circ = 188^\circ$$

$$\therefore x = \frac{188^\circ}{4} = 47^\circ$$

$$4 \therefore x = 60^\circ \text{ (Alternate angles) ,}$$

$$y = 61^\circ \text{ (Corresponding angles)}$$

$$5 \therefore 3x + 90^\circ = 180^\circ$$

$$\therefore 3x = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore x = \frac{90^\circ}{3} = 30^\circ \quad \therefore 4y + 5y = 180^\circ$$

$$\therefore 9y = 180^\circ \quad \therefore y = \frac{180^\circ}{9} = 20^\circ$$

$$6 \therefore m(\angle ABC) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore x = 60^\circ \text{ (Corresponding angles)}$$

$$\therefore 3y + 6^\circ = x \text{ (Corresponding angles)}$$

$$\therefore 3y = 60^\circ - 6^\circ = 54^\circ$$

$$\therefore y = \frac{54^\circ}{3} = 18^\circ$$

29

$$\therefore m(\angle AMB) + m(\angle AMD) = 115^\circ + 45^\circ = 160^\circ$$

$$\therefore m(\angle BMD) = 160^\circ$$

$$\therefore m(\angle BMD) + m(\angle BMC) + m(\angle CMD) = 360^\circ$$

$$\therefore m(\angle BMC) + m(\angle CMD) = 360^\circ - 160^\circ = 200^\circ$$

$$\therefore 3x + 5x = 200 \quad \therefore 8x = 200$$

$$\therefore x = 25^\circ \text{ (First req.) } \therefore m(\angle BMC) = 25 \times 3 = 75^\circ$$

$$\therefore m(\angle BMC) + m(\angle AMB) = 75^\circ + 115^\circ = 190^\circ$$

 $A, M$  and  $C$  are not collinear (Second req.)

30

Yes  $\Delta ADE = \Delta CBF$  (Two angles and side) (Q.E.D. 1)Then we deduce that  $DE = BF$ 

$$\therefore AE = FC$$

$$\therefore m(\angle AED) = m(\angle CFB)$$

$$\therefore m(\angle DEF) = m(\angle BFE)$$

 $\Delta \Delta DEF, BFE$  in them

$$\begin{cases} DE = BF \\ \overline{FE} \text{ is a common side} \end{cases}$$

$$m(\angle DEF) = m(\angle BFE)$$

$$\therefore \Delta DEF = \Delta BFE \quad (\text{Q.E.D. 2})$$

Then we deduce that :  $DF = BE$ 

$$m(\angle BEF) = m(\angle DFE)$$

$$\therefore m(\angle BEA) = m(\angle DFC)$$

 $\Delta \Delta ABE, CDF$  in them

$$\begin{cases} AE = FC \\ DF = BE \end{cases}$$

$$m(\angle BEA) = m(\angle DFC)$$

$$\therefore \Delta ABE = \Delta CDF \quad (\text{Q.E.D. 3})$$



## Geometry and Measurement



Yes  $\triangle PAQ \cong \triangle QBO$

"two sides and included angle"

• then we deduce that :  $PQ = QO$

•  $\triangle PQR \cong \triangle QOR$  in them

$PQ = QO$

$QR$  is a common side

$m(\angle QPR) = m(\angle ROQ) = 90^\circ$

•  $\triangle PQR \cong \triangle QOR$

• then we deduce that :  $PR = OR$

•  $\triangle PAQ \cong \triangle QBO$

•  $m(\angle 5) = m(\angle 6)$

•  $m(\angle 5) + m(\angle 4) = 90^\circ$

•  $m(\angle 6) + m(\angle 4) = 90^\circ$

•  $m(\angle 4) + m(\angle 1) = 90^\circ$

•  $m(\angle 6) = m(\angle 1)$

•  $m(\angle 6) + m(\angle 7) = 90^\circ$

•  $m(\angle 3) + m(\angle 7) = 90^\circ$

•  $m(\angle 6) = m(\angle 3)$

From (1) and (2) :

•  $m(\angle 1) = m(\angle 3)$

• From  $\triangle PDR \cong \triangle RCO$

$m(\angle 1) = m(\angle 3)$  ;  $m(\angle C) = m(\angle D)$  ;

$m(\angle 4) = m(\angle 2)$

• In  $\triangle PDR \cong \triangle RCO$

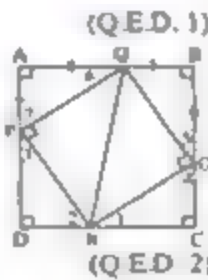
$m(\angle 1) = m(\angle 3)$

$m(\angle 2) = m(\angle 4)$

$PR = RO$

•  $\triangle PDR \cong \triangle RCO$

(Q.E.D. 3)



(Q.E.D. 1)

(Q.E.D. 2)

## Answers of Exercise 2

1

(a) All its sides are equal in length

(b) All its angles are equal in measure

2 360° 3 540° 4 720° 5 900°

6 108° ; 128° 7 360°

8 5 ; 120° 9 135°

2

1 (b) 2 (c) 3 (c) 4 (d)

5 (d) 6 (c) 7 (c)

3

• The number of diagonals of the polygon of  $n$  sides =  $\frac{n(n-3)}{2}$

1 The number of diagonals of the triangle

$$= \frac{3(3-3)}{2} = \text{zero}$$

2 The number of diagonals of the quadrilateral

$$= \frac{4(4-3)}{2} = 2$$

3 The number of diagonals of the pentagon

$$= \frac{5(5-3)}{2} = 5$$

4

Fig (1) :  $m(\angle D) = 360^\circ - (90^\circ + 100^\circ + 80^\circ) = 90^\circ$

Fig. (2) :  $m(\angle D) = 540^\circ - (90^\circ + 90^\circ + 112^\circ + 130^\circ) = 118^\circ$

(1)

5

Given :  $\overline{AE} \parallel \overline{BC}$  ;  $m(\angle B) = 70^\circ$  ;

$m(\angle C) = 150^\circ$  ;  $m(\angle D) = 80^\circ$

(2)

R.T.F.  $m(\angle E)$

Proof :

•  $\overline{AE} \parallel \overline{BC}$  ;  $\overline{AB}$  is a transversal to them

•  $m(\angle A) + m(\angle B) = 180^\circ$

(Two interior angles in the same side of the transversal)

•  $m(\angle B) = 70^\circ$

$m(\angle A) = 180^\circ - 70^\circ = 110^\circ$

•  $ABCDE$  is a pentagon

• The sum of measures of its interior angles

$$= (5-2) \times 180^\circ = 540^\circ$$

•  $m(\angle E) = 540^\circ - (70^\circ + 150^\circ + 80^\circ + 110^\circ)$

$$= 130^\circ$$

(The req.)

6

Given :  $\overline{CE} \cap \overline{AD} = \{B\}$  ;  $m(\angle A) = 50^\circ$  ;

$m(\angle C) = 70^\circ$  ;  $m(\angle D) = 130^\circ$  ;

$m(\angle F) = 90^\circ$

R.T.F. :  $m(\angle E)$

Proof :

• The sum of measures of the interior angles

of the triangle =  $180^\circ$

• In  $\triangle ABC$

$m(\angle ABC) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$

•  $\overline{CE} \cap \overline{AD} = \{B\}$

$$\therefore m(\angle EBD) = m(\angle ABC) = 60^\circ \text{ (V.O.A.)}$$

• The sum of measures of the interior angles of the quadrilateral =  $360^\circ$

$$\therefore m(\angle E) = 360^\circ - (90^\circ + 130^\circ + 60^\circ) = 80^\circ \text{ (The req.)}$$

7

$$Z \in \overline{YF}$$

$$m(\angle LZY) = 180^\circ - 120^\circ = 60^\circ$$

From the quadrilateral XYZL

$$\therefore m(\angle X) = 360^\circ - (70^\circ + 60^\circ + 90^\circ) = 140^\circ \text{ (The req.)}$$

8

From the hexagon ABCDEF

$$m(\angle A) + m(\angle C) + m(\angle B) + m(\angle D) + m(\angle E) + m(\angle F) = 720^\circ$$

$$m(\angle A) + m(\angle C)$$

$$= 720^\circ - (90^\circ + 115^\circ + 165^\circ + 110^\circ) = 240^\circ$$

$$\therefore m(\angle FAB) = m(\angle DCB) = \frac{240^\circ}{2} = 120^\circ$$

$$x = 120^\circ \text{ (The req.)}$$

9

From the quadrilateral ABCD

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^\circ$$

$$\therefore 90^\circ + 3x + 5x + 2x = 360^\circ$$

$$\therefore 10x = 360^\circ - 90^\circ = 270^\circ$$

$$x = \frac{270^\circ}{10} = 27^\circ \text{ (The req.)}$$

10

From the figure (The quadrilateral ABCD)

$$m(\angle C) = 360^\circ - (120^\circ + 85^\circ + 90^\circ) = 65^\circ$$

$\therefore \overline{BE} \parallel \overline{CD}$ ,  $\overline{BC}$  is a transversal to them

$$\therefore m(\angle C) + m(\angle EBC) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle EBC) = 180^\circ - 65^\circ = 115^\circ$$

$$m(\angle ABE) = 115^\circ - 85^\circ = 30^\circ \text{ (The req.)}$$

11

$\therefore \triangle DFE$  is an equilateral triangle

$$m(\angle FDE) = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore \overline{AE} \cap \overline{CF} = \{D\}$$

$$m(\angle ADC) = m(\angle FDE) = 60^\circ \text{ (V.O.A.)}$$

• From the quadrilateral ABCD

$$\therefore m(\angle B) = 360^\circ - (120^\circ + 60^\circ + 105^\circ) = 75^\circ$$

(The req.)

12

From the quadrilateral ARFE

$$\therefore m(\angle RAE) = 360^\circ - (120^\circ + 45^\circ + 105^\circ) = 90^\circ$$

$$\therefore \overline{ED} \cap \overline{RB} = \{A\}$$

$$\therefore m(\angle DAB) = m(\angle RAE) = 90^\circ \text{ (V.O.A.)}$$

From the quadrilateral ABCD

$$m(\angle B) = 360^\circ - (90^\circ + 130^\circ + 80^\circ) = 60^\circ \text{ (The req.)}$$

13

From  $\triangle ABC$

$$m(\angle B) = 180^\circ - (30^\circ + 58^\circ) = 92^\circ$$

$\overline{AB} \parallel \overline{DE}$ ,  $\overline{BD}$  is a transversal to them

$$m(\angle D) = m(\angle B) = 92^\circ \text{ (alternate angles)}$$

$$\therefore \overline{AF} \cap \overline{BD} = \{C\}$$

$$\therefore m(\angle DCF) = m(\angle ACB) = 58^\circ \text{ (V.O.A.)}$$

From the quadrilateral CFED

$$m(\angle E) = 360^\circ - (58^\circ + 125^\circ + 92^\circ) = 85^\circ \text{ (The req.)}$$

14

From the quadrilateral EOXY

$$m(\angle OCY) = 360^\circ - (50^\circ + 90^\circ + 90^\circ) = 130^\circ$$

$$\therefore \overline{DY} \cap \overline{OA} = \{C\}$$

$$\therefore m(\angle DCA) = m(\angle OCY) = 130^\circ \text{ (V.O.A.)}$$

$\therefore \overline{CD} \parallel \overline{AB}$ ,  $\overline{AC}$  is a transversal to them

$$\therefore m(\angle DCA) + m(\angle BAC) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore x = 180^\circ - 130^\circ = 50^\circ \text{ (The req.)}$$

15

From the quadrilateral ABCD

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360^\circ$$

$$\therefore m(\angle B) + m(\angle D) = 360^\circ - (95^\circ + 85^\circ) = 180^\circ$$

$$\therefore m(\angle B) = \frac{1}{2} m(\angle D)$$

$$\therefore m(\angle D) + \frac{1}{2} m(\angle D) = 180^\circ$$

$$\therefore \frac{3}{2} m(\angle D) = 180^\circ \therefore m(\angle D) = \frac{180^\circ \times 2}{3} = 120^\circ$$

$$\therefore m(\angle B) = 60^\circ \text{ (The req.)}$$

16

$$1) x = \frac{(5-2) \times 180^\circ}{5} = 108^\circ$$

$$2) y = \frac{(8-2) \times 180^\circ}{8} = 135^\circ, x = 180^\circ - 135^\circ = 45^\circ$$

## Geometry and Measurement

$$\therefore 2x = \frac{(6-2) \times 180^\circ}{6} = 120^\circ$$

$$\therefore x = \frac{120^\circ}{2} = 60^\circ \quad \therefore y = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore z = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore y = \frac{(8-2) \times 180^\circ}{8} = 135^\circ \quad \therefore x = \frac{135^\circ}{2} = 67.5^\circ$$

$$\therefore z = 180^\circ - 135^\circ = 45^\circ \quad \therefore a = 360^\circ - 135^\circ = 225^\circ$$

17

1. The figure is a pentagon

$$\therefore a + 2a + 2a + a + 2a = 540^\circ$$

$$\therefore 8a = 540^\circ$$

$$\therefore a = 67.5^\circ$$

2. The figure is a pentagon

$$\therefore a + a + a + 90^\circ + 90^\circ = 540^\circ$$

$$\therefore 3a + 180^\circ = 540^\circ$$

$$\therefore 3a = 540^\circ - 180^\circ = 360^\circ$$

$$\therefore a = 120^\circ$$

3.  $b = 180^\circ - (90^\circ + 42^\circ) = 48^\circ$ 

$$a = 90^\circ \text{ (alternate angle)}$$

$$c = 180^\circ - (90^\circ + 38^\circ) = 52^\circ$$

4.  $a = 180^\circ - (2 \times 62^\circ) = 56^\circ$ 

$$b = \frac{180^\circ - 56^\circ}{2} = 62^\circ$$

18

From the quadrilateral ABCD.

$$m(\angle DCB) = 360^\circ - (110^\circ + 130^\circ + 60^\circ) = 60^\circ$$

 $\therefore C \in \overline{BE}$ 

$$\therefore m(\angle DCE) = 180^\circ - 60^\circ = 120^\circ$$

 $\therefore \overline{CF}$  bisects  $\angle DCE$ 

$$\therefore m(\angle FCE) = \frac{120^\circ}{2} = 60^\circ$$

$$\therefore m(\angle FCE) = m(\angle B) = 60^\circ$$

(And they are two corresponding angles)

 $\therefore \overline{CF} \parallel \overline{AB}$  (Q.E.D.)

19

From  $\triangle XYB$ 

$$m(\angle XBY) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

The figure ABCDEF is a regular hexagon

$$\therefore m(\angle ABC) = \frac{(6-2) \times 180^\circ}{6} = 120^\circ$$

 $\therefore B \in \overline{YC}$ 

$$\therefore m(\angle ABY) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle ABY) = m(\angle YBX) + m(\angle XBA)$$

$$\therefore 60^\circ = 30^\circ + m(\angle XBA)$$

$$\therefore m(\angle XBA) = 30^\circ$$

 $\therefore \overline{BX}$  bisects  $\angle ABY$  (Q.E.D.)

20

 $\therefore \angle EBC$  is an exterior angle of  $\triangle EFB$ 

$$\therefore m(\angle EBC) = 45^\circ + 60^\circ = 105^\circ$$

From  $\triangle ABC$ 

$$\therefore m(\angle C) = 180^\circ - (105^\circ + 30^\circ) = 45^\circ$$

 $\therefore E \in \overline{FD}$ 

$$m(\angle BED) = 180^\circ - 45^\circ = 135^\circ$$

From the quadrilateral EBCD

$$m(\angle EDC) = 360^\circ - (135^\circ + 105^\circ + 45^\circ) = 75^\circ$$

(The req.)

21

Let the measures of the interior angles of the pentagon be  $3x, 3x, 2x, 3x, 4x$  $\therefore$  The sum of the measures of the interior angles of the pentagon  $= (5-2) \times 180^\circ = 540^\circ$ 

$$\therefore 3x + 3x + 2x + 3x + 4x = 540^\circ$$

$$15x = 540^\circ$$

$$\therefore x = \frac{540^\circ}{15} = 36^\circ$$

 $\therefore$  The greatest measure  $= 4 \times 36^\circ = 144^\circ$  (The req.)

22

Let the number of the sides of the polygon be  $(n)$  $\therefore$  The measure of the exterior angle of the polygon  $= 30^\circ$  $\therefore$  The measure of the interior angle of the polygon  $= 180^\circ - 30^\circ = 150^\circ$ 

$$\therefore \frac{(n-2) \times 180^\circ}{n} = 150^\circ \quad \therefore 150^\circ n = 180^\circ n - 360^\circ$$

$$\therefore 360^\circ = 180^\circ n - 150^\circ n \quad \therefore 360^\circ = 30^\circ n$$

$$\therefore n = \frac{360^\circ}{30^\circ} = 12$$

The sum of the measures of the interior angles

$$= (12-2) \times 180^\circ = 1800^\circ \quad \text{(The req.)}$$

23

Let the number of sides of the polygon be  $(n)$



$$\frac{(n-2) \times 180^\circ}{n} = 100^\circ$$

$$\therefore 100^\circ n = 180^\circ n - 360^\circ$$

$$\therefore 360^\circ = 180^\circ n - 100^\circ n \quad \therefore 360^\circ = 80^\circ n$$

$$\therefore n = \frac{360^\circ}{80^\circ} = 4.5 \notin \mathbb{Z}$$

$\therefore$  There is no regular polygon of interior angles each of them is of measure  $100^\circ$  (The req.)

20.

$\therefore$  The sum of measures of the interior angles of the polygon of 9 sides

$$= (9-2) \times 180^\circ = 1260^\circ$$

$\therefore$  The measure of the remained angle

$$= 1260^\circ - 1140^\circ = 120^\circ \quad (\text{First req.})$$

$\therefore$  The measure of the interior angle of a regular polygon of 9 sides  $= \frac{(9-2) \times 180^\circ}{9} = 140^\circ$

$\therefore$  The polygon has an interior angle of measure  $120^\circ$

$$\therefore 120^\circ \neq 140^\circ$$

$\therefore$  The polygon is not regular. (Second req.)

21.

1 The sum of the measures of the interior angles of the polygon of 15 sides  $= (15-2) \times 180^\circ = 2340^\circ$

2 The sum of the measures of the interior and exterior angles at five vertices  $= 5 \times 180^\circ = 900^\circ$

$\therefore$  The sum of the measures of the interior angles at these vertices  $= 900^\circ - 200^\circ = 700^\circ$

$\therefore$  The sum of the measures of the ten remained interior angles  $= 2340^\circ - 700^\circ = 1640^\circ$  (The req.)

22.

In  $\triangle AMD$ ,

$$\therefore m(\angle AMD) = 71^\circ$$

$$\therefore m(\angle MAD) + m(\angle MDA) = 180^\circ - 71^\circ = 109^\circ$$

$\therefore \overline{AM}$  bisects  $\angle BAD$ ,  $\overline{DM}$  bisects  $\angle ADC$

$$\begin{aligned} \therefore m(\angle BAD) + m(\angle ADC) \\ &= 2[m(\angle MAD) + m(\angle MDA)] \\ &= 2 \times 109^\circ = 218^\circ \end{aligned}$$

$\therefore$  The sum of measures of the interior angles of the quadrilateral  $= 360^\circ$

$$\therefore m(\angle B) + m(\angle C) = 360^\circ - 218^\circ = 142^\circ \quad (\text{Q.E.D.})$$

27.

$\overline{AE} \parallel \overline{BC}$ ,  $\overline{AB}$  is a transversal to them

$$\therefore m(\angle A) + m(\angle B) = 180^\circ \quad (1)$$

(Two interior angles in the same side of the transversal)

From the pentagon ABCDE

$$\begin{aligned} \therefore m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) \\ + m(\angle E) = 540^\circ \end{aligned}$$

$$\therefore m(\angle A) + m(\angle B) = 180^\circ \quad \text{"from (1)"} \quad \therefore m(\angle C) + m(\angle D) + m(\angle E) = 540^\circ - 180^\circ = 360^\circ$$

$$\therefore m(\angle C) + m(\angle D) + m(\angle E) = 360^\circ$$

$$\therefore m(\angle C) = m(\angle D) = m(\angle E) = \frac{360^\circ}{3} = 120^\circ$$

$$\therefore m(\angle A) = 120^\circ$$

$$\therefore m(\angle B) = 180^\circ - 120^\circ = 60^\circ \quad (\text{The req.})$$

### Answers of Exercise 3

1 parallel and equal in length

2 equal in measure

3 supplementary

4 bisect each other

5 a trapezium

6 each two opposite sides are parallel

7  $130^\circ$

8  $120^\circ$

9 6, 2

10  $105^\circ, 75^\circ, 75^\circ$

11 16

Given:  $XYZL$  is a parallelogram,  $XY = 3$  cm,  $YZ = 4$  cm,  $m(\angle LXZ) = 31^\circ$  and  $m(\angle LZX) = 43^\circ$

R.T.P.: (1)  $m(\angle Y)$

(2) The perimeter of the parallelogram  $XYZL$ .

Proof:  $\because \triangle XLZ$  is which,

$$m(\angle LXZ) = 31^\circ, m(\angle LZX) = 43^\circ$$

$$\therefore m(\angle L) = 180^\circ - (31^\circ + 43^\circ) = 106^\circ$$

$\therefore$  The figure  $XYZL$  is a parallelogram.

$$\therefore m(\angle Y) = m(\angle L) = 106^\circ \quad (\text{First req.})$$

$\therefore$  the perimeter of the parallelogram  $XYZL$

$$= (XY + YZ) \times 2 = (3 + 4) \times 2 = 7 \times 2 = 14 \text{ cm.} \quad (\text{Second req.})$$

28.

Given:  $ABCD$  is a quadrilateral,  $\overline{AC} \cap \overline{BD} = \{M\}$ ,  $MA = MC$ ,  $MB = MD$ ,  $m(\angle AMB) = 110^\circ$  and  $m(\angle MBA) = 25^\circ$



## Geometry and Measurement

R.T.P. : ABCD is a parallelogram

R.T.F. :  $m(\angle ACD)$

Proof : In the figure ABCD

MA = MC (Given) , MB = MD (Given)

∴ Its diagonals bisect each other

∴ The figure ABCD is a parallelogram.

(First req.)

In  $\triangle MBA$

∴  $m(\angle AMB) = 110^\circ$  ,  $m(\angle MBA) = 25^\circ$

∴  $m(\angle MAB) = 180^\circ - (110^\circ + 25^\circ) = 45^\circ$

∴ The figure ABCD is a parallelogram

∴  $\overline{AB} \parallel \overline{CD}$

∴  $\overline{CA}$  is a transversal to them.

$m(\angle ACD) = m(\angle CAB)$

$= 45^\circ$  (alternate angles) (Second req.)

5

∴ ABCD is a parallelogram

∴ The two diagonals bisect each other

MB = MD = 1.8 cm

MA = MC = 2.5 cm , AB = CD = 2 cm

∴ The perimeter of  $\triangle AMB$

$= 2 + 2.5 + 1.8 = 6.3$  cm. (The req.)

6

From  $\triangle XYZ$  :

$m(\angle YXZ) = 180^\circ - (118^\circ + 27^\circ) = 35^\circ$  (First req.)

∴  $XYZL$  is a parallelogram

∴  $\overline{XY} \parallel \overline{LZ}$  ,  $\overline{XZ}$  is a transversal

∴  $m(\angle LXZ) = m(\angle YXZ) = 35^\circ$  (alternate angles)

(Second req.)

∴  $\overline{XL} \parallel \overline{YZ}$  ,  $\overline{XZ}$  is a transversal

∴  $m(\angle LXZ) = m(\angle XZY) = 27^\circ$  (alternate angles)

(Third req.)

∴  $m(\angle L) = m(\angle Y) = 118^\circ$  (Fourth req.)

7

∴ ABCD is a parallelogram

∴  $\overline{AD} \parallel \overline{BC}$  ,  $\overline{AB}$  is a transversal

∴  $m(\angle ABL) = m(\angle A) = 50^\circ$  (alternate angles) (1)

∴ BHOC is a parallelogram

∴  $\overline{CO} \parallel \overline{BH}$  ,  $\overline{CB}$  is a transversal

∴  $m(\angle LBH) = m(\angle BCO)$

$= 60^\circ$  (Corresponding angles) (2)

From (1) + (2)

∴  $m(\angle ABH) = m(\angle ABL) + m(\angle LBH)$

$= 50^\circ + 60^\circ = 110^\circ$  (The req.)

8

∴  $\overline{AD} \parallel \overline{BC}$  ,  $\overline{AB}$  is a transversal

∴  $m(\angle B) = m(\angle EAB)$

$= 110^\circ$  (alternate angles)

∴  $m(\angle B) + m(\angle C) = 110^\circ + 70^\circ = 180^\circ$

and they are interior angles on the same side of the transversal

$\overline{AB} \parallel \overline{CD}$

∴  $\overline{AD} \parallel \overline{BC}$

ABCD is a parallelogram

(Q.E.D.)

9

ABCD is a parallelogram

∴  $m(\angle B) + m(\angle C) = 180^\circ$

∴  $m(\angle C) = 180^\circ - 120^\circ = 60^\circ$

From  $\triangle DHC$  :

$m(\angle HDC) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$  (The req.)

10

∴ ABCD is a parallelogram

$m(\angle A) + m(\angle ABC) = 180^\circ$

$m(\angle ABC) = 180^\circ - 60^\circ = 120^\circ$

∴  $m(\angle ABO) = m(\angle ABC) - m(\angle OBC)$

$= 120^\circ - 40^\circ = 80^\circ$  (The req.)

11

∴ ABCD is a parallelogram

$m(\angle A) + m(\angle B) = 180^\circ$

(1)

∴ The sum of measures of the interior angles

of the quadrilateral DEBF =  $360^\circ$

$m(\angle EDF) + m(\angle B) = 360^\circ - (90^\circ + 90^\circ) = 180^\circ$  (2)

From (1) and (2)

$m(\angle EDF) = m(\angle A)$  (Q.E.D.)

12

∴  $M \in \overline{AC}$

∴  $m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$

∴ In  $\triangle BMC$  :

$m(\angle MCB) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$

∴  $m(\angle MCB) = m(\angle MAD)$

and they are alternate angles

- $\overline{AD} \parallel \overline{BC}$
- $\therefore \overline{AB} \parallel \overline{DC}$
- $\therefore ABCD$  is a parallelogram (Q.E.D.)

13

- $ABCD$  is a parallelogram
- $m(\angle A) = m(\angle C)$  (1)
- $\therefore XYCL$  is a parallelogram
- $m(\angle X) = m(\angle C)$  (2)
- From (1) and (2):
- $m(\angle A) = m(\angle X)$  (Q.E.D.)

14

- $ABCD$  is a parallelogram
- $BC = AD$  (1)
- $\therefore HBCO$  is a parallelogram
- $\therefore BC = HO$  (2)
- From (1) + (2):
- $\therefore AD = HO$  (Q.E.D.)

15

- $ABCD$  is a parallelogram.
- $AD = BC$
- $\therefore AD = DE$
- $\therefore DE = BC$  (1)
- $\therefore \overline{AD} \parallel \overline{BC}$
- (Because  $ABCD$  is a parallelogram)
- $\therefore \overline{DE} \parallel \overline{BC}$  (2)
- From (1) and (2):
- $\therefore DECB$  is a parallelogram
- The two diagonals bisect each other
- $\overline{DC}$  and  $\overline{BE}$  bisect each other (Q.E.D.)



16

- $m(\angle ABE) = m(\angle C)$
- and they are corresponding angles
- $\therefore \overline{AB} \parallel \overline{CD}$  (1)
- $\therefore m(\angle A) = m(\angle ABE)$  and they are alternate angles
- $\therefore \overline{AD} \parallel \overline{BC}$  (2)
- From (1) + (2):
- $ABCD$  is a parallelogram (Q.E.D.)

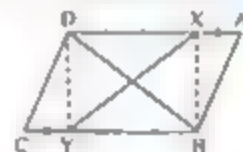
17

- $\therefore ABCD$  is a parallelogram  $AB = DC$
- $\therefore \frac{1}{2} AB = \frac{1}{2} DC$

- $\therefore H$  is the midpoint of  $\overline{AB}$  +  $O$  is the midpoint of  $\overline{CD}$
- $\therefore HB = DO$  (1)
- $\therefore \overline{AB} \parallel \overline{CD}$
- i.e.  $\therefore \overline{HB} \parallel \overline{DO}$  (2)
- From (1) + (2)
- $HBO$  is a parallelogram (Q.E.D.)

18

- $ABCD$  is a parallelogram
- $AD = BC$
- $\therefore AX = CY$
- $\therefore AD - AX = BC - CY$
- $\therefore XD = BY$   $\therefore \overline{XD} \parallel \overline{BY}$
- The figure  $XYBD$  is a parallelogram
- $\therefore \overline{XY}$  and  $\overline{BD}$  bisect each other. (Q.E.D.)



19

- $ABCD$  is a parallelogram
- $AB = DC$
- $\therefore \triangle DHC$  is equilateral
- $DC = HC$   $\therefore AB = HC$  (First req)
- $\therefore \triangle DHC$  is equilateral
- $\therefore m(\angle C) = 60^\circ$
- $\therefore m(\angle B) + m(\angle C) = 180^\circ$
- (Two consecutive angles in  $\square ABCD$ )
- $\therefore m(\angle B) + 60^\circ = 180^\circ$
- $m(\angle B) = 180^\circ - 60^\circ = 120^\circ$  (Second req)
- $\therefore \overline{AD} \parallel \overline{BC}$  +  $\overline{DH}$  is a transversal
- $m(\angle HDA) = m(\angle DHC)$  (alternate angles)
- $\therefore m(\angle DHC) = 60^\circ$
- $m(\angle HDA) = 60^\circ$  (Third req)

20

- $\therefore \overline{DH}$  bisects  $\angle ADC$   $\therefore m(\angle ADH) = 64^\circ$
- $ABCD$  is a parallelogram
- $\overline{AD} \parallel \overline{BC}$  +  $\overline{DH}$  is a transversal
- $m(\angle ADH) + m(\angle DHB) = 180^\circ$
- $\therefore m(\angle DHB) = 180^\circ - 64^\circ = 116^\circ$  (First req)
- $ABCD$  is a parallelogram
- $m(\angle ABC) = m(\angle ADC) = 2m(\angle HDC)$
- $= 2 \times 64^\circ = 128^\circ$  (Second req)

21

- $E \in \overline{BC}$
- $\therefore m(\angle AEC) = 180^\circ - 70^\circ = 110^\circ$

## Geometry and Measurement

∴ AECD is a quadrilateral.

$$\therefore m(\angle EAD) = 360^\circ - (110^\circ + 115^\circ + 65^\circ) = 70^\circ$$

$$\therefore m(\angle BAD) = 70^\circ + 45^\circ = 115^\circ$$

$$\therefore m(\angle BAD) = m(\angle C) \quad (1)$$

In  $\triangle ABE$ :

$$m(\angle B) = 180^\circ - (45^\circ + 70^\circ) = 65^\circ$$

$$\therefore m(\angle B) = m(\angle D) \quad (2)$$

From (1) and (2):

∴ ABCD is a parallelogram. (Q.E.D.)

22

∴ ABCD is a parallelogram

$$\therefore m(\angle ABC) = m(\angle ADC) = 115^\circ$$

∴ HBCO is a parallelogram

$$\therefore m(\angle HBC) = m(\angle O) = 50^\circ$$

$$\therefore m(\angle ABH) = m(\angle ABC) - m(\angle HBC) \\ = 115^\circ - 50^\circ = 65^\circ \quad (\text{The req.})$$

23

∴  $\overline{AB} \parallel \overline{CD}$

∴  $\overline{AC}$  is a transversal

$$\therefore m(\angle BAC) = m(\angle ACD) \\ (\text{alternate angles})$$

$$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle ACD)$$

∴  $m(\angle EAC) = m(\angle ACF)$  and they are alternate angles

∴  $\overline{AE} \parallel \overline{CF}$

∴  $AE = CF$

∴ AECF is a parallelogram (Q.E.D.)



24

∴ ABCD is a parallelogram

$$\therefore m(\angle DAB) + m(\angle ABC) \\ = 180^\circ$$

$$\therefore x^\circ + 2x^\circ + 5x^\circ = 180^\circ$$

$$\therefore 8x^\circ = 180^\circ \quad \therefore x^\circ = 22.5^\circ$$

$$\therefore m(\angle BCD) = m(\angle BAD) \\ = 3x^\circ = 3 \times 22.5^\circ = 67.5^\circ \quad (\text{First req.})$$

$$\therefore m(\angle ADC) = m(\angle ABC) = 5x^\circ = 5 \times 22.5^\circ = 112.5^\circ \\ (\text{Second req.})$$

25 1 (a) 2 (c) 3 (c) 4 (d) 5 (b)

26

To decide the place of the building D, we can do this by two methods

First:

- Draw  $\overline{AC}$  and determine its midpoint M
- Draw  $\overline{BM}$  and determine the point D on it where  $BM = MD$
- Then D is the required position



Second:

- Draw  $\overline{BC}$  and determine its midpoint M
- Draw  $\overline{AM}$  and determine the point D on it where  $AM = MD$
- Then D is the required position



27

∴ ABCD is a parallelogram

∴  $AB = CD$

∴ E and F are the midpoints of  $\overline{AB}$  and  $\overline{CD}$  respectively

$$BE = DF \quad (1)$$

∴  $\overline{AB} \parallel \overline{CD}$

$$BE \parallel DF \quad (2)$$

From (1) and (2)

∴ The figure BEDF is a parallelogram.

$$\therefore \overline{ED} \parallel \overline{BF} \quad (3) \quad (\text{Q.E.D. 1})$$

By the same way we can prove that

AECF is a parallelogram

$$\therefore \overline{AF} \parallel \overline{EC} \quad (4)$$

∴ from (3) and (4):

$$\therefore \overline{EM} \parallel \overline{NF}, \overline{EN} \parallel \overline{MF}$$

∴ FMEN is a parallelogram. (Q.E.D. 2)



28

∴ XYZL is a parallelogram

$$\therefore m(\angle X) + m(\angle Y) = 180^\circ$$

$$\therefore m(\angle Y) = 3m(\angle X)$$

$$\therefore m(\angle X) + 3m(\angle X) = 180^\circ$$

$$\therefore 4m(\angle X) = 180^\circ$$

$$\therefore m(\angle X) = \frac{180^\circ}{4} = 45^\circ$$

$$\therefore m(\angle Y) = 3m(\angle X) = 3 \times 45^\circ = 135^\circ$$

$$\therefore m(\angle Z) = m(\angle X) = 45^\circ$$

$$\therefore m(\angle L) = m(\angle Y) = 135^\circ \quad (\text{The req.})$$





## Answers of Exercise 4

1

- 1 rhombus. 2 equal in length  
3 square. 4 rhombus  
5 parallelogram 6 parallelogram  
7 parallelogram 8 rhombus  
9 square 10 square  
11 square 11 parallelogram  
12  $\overline{AC} \perp \overline{BD}$   
13 side length  $\times 4 = (\text{length} + \text{width}) \times 2$ ,  
side length  $\times 4$  14 10.5

2

- 1 (b) 2 (n) 3 (c) 4 (b)  
5 (b) 6 (b) 7 (a) 8 (c)

3

- 1 2 2 3 3 8

4

- 1 16 2 135° 3 110°

5

Fig (1) :  $m(\angle ADB) = 40^\circ$  ,  $m(\angle CDB) = 80^\circ$  ,  
 $m(\angle A) = 60^\circ$  ,  $m(\angle C) = 60^\circ$

Fig. (2) :  $m(\angle MFE) = 45^\circ$  ,  $m(\angle L) = 90^\circ$  ,  
 $m(\angle LMF) = 45^\circ$  ,  $m(\angle FME) = 45^\circ$  ,  
 $m(\angle E) = 90^\circ$

Fig. (3) :  $m(\angle XZY) = 35^\circ$  ,  $m(\angle ZXH) = 35^\circ$  ,  
 $m(\angle HZX) = 35^\circ$  ,  $m(\angle H) = 110^\circ$

Fig (4) :  $m(\angle NWK) = 60^\circ$  ,  $m(\angle WMK) = 30^\circ$  ,  
 $m(\angle KMN) = 60^\circ$  ,  $m(\angle MNW) = 60^\circ$  ,  
 $m(\angle MZN) = 60^\circ$  ,  $m(\angle WNK) = 30^\circ$  ,  
 $m(\angle NZK) = 120^\circ$

6

Given : ABCD is a square ,  $AB = 5$  cm. ,  
 $E \in \overline{AC}$  ,  $m(\angle EDC) = 30^\circ$

R.T.P. : (1) The perimeter of the square ABCD  
(2)  $m(\angle AED)$

Proof :  $\because$  The perimeter of the square = side length  $\times 4$   
 $\therefore$  The perimeter of the square ABCD  
 $= 5 \times 4 = 20$  cm. (First req.)

$\because$  ABCD is a square ,  $\overline{AC}$  is a diagonal  
 $\therefore m(\angle ACD) = 45^\circ$

In  $\triangle DEC$  :

$$m(\angle DEC) = 180^\circ - (45^\circ + 30^\circ) = 105^\circ$$

$\because E \in \overline{AC}$

$$\therefore m(\angle AED) = 180^\circ - 105^\circ = 75^\circ \text{ (Second req.)}$$

7

Given : ABCD is a parallelogram ,  $E \in \overline{CB}$  ,  
 $BC = BE$  ,  $DE = DC$

R.T.P. : The figure AEBD is a rectangle.

Proof :  $\because$  ABCD is a parallelogram.

$$\therefore AD = BC , \overline{AD} \parallel \overline{BC}$$

$$\because ES = BC , E \in \overline{CB} \text{ (Given)}$$

$$\therefore AD = ES , \overline{AD} \parallel \overline{ES}$$

$\therefore$  The figure AEBD is a parallelogram.

$$\because DE = DC \text{ (Given)}$$

$$\because AB = DC \text{ (Properties of parallelogram)}$$

$$\therefore DE = AB$$

$\therefore$  The two diagonals of the parallelogram AEBD  
are equal in length

$\therefore$  The figure AEBD is a rectangle. (Q.E.D.)

8

ABCD is a rectangle  $\therefore m(\angle B) = 90^\circ$

From  $\triangle ABH$  :

$$m(\angle AHB) = 180^\circ - (44^\circ + 90^\circ) = 46^\circ$$

$$\because H \in \overline{BC}$$

$$\therefore m(\angle AHB) + m(\angle AHD) + m(\angle DHC) = 180^\circ$$

$$\therefore 46^\circ + m(\angle AHD) + 46^\circ = 180^\circ$$

$$\therefore m(\angle AHD) = 88^\circ \text{ (The req.)}$$

9

From  $\triangle AHM$  :

$$m(\angle MAH) = 180^\circ - (90^\circ + 32^\circ) = 58^\circ$$

$\because$  ABCD is a rhombus ,  $\overline{AC}$  is a diagonal

$$\therefore m(\angle BAD) = 2 m(\angle BAM) = 2 \times 58^\circ = 116^\circ$$

$$\therefore m(\angle BCD) = 116^\circ$$

$$\therefore m(\angle ABC) = 180^\circ - 116^\circ = 64^\circ$$

$$\therefore m(\angle ADC) = 64^\circ \text{ (The req.)}$$

10

$\because$  ABCD is a rhombus and  $\overline{BD}$  is a diagonal in it.

$$\therefore m(\angle ABC) = 2 m(\angle ABD) = 2 \times 62^\circ = 124^\circ$$

$$\therefore m(\angle A) = 180^\circ - 124^\circ = 56^\circ \text{ (The req.)}$$

11

$\because$  XYZL is a rhombus

$$\therefore m(\angle XYL) = m(\angle XLY)$$

$\because \overline{DH} \parallel \overline{YX}$  ,  $\overline{YL}$  is a transversal

$$\therefore m(\angle HDL) = m(\angle XYL) \text{ (corresponding angles)}$$

$$\therefore m(\angle HDL) = m(\angle HLD) \text{ (Q.E.D.)}$$



## Geometry and Measurement

12

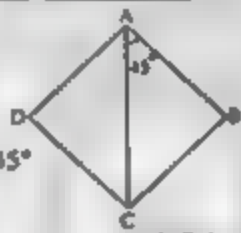
- $\therefore$  ABCD is a rectangle  $\therefore AD = BC$  (1)  
 $\therefore$  OBCH is a parallelogram  $\therefore OH = BC$  (2)  
 From (1) & (2):  $\therefore AD = OH$   
 by subtracting AH from both sides  
 $\therefore AD - AH = OH - AH$   
 $\therefore HD = AO$  (Q.E.D.)

13

- $\therefore$  ABCD is a square.  $\therefore \overline{AD} \parallel \overline{BC}$   
 $\therefore E \in \overline{BC}$   $\therefore \overline{AD} \parallel \overline{CE}$   
 $\therefore \overline{AC} \parallel \overline{DE}$   
 $\therefore$  ACED is a parallelogram. (First req.)  
 $\therefore$  ABCD is a square and  $\overline{AC}$  is a diagonal in it.  
 $\therefore m(\angle DAC) = 45^\circ$   
 $\therefore$  ACED is a parallelogram.  
 $\therefore m(\angle ACE) = 180^\circ - 45^\circ = 135^\circ$  (Second req.)

14

- $\therefore$  ABCD is a rhombus  
 $\therefore \overline{AC}$  bisects  $\angle BAD$   
 $\therefore m(\angle BAC) = m(\angle CAD) = 45^\circ$   
 $\therefore m(\angle BAD) = 90^\circ$   
 $\therefore$  ABCD is a square (Q.E.D.)



15

- $\therefore$  ABCD is a rectangle  $\therefore \overline{AB} \parallel \overline{DC}$   
 $\therefore$  ABYX is a square  $\therefore \overline{AB} \parallel \overline{XY}$   
 $\therefore \overline{XY} \parallel \overline{DC}$ ,  $\overline{YD}$  is a transversal  
 $\therefore m(\angle XYD) = m(\angle YDC) = 52^\circ$  (alternate angles)  
 $\therefore$  ABYX is a square,  $\overline{AY}$  is a diagonal  
 $\therefore m(\angle AYX) = 45^\circ$   
 $\therefore m(\angle AYD) = m(\angle AYX) + m(\angle XYD)$   
 $= 45^\circ + 52^\circ = 97^\circ$  (The req.)

16

- $\therefore$  ABCD is a square and  $\overline{BD}$  is a diagonal in it.  
 $\therefore m(\angle BDC) = 45^\circ$  &  $m(\angle C) = 90^\circ$   
 $\therefore$  In the quadrilateral DEFC:  
 $m(\angle EFC) = 360^\circ - (45^\circ + 90^\circ + 102^\circ) = 123^\circ$   
 $\therefore y = 123^\circ$   
 $\therefore F \in \overline{BC}$   
 $\therefore m(\angle AFB) = 180^\circ - 123^\circ = 57^\circ$

- $\therefore$  In the right-angled triangle ABF at B:  
 $m(\angle BAF) = 180^\circ - (90^\circ + 57^\circ) = 33^\circ$   
 $\therefore x = 33^\circ$  (The req.)

17

- $\therefore$  ABCD is a square.  $\therefore AM = CM$   
 $\therefore AE = CF$  and by subtracting  $\therefore EM = FM$   
 $\therefore BM = MD$  (properties of the square)  
 $\therefore$  The figure EBFD is a parallelogram  
 $\therefore$  ABCD is a square.  $\therefore \overline{AC} \perp \overline{BD}$   
 $\therefore \overline{EF} \perp \overline{BD}$   
 $\therefore$  The parallelogram EBFD is a rhombus. (Q.E.D.)

18

- $\therefore$  ABCD is a rectangle  $\therefore \overline{AB} \parallel \overline{DC}$   
 $\therefore E \in \overline{DC}$   $\therefore \overline{AB} \parallel \overline{CE}$   
 $\therefore \overline{AC} \parallel \overline{BE}$   
 $\therefore$  The figure ABEC is a parallelogram  
 $\therefore AB = CE$  but  $AB = DC$  (Q.E.D. 1)  
 $\therefore DC = CE$   
 $\therefore$  The two diagonals of the parallelogram ABEC bisect each other  
 $\therefore MC = \frac{1}{2} BC$   
 $\therefore AD = BC$   $\therefore MC = \frac{1}{2} AD$  (Q.E.D. 2)

19

- 1 square 2 parallelogram  
 3 rhombus 4 rectangle

20

- 1 all 2 some 3 all  
 4 some 5 all 6 some

21

- $\therefore$  ABCD is a parallelogram.  
 $\therefore AB = CD$  but  $AB = GF$   
 $\therefore CD = GF$  (1)  
 $\therefore AD = BC$  but  $AD = CG$   
 $\therefore BC = CG$  (2)  
 $\therefore$  The figure CEFG is a rhombus.  
 $\therefore CE = EF = FG = GC$  (3)  
 From (1) & (2) and (3):  
 $\therefore CD = BC = CG = CE$   
 $\therefore$  The two diagonals are equal in length and bisect each other.  
 $\therefore BEGD$  is a rectangle. (Q.E.D.)

### Answers of exams on the first part of unit three

#### Model 1

1 (b) 2 (b) 3 (d) 4 (c) 5 (b) 6 (d)

1 900° 2 360° 3 120°  
4 rhombus 5 trapezium

3 [a]  $a = 120^\circ$  [b] Prove by yourself

4 [a]  $m(\angle B) = 75^\circ$  [b]  $m(\angle A) = 72^\circ$

5 [a] Prove by yourself  
[b] 1 Prove by yourself  
2  $m(\angle ACE) = 135^\circ$

#### Model 2

1 1 (c) 2 (b) 3 (b) 4 (c) 5 (d) 6 (b)

2 1  $144^\circ$  2 bisect each other  
3 parallelogram 4  $125^\circ$  5  $118^\circ$

3 [a]  $x = 30^\circ$   
[b] Prove by yourself.

4 [a]  $x = 30^\circ$  ,  $y = 120^\circ$   
[b]  $m(\angle E) = 150^\circ$

5 [a] 1  $MD = 4 \text{ cm}$  ,  $AB = 3 \text{ cm}$ .  
2 The perimeter of  $\triangle ABM = 11 \text{ cm}$   
[b] Prove by yourself

#### Answers of Exercise 5

1 180°  
2 the measures of its nonadjacent interior angles  
3 right-angled 4 obtuse-angled 5 90°  
6 obtuse 7 60°

2 1 (a) 2 (b) 3 (a) 4 (c) 5 (c) 6 (b)

3

Fig (1) :  $m(\angle C) = 180^\circ - (70^\circ + 65^\circ) = 45^\circ$

Fig (2) :  $m(\angle A) = 180^\circ - (90^\circ + 50^\circ) = 40^\circ$

Fig (3) :  $m(\angle B) = 180^\circ - (28^\circ + 124^\circ) = 28^\circ$

Fig (4) :  $m(\angle ACB) = 52^\circ$  (V.O.A.)  
 $\therefore m(\angle A) = 180^\circ - (90^\circ + 52^\circ) = 38^\circ$

Fig (5) :  $m(\angle ACD) = 50^\circ + 60^\circ = 110^\circ$

Fig (6) :  $m(\angle ACD) = 67^\circ + 42^\circ = 109^\circ$

Fig (7) :  $m(\angle B) = 120^\circ - 60^\circ = 60^\circ$

Fig (8) :  $m(\angle A) = 125^\circ - 35^\circ = 90^\circ$

Fig (9) :  $m(\angle A) = \frac{100^\circ}{2} = 50^\circ$

Fig (10) :  $m(\angle DAB) = 60^\circ + 60^\circ = 120^\circ$

Fig (11) :  $m(\angle C) = \frac{180^\circ - 30^\circ}{2} = 75^\circ$

$\therefore m(\angle ABD) = 30^\circ + 75^\circ = 105^\circ$

Fig (12) :  $m(\angle XAC) = m(\angle ABC) + m(\angle ACB)$   
 $= (180^\circ - 110^\circ) + (180^\circ - 130^\circ)$   
 $= 70^\circ + 50^\circ = 120^\circ$

Fig (13) :  $m(\angle ACB) = 180^\circ - 120^\circ = 60^\circ$   
 $\therefore m(\angle A) = 100^\circ - 60^\circ = 40^\circ$

Fig (14) :  $m(\angle ABC) = 180^\circ - 150^\circ = 30^\circ$   
 $m(\angle ACD) = 25^\circ + 30^\circ = 55^\circ$

Fig (15) :  $m(\angle BCE) = 72^\circ + 48^\circ = 120^\circ$   
 $m(\angle ACB) = 180^\circ - 120^\circ = 60^\circ$   
 $m(\angle BAC) = 120^\circ - 53^\circ = 67^\circ$

Fig (16) :  $m(\angle ABC) = 115^\circ - 50^\circ = 65^\circ$   
 $\therefore m(\angle EBG) = 65^\circ$  (V.O.A.)

Fig (17) :  $\therefore m(\angle ACB) = 42^\circ$  (alternate angles)  
 $\therefore m(\angle BAC) = 180^\circ - (42^\circ + 63^\circ) = 75^\circ$

Fig (18) :  $m(\angle ABC) = m(\angle ACB)$   
 $= \frac{180^\circ - 70^\circ}{2} = 55^\circ$   
 $\therefore m(\angle ABD) = 180^\circ - (73^\circ + 55^\circ) = 52^\circ$   
 $\therefore m(\angle ACE) = 180^\circ - (82^\circ + 55^\circ) = 43^\circ$

4

Given :

$\overline{BD} \cap \overline{AE} = \{C\}$  ,  $\overline{AF} \parallel \overline{BC}$  ,

$m(\angle BAF) = 65^\circ$  ,  $m(\angle DCE) = 55^\circ$

R.T.F. : The measures of the interior angles of  $\triangle ABC$

Proof :

$\therefore \overline{BD} \cap \overline{AE} = \{C\}$  (given)

$\therefore m(\angle ACB) = m(\angle DCE) = 55^\circ$  (V.O.A.)

## Geometry and Measurement

- $\therefore \overline{AF} \parallel \overline{BC}$ ,  $\overline{AB}$  is a transversal to them  
 $\therefore m(\angle FAB) = m(\angle B) = 65^\circ$  (alternate angles)  
 The sum of measures of the interior angles of the triangle =  $180^\circ$   
 $\therefore m(\angle BAC) = 180^\circ - (55^\circ + 65^\circ) = 60^\circ$  (The req.)

5

Given :

$A \in \overline{DC}$ ,  $\overline{DE} \parallel \overline{CB}$ ,  $m(\angle D) = 100^\circ$ ,  
 $m(\angle B) = 40^\circ$

R.T.P. :  $m(\angle BAD)$ 

Proof :

- $\therefore \overline{DE} \parallel \overline{CB}$ ,  $\overline{CD}$  is a transversal  
 $\therefore m(\angle D) + m(\angle C) = 180^\circ$   
 (two interior angles in the same side of the transversal)  
 $\therefore m(\angle D) = 100^\circ$   
 $\therefore m(\angle C) = 180^\circ - 100^\circ = 80^\circ$   
 $\therefore \angle BAD$  is an exterior angle of  $\triangle ABC$   
 $\therefore m(\angle BAD) = m(\angle B) + m(\angle C)$   
 $= 40^\circ + 80^\circ = 120^\circ$  (The req.)

6

- $\therefore \overline{BD} \parallel \overline{CA}$ ,  $\overline{AB}$  is a transversal to them  
 $\therefore m(\angle A) = m(\angle ABD) = 75^\circ$  (alternate angles)  
 In  $\triangle ABC$ :  
 $m(\angle ABC) = 180^\circ - (75^\circ + 45^\circ) = 60^\circ$  (The req.)

7

- $\therefore \angle CAD$  is an exterior angle of  $\triangle ABC$   
 $\therefore m(\angle DAC) = m(\angle ACB) + m(\angle B)$   
 $\therefore 112^\circ = m(\angle ACB) + 58^\circ$   
 $\therefore m(\angle ACB) = 54^\circ$   $\therefore \overline{CE}$  bisects  $\angle ACB$   
 $\therefore m(\angle ECB) = \frac{54^\circ}{2} = 27^\circ$   
 $\therefore \angle AEC$  is an exterior angle of  $\triangle BCE$   
 $\therefore m(\angle AEC) = 58^\circ + 27^\circ = 85^\circ$  (The req.)

8

- $\therefore \overline{DE} \parallel \overline{BC}$ ,  $\overline{DB}$  is a transversal to them  
 $\therefore m(\angle B) + m(\angle D) = 180^\circ$   
 (two interior angles in the same side of the transversal)  
 $\therefore m(\angle B) = 180^\circ - 100^\circ = 80^\circ$   
 $\therefore$  In  $\triangle ABC$ :  
 $m(\angle BAC) = 180^\circ - (80^\circ + 30^\circ) = 70^\circ$  (The req.)

9

- $\therefore \overline{EF} \parallel \overline{CD}$ ,  $\overline{BE}$  is a transversal  
 $\therefore m(\angle ABC) = m(\angle E) = 50^\circ$  (alternate angles)  
 $\therefore$  In  $\triangle ABC$ :  
 $m(\angle BAC) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$  (First req.)  
 $\therefore \angle ABD$  is an exterior angle of  $\triangle ABC$   
 $\therefore m(\angle ABD) = m(\angle BAC) + m(\angle C)$   
 $= 100^\circ + 30^\circ = 130^\circ$  (Second req.)

10

- $\therefore \overline{FR} \parallel \overline{BC}$ ,  $\overline{FB}$  is a transversal to them  
 $\therefore m(\angle B) + m(\angle F) = 180^\circ$   
 (two interior angles in the same side of the transversal)  
 $\therefore m(\angle B) = 180^\circ - 135^\circ = 45^\circ$   
 $\therefore \overline{DE} \parallel \overline{BC}$  and  $\overline{DC}$  is a transversal to them  
 $\therefore m(\angle C) + m(\angle D) = 180^\circ$   
 (two interior angles in the same side of the transversal)  
 $\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$   
 $\therefore$  In  $\triangle ABC$ :  
 $m(\angle BAC) = 180^\circ - (45^\circ + 60^\circ) = 75^\circ$  (The req.)

11

- $\therefore \overline{EF} \parallel \overline{AB}$ ,  $\overline{AE}$  is a transversal to them  
 $\therefore m(\angle A) + m(\angle E) = 180^\circ$   
 $\therefore m(\angle A) = 180^\circ - 110^\circ = 70^\circ$   
 Similarly :  $m(\angle B) + m(\angle D) = 180^\circ$   
 $\therefore m(\angle B) = 180^\circ - 120^\circ = 60^\circ$   
 In  $\triangle ABG$   
 $m(\angle BGA) = 180^\circ - (60^\circ + 70^\circ) = 50^\circ$   
 $\therefore \overline{AE} \cap \overline{BD} = \{G\}$   
 $\therefore m(\angle EGD) = m(\angle BGA) = 50^\circ$  (The req.)

12

- $\therefore \overline{DE} \parallel \overline{BC}$ ,  $\overline{DB}$  is a transversal to them.  
 $\therefore m(\angle ADE) = m(\angle B) = 60^\circ$  (corresponding angles)  
 $\therefore$  In  $\triangle ADE$   
 $m(\angle AED) = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$   
 $m(\angle DEC) = 180^\circ - 40^\circ = 140^\circ$  (The req.)



13

- $\overline{DF} \parallel \overline{BC}$ ,  $\overline{EC}$  is a transversal to them
- $\therefore m(\angle C) = m(\angle FEC) = 70^\circ$  (alternate angles)
- $\overline{DF} \parallel \overline{BC}$ ,  $\overline{AB}$  is a transversal to them
- $\therefore m(\angle B) = m(\angle ADE) = 60^\circ$  (corresponding angles)
- $\therefore$  In  $\triangle ABC$ ,
- $m(\angle A) = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$  (The req.)

14

- In  $\triangle ABC$ :
- $m(\angle C) = 180^\circ - (64^\circ + 52^\circ) = 64^\circ$
- $\therefore m(\angle DEC) + m(\angle C) = 116^\circ + 64^\circ = 180^\circ$
- (but they are interior angles in the same side of the transversal)
- $\therefore \overline{DE} \parallel \overline{BC}$  (Q.E.D.)

15

- $\therefore \overline{AY} \cap \overline{BE} = \{C\}$
- $\therefore m(\angle XCF) = m(\angle ACB) = 42^\circ$  (V.O.A.)
- Similarly:  $m(\angle XFC) = m(\angle YFZ) = 53^\circ$
- $\therefore$  In  $\triangle XFC$ ,
- $m(\angle FXC) = 180^\circ - (42^\circ + 53^\circ) = 85^\circ$
- $\therefore \overline{EB} \cap \overline{DF} = \{X\}$
- $\therefore m(\angle DXE) = m(\angle FXC) = 85^\circ$  (V.O.A.) (First req.)
- $\therefore X \in \overline{EC}$
- $\therefore m(\angle DXC) = 180^\circ - 85^\circ = 95^\circ$  (Second req.)
- $\therefore \overline{DF} \cap \overline{EC} = \{X\}$
- $\therefore m(\angle EXF) = m(\angle DXC) = 95^\circ$  (V.O.A.) (Third req.)

16

- $\overline{ED} \parallel \overline{BF}$ ,  $\overline{AE}$  is a transversal to them
- $\therefore m(\angle E) + m(\angle CAF) = 180^\circ$
- (two interior angles in the same side of the transversal)
- $m(\angle E) = 180^\circ - 110^\circ = 70^\circ$
- $\therefore$  In  $\triangle DCE$ :
- $m(\angle D) = 180^\circ - (60^\circ + 70^\circ) = 50^\circ$
- $\overline{ED} \parallel \overline{BA}$ ,  $\overline{DB}$  is a transversal to them
- $\therefore m(\angle B) = m(\angle D) = 50^\circ$  (alternate angles)
- $\therefore m(\angle ACB) = m(\angle DCE) = 60^\circ$  (V.O.A.)
- $\therefore$  In  $\triangle ABC$ :
- $m(\angle BAC) = 180^\circ - (50^\circ + 60^\circ) = 70^\circ$  (The req.)

17

- ABCD is a rectangle
- $\therefore m(\angle BAD) = m(\angle ADC) = 90^\circ$
- $m(\angle EAD) = 90^\circ - 30^\circ = 60^\circ$
- $m(\angle EDA) = 90^\circ - 60^\circ = 30^\circ$
- $\therefore$  In  $\triangle AED$ ,
- $m(\angle AED) = 180^\circ - (60^\circ + 30^\circ) = 90^\circ$  (The req.)

18

- In  $\triangle ABC$ ,
- $m(\angle ABC) + m(\angle ACB) = 180^\circ - 80^\circ = 100^\circ$
- $\therefore \overline{BM}$  bisects  $\angle ABC$ ,  $\overline{CM}$  bisects  $\angle ACB$
- $\therefore m(\angle MBC) + m(\angle MCB) = \frac{1}{2} \times 100^\circ = 50^\circ$
- $\therefore$  In  $\triangle MBC$ ,
- $m(\angle BMC) = 180^\circ - 50^\circ = 130^\circ$
- $m(\angle EMD) = m(\angle BMC) = 130^\circ$  (V.O.A.) (The req.)

19

- $\therefore \angle EBD$  is an exterior angle of  $\triangle ABC$
- $\therefore m(\angle EBD) = m(\angle A) + m(\angle C) = 63^\circ + 56^\circ = 119^\circ$
- $\therefore \angle EDX$  is an exterior angle of  $\triangle EDB$
- $\therefore m(\angle EDX) = m(\angle DEB) + m(\angle EBD)$
- $= 34^\circ + 119^\circ = 153^\circ$  (The req.)

20

- $\angle ADC$  is an exterior angle of  $\triangle ABD$
- $\therefore m(\angle ADC) = m(\angle B) + m(\angle BAD)$
- $= 35^\circ + 25^\circ = 60^\circ$
- $\angle XAC$  is an exterior angle of  $\triangle ADC$
- $\therefore m(\angle XAC) = m(\angle ADC) + m(\angle C)$
- $= 60^\circ + 60^\circ = 120^\circ$  (The req.)

21

- $\therefore A \in \overline{BF}$
- $\therefore m(\angle BAC) + m(\angle CAF) = 180^\circ$
- $\therefore m(\angle BAC) = 180^\circ - 110^\circ = 70^\circ$
- Similarly:  $m(\angle EAB) = 180^\circ - 130^\circ = 50^\circ$
- $\overline{AE} \parallel \overline{BC}$ ,  $\overline{AB}$  is a transversal to them
- $\therefore m(\angle B) = m(\angle EAB) = 50^\circ$  (alternate angles)
- $\therefore \angle ACD$  is an exterior angle of  $\triangle ABC$
- $\therefore m(\angle ACD) = m(\angle BAC) + m(\angle B)$
- $= 70^\circ + 50^\circ = 120^\circ$  (The req.)

37



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22

$\overline{OK} \parallel \overline{ED}$ ,  $\overline{EO}$  is a transversal to them

$$m(\angle DEO) + m(\angle O) = 180^\circ$$

(Two interior angles on the same side of the transversal)

$$\therefore m(\angle DEO) = 180^\circ - 120^\circ = 60^\circ$$

$\therefore \overline{ED}$  bisects  $\angle AEO$

$$\therefore m(\angle AED) = 60^\circ$$

$\therefore \overline{AB} \parallel \overline{ED}$ ,  $\overline{AE}$  is a transversal to them

$$\therefore m(\angle BAE) = m(\angle AED) = 60^\circ \text{ (Alternate angles)}$$

$\therefore \overline{AC}$  bisects  $\angle BAE$

$$\therefore m(\angle CAE) = 30^\circ$$

$\therefore \angle AED$  is an exterior angle of  $\triangle ACE$

$$m(\angle C) = 60^\circ - 30^\circ = 30^\circ \quad (\text{The req.})$$

23

$\therefore \overline{AC} \parallel \overline{DF}$ ,  $\overline{FC}$  is a transversal to them.

$$\therefore m(\angle ACB) = m(\angle F) \text{ (corresponding angles)}$$

In  $\triangle ABC$ ,  $DEF$

$$m(\angle B) = m(\angle DEF) = 90^\circ \text{ (given)}$$

$$\therefore m(\angle ACB) = m(\angle F) \text{ (by proof)}$$

$$m(\angle A) = m(\angle D) \quad (\text{Q.E.D.})$$

24

$\overline{AD}$  bisects  $\angle BAC$

$$m(\angle BAD) = m(\angle CAD)$$

$$m(\angle B) = m(\angle C) \quad m(\angle ADB) = m(\angle ADC)$$

$\triangle ABD$ ,  $\triangle ACD$  are them

$$\begin{cases} m(\angle BAD) = m(\angle CAD) \\ m(\angle ADB) = m(\angle ADC) \end{cases}$$

$$\overline{AD} \text{ is a common side}$$

$\therefore \triangle ABD \cong \triangle ACD$ , then we deduce that  $AB = AC$   
(Q.E.D.)

25

$\overline{AB} \parallel \overline{ED}$ ,  $\overline{AD}$  is a transversal

$$\therefore m(\angle BAC) = m(\angle ADE) \text{ (alternate angles)}$$

$\therefore$  In  $\triangle ABC$  and  $DEA$

$$\therefore m(\angle B) = m(\angle E), m(\angle BAC) = m(\angle ADE)$$

$$\therefore m(\angle ACB) = m(\angle DAE)$$

(and they are alternate angles).

$$\therefore \overline{BC} \parallel \overline{AE}$$

(Q.E.D.)

26

In  $\triangle XYZ$  and  $\triangle XML$

$$m(\angle XML) = m(\angle Y), \angle X \text{ is a common angle}$$

$$m(\angle XLM) = m(\angle Z) = 52^\circ \quad (\text{The req.})$$

27

$\angle ACB$  is an exterior angle of  $\triangle ACD$

$$\therefore m(\angle ACB) > m(\angle D)$$

$$m(\angle ACB) = m(\angle B)$$

$$m(\angle B) > m(\angle D) \quad (\text{Q.E.D.})$$

28

$\angle DAC$  is an exterior angle of  $\triangle ABC$

$$\therefore m(\angle DAC) = m(\angle B) + m(\angle C)$$

$$m(\angle B) = m(\angle C)$$

$$\frac{1}{2} m(\angle DAC) = m(\angle B)$$

$$\therefore m(\angle DAE) = m(\angle B)$$

(and they are corresponding angles)

$$\therefore \overline{AE} \parallel \overline{BC} \quad (\text{Q.E.D.})$$

29

$$m(\angle 1) = m(\angle A) + m(\angle 2) = m(\angle C)$$

$$\therefore m(\angle 1) + m(\angle 2) = m(\angle A) + m(\angle C)$$

$$m(\angle ABC) = m(\angle A) + m(\angle C)$$

$$\angle ABC \text{ is a right angle} \quad (\text{Q.E.D.})$$

30

In  $\triangle ABC$

$$m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

$$2m(\angle C) + 4m(\angle C) + m(\angle C) = 180^\circ$$

$$\therefore 7m(\angle C) = 180^\circ \quad m(\angle C) = \frac{180^\circ}{7}$$

$$m(\angle B) = 4 \times \frac{180^\circ}{7} = 102 \frac{6}{7}^\circ$$

$$\therefore \angle B \text{ is an obtuse angle} \quad (\text{Q.E.D.})$$

31

In  $\triangle ABC$

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

$$\therefore 4x + 2x + 2^\circ + 28^\circ = 180^\circ$$

$$\therefore 6x + 30^\circ = 180^\circ$$

$$\therefore 6x = 150^\circ$$

$$\therefore x = 25^\circ$$

$$\therefore m(\angle A) = 4 \times 25^\circ = 100^\circ$$

$$\therefore m(\angle B) = 2 \times 25^\circ + 2 = 52^\circ$$

$$\therefore m(\angle C) = 2 \times 25^\circ + 2 = 52^\circ \quad (\text{The req.})$$

32

Comparing the measures of the angles of the two triangles ABC and ADE

$$\therefore m(\angle B) = m(\angle EDA) = 90^\circ$$

$$m(\angle ACB) = m(\angle E)$$

$$\therefore m(\angle CAD) = m(\angle DAE)$$

(1)

$\therefore \angle CDB$  is an exterior angle of  $\triangle CDA$

$$\therefore m(\angle CDB) > m(\angle CAD)$$

$$\text{From (1): } \therefore m(\angle CDB) > m(\angle DAE) \quad (\text{Q.E.D.})$$

### Answers of Exercise 6

1

1) bisects the third side.

2) parallel to

3) half the length of the third side

4)  $E \neq AC$

$$5) \overline{DE} \parallel \overline{BC}$$

$$6) 3$$

$$7) 90^\circ$$

$$8) 12$$

$$9) 17$$

$$10) 2.5$$

$$11) 3$$

$$12) 3, 45^\circ$$

2

Given: ABCD is a parallelogram,  $DC = 5$  cm,

$F \in \overline{BC}$ ,  $\overline{DF} \cap \overline{AB} = \{E\}$ ,  $DF = FE$

R.T.F.: The length of  $\overline{BE}$

Proof: ABCD is a parallelogram

$$\therefore \overline{AD} \parallel \overline{BC} \quad \therefore \overline{BF} \parallel \overline{AD}$$

In  $\triangle AED$ :

$$\therefore F \text{ is the midpoint of } \overline{DE} \therefore \overline{BF} \parallel \overline{AD}$$

$$\therefore B \text{ is the midpoint of } \overline{AE}$$

$$\therefore AB = BE$$

$$\therefore AB = DC = 5 \text{ cm. (properties of parallelogram)}$$

$$\therefore BE = 5 \text{ cm. (The req.)}$$

3

Given: ABC is a triangle in which  $CA = CB$ ,

E is the midpoint of  $\overline{AB}$ ,  $\overline{EF} \parallel \overline{AC}$ ,

H and G are the two midpoints of  $\overline{BD}$

and  $\overline{CD}$  respectively.

R.T.P.:  $EF = GH$

Proof: In  $\triangle ABC$ :

$$\therefore E \text{ is the midpoint of } \overline{AB} \therefore \overline{EF} \parallel \overline{AC}$$

$$\therefore F \text{ is the midpoint of } \overline{BC} \therefore EF = \frac{1}{2} AC$$

In  $\triangle BDC$ :

$$\therefore H \text{ is the midpoint of } \overline{BD} \therefore G \text{ is the midpoint of } \overline{CD}$$

$$\therefore GH = \frac{1}{2} BC \quad \because CA = BC$$

$$\therefore EF = GH \quad (\text{Q.E.D.})$$

4

In  $\triangle ABC$ :

$$\therefore E \text{ is the midpoint of } \overline{AB} \therefore \overline{EX} \parallel \overline{BC}$$

$$\therefore X \text{ is the midpoint of } \overline{AC}$$

In  $\triangle ACD$ :

$$\therefore X \text{ is the midpoint of } \overline{AC} \therefore \overline{XY} \parallel \overline{CD}$$

$$\therefore Y \text{ is the midpoint of } \overline{AD}$$

$$\therefore AY = \frac{6}{2} = 3 \text{ cm.}$$

(The req.)

5

In  $\triangle DBC$

$$X \text{ is the midpoint of } \overline{DC} \therefore \overline{XY} \parallel \overline{BC}$$

$$\therefore Y \text{ is the midpoint of } \overline{DB}$$

In  $\triangle ADB$ :

$$Y \text{ is the midpoint of } \overline{DB} \therefore Z \text{ is the midpoint of } \overline{AB}$$

$$\therefore \overline{ZY} \parallel \overline{AD}$$

$$\therefore \overline{DB} \text{ is a transversal to them}$$

$$\therefore m(\angle ZYB) = m(\angle ADB) = 40^\circ$$

(corresponding angles)

(The req.)

6

1 The perimeter of the parallelogram

$$ABCD = 2(12 + 8) = 40 \text{ cm}$$

2  $\therefore$  ABCD is a parallelogram, M is the point

of intersection of its diagonals

$$\therefore M \text{ is the midpoint of } \overline{BD}$$

In  $\triangle ABD$ :

$$\therefore M \text{ is the midpoint of } \overline{BD} \therefore \overline{MO} \parallel \overline{AD}$$

$$\therefore O \text{ is the midpoint of } \overline{AB}$$

$$\therefore AO = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4 \text{ cm} \quad (\text{The req.})$$

7

ABCD is a parallelogram, M is the point of

intersection of its diagonals

$$\therefore M \text{ is the midpoint of } \overline{BD} \therefore \overline{MX} \parallel \overline{AB}$$

$$\therefore X \text{ is the midpoint of } \overline{AD} \quad (\text{First req.})$$

$$\therefore MX = \frac{1}{2} AB$$

$$\therefore AB = 2MX = 2 \times 5 = 10 \text{ cm.}$$

$$\therefore DC = AB = 10 \text{ cm.} \quad (\text{Second req.})$$

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∴ ABCD is a parallelogram.

• M is the intersection point of its diagonals.

∴ M is the midpoint of  $\overline{BD}$

In  $\triangle ABD$ :

∴ M is the midpoint of  $\overline{BD}$ ,  $\overline{ME} \parallel \overline{AB}$

∴ E is the midpoint of  $\overline{AD}$

∴  $ME = \frac{1}{2} AB$  ∵  $AB = DC$

∴  $ME = \frac{1}{2} DC$  (Q.E.D.)



9

∴ ABCD is a parallelogram.

∴  $\overline{AB} \parallel \overline{DC}$  ∴  $\overline{OC} \parallel \overline{AB}$

In  $\triangle ABH$ : ∵ C is the midpoint of  $\overline{BH}$ ,  $\overline{CO} \parallel \overline{AB}$

∴ O is the midpoint of  $\overline{AH}$

∴  $AO = OH$  (Q.E.D.)

10

∴ ABCD is a trapezium.

∴  $\overline{AD} \parallel \overline{BC}$

∴  $\overline{AD} \parallel \overline{XY}$

∴  $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$

∴  $AX = XB$

∴  $DY = YC$

∴ Y is the midpoint of  $\overline{DC}$

In  $\triangle HDC$ : ∵ Y is the midpoint of  $\overline{DC}$

∴  $\overline{YZ} \parallel \overline{DH}$  ∴ Z is the midpoint of  $\overline{CH}$

∴  $CZ = ZH$  (Q.E.D.)

11

∴ D, E are the midpoints of  $\overline{AB}$ ,  $\overline{AC}$  respectively

∴  $\overline{DE} \parallel \overline{BC}$  ∴  $\overline{AX} \parallel \overline{BC}$

∴  $\overline{DE} \parallel \overline{AX}$

∴  $Y \in \overline{DE}$  ∴  $\overline{EY} \parallel \overline{AX}$

In  $\triangle ACX$ :

∴  $\overline{EY} \parallel \overline{AX}$  ∴ E is the midpoint of  $\overline{AC}$

∴ Y is the midpoint of  $\overline{XC}$  (Q.E.D.)

12

In  $\triangle ACD$ :

∴ Z is the midpoint of  $\overline{AD}$ ,  $\overline{ZY} \parallel \overline{CD}$

∴ Y is the midpoint of  $\overline{AC}$

∴  $AY = YC = 4$  cm. (First req.)

In  $\triangle ABC$ :

∴ X is the midpoint of  $\overline{AB}$

∴ Y is the midpoint of  $\overline{AC}$

∴  $XY = \frac{1}{2} BC = \frac{1}{2} \times 6 = 3$  cm

∴ The perimeter of  $\triangle AXY = 5 + 3 + 4 = 12$  cm (Second req.)

13

In  $\triangle ABC$ :

∴ D is the midpoint of  $\overline{AB}$  ∴ E is the midpoint of  $\overline{BC}$

∴  $DE = \frac{1}{2} AC = 3.5$  cm.

∴ E is the midpoint of  $\overline{BC}$  ∴ F is the midpoint of  $\overline{AC}$

∴  $EF = \frac{1}{2} AB = 2.5$  cm.

∴ F is the midpoint of  $\overline{AC}$  ∴ D is the midpoint of  $\overline{AB}$

∴  $DF = \frac{1}{2} BC = 4$  cm

∴ The perimeter of  $\triangle DEF = 3.5 + 2.5 + 4 = 10$  cm. (The req.)

14

In  $\triangle XYZ$ :

H is the midpoint of  $\overline{XY}$ , G is the midpoint of  $\overline{XZ}$

∴  $HG = \frac{1}{2} YZ$  ∴  $YZ = 2 HG$  (1)

∴ H is the midpoint of  $\overline{XY}$

∴ O is the midpoint of  $\overline{YZ}$

∴  $HO = \frac{1}{2} XZ$  ∴  $XZ = 2 HO$  (2)

∴ O is the midpoint of  $\overline{YZ}$  ∴ G is the midpoint of  $\overline{XZ}$

∴  $OG = \frac{1}{2} XY$  ∴  $XY = 2 OG$  (3)

∴ The perimeter of  $\triangle XYZ = XY + YZ + XZ$

∴ From (1) + (2) + (3) ∴ The perimeter of  $\triangle XYZ$

$= 2 OG + 2 HG + 2 HO = 2 (OG + HG + HO)$

$= 2$  the perimeter of  $\triangle HOG = 2 \times 18 = 36$  cm.

(The req.)

15

In  $\triangle ABC$ :

∴ X is the midpoint of  $\overline{AB}$

∴ Z is the midpoint of  $\overline{AC}$

∴  $XZ = \frac{1}{2} BC$  (1)

∴ X is the midpoint of  $\overline{AB}$

∴ Y is the midpoint of  $\overline{BC}$  ∴  $XY = \frac{1}{2} AC$  (2)





∵ Y is the midpoint of  $\overline{BC}$  ∴ Z is the midpoint of  $\overline{AC}$

$$\therefore YZ = \frac{1}{2} AB \quad (3)$$

By adding (1) + (2) + (3) :

$$\therefore XZ + XY + YZ = \frac{1}{2} (BC + AC + AB)$$

∴ The perimeter of  $\triangle XYZ$

$$= \frac{1}{2} \text{ the perimeter of } \triangle ABC \quad (\text{Q.E.D.})$$

16

In  $\triangle ABC$  :

D is the midpoint of  $\overline{AB}$  ∴ F is the midpoint of  $\overline{AC}$

$$\therefore DF = \frac{1}{2} BC = 6 \text{ cm.}$$

Similarly : ∵ D is the midpoint of  $\overline{AB}$

∴ E is the midpoint of  $\overline{BC}$

$$\therefore DE = \frac{1}{2} AC = 5 \text{ cm}$$

∴ The perimeter of the figure DECF

$$= 6 + 5 + 6 + 5 = 22 \text{ cm.} \quad (\text{The req.})$$

17

In  $\triangle DBC$  :

∵ H is the midpoint of  $\overline{BD}$

∴ O is the midpoint of  $\overline{DC}$

$$\therefore HO \parallel BC$$

$$\therefore AD \parallel BC \quad \therefore HO \parallel AD \quad (1)$$

$$\therefore HO = \frac{1}{2} BC$$

$$\therefore AD = \frac{1}{2} BC \quad \therefore HO = AD \quad (2)$$

From (1) + (2) :

∴ AHOD is a parallelogram. (Q.E.D.)

18

In  $\triangle ABC$  :

∴ D is the midpoint of  $\overline{AB}$  ∴ E is the midpoint of  $\overline{AC}$

$$\therefore DE \parallel BC \quad \therefore F \in \overline{CB}$$

$$\therefore DE \parallel BF \quad (1)$$

$$\therefore DE = \frac{1}{2} BC \quad BF = \frac{1}{2} BC$$

$$\therefore DE = BF \quad (2)$$

From (1) and (2) :

∴ BEDF is a parallelogram. (Q.E.D.)

19

In  $\triangle MBC$  :

∵ H is the midpoint of  $\overline{MB}$  ∴ O is the midpoint of  $\overline{MC}$

$$\therefore HO \parallel BC$$

$$\therefore BC \parallel AD$$

$$\therefore HO \parallel AD$$

(1)

$$\therefore HO = \frac{1}{2} BC$$

$$\therefore AD = \frac{1}{2} BC$$

$$\therefore HO = AD \quad (2)$$

From (1) + (2) :

∴ AHOD is a parallelogram. (Q.E.D.)

20

In  $\triangle ABC$  :

∴ D is the midpoint of  $\overline{AB}$  ∴ E is the midpoint of  $\overline{AC}$

$$\therefore DE \parallel BC \quad DE = \frac{1}{2} BC = 6 \text{ cm.}$$

In  $\triangle FDE$  :

∵ X is the midpoint of  $\overline{DF}$  ∴  $\overline{XY} \parallel \overline{DE}$

∴ Y is the midpoint of  $\overline{EF}$

$$\therefore XY = \frac{1}{2} DE = 3 \text{ cm.} \quad (\text{The req.})$$

21

In  $\triangle ABH$  :

X is the midpoint of  $\overline{AB}$  ∴  $\overline{XY} \parallel \overline{BH}$

∴ Y is the midpoint of  $\overline{AH}$

∵ ABCD is a parallelogram

$$\therefore \overline{AB} \parallel \overline{CD}$$

In  $\triangle ABH$  : ∵ Y is the midpoint of  $\overline{AH}$  ∴  $\overline{YC} \parallel \overline{AB}$

∴ C is the midpoint of  $\overline{BH}$  (Q.E.D.)

22

In  $\triangle ABC$  :

D is the midpoint of  $\overline{AB}$  ∴  $\overline{DH} \parallel \overline{BC}$

∴ H is the midpoint of  $\overline{AC}$

$$\therefore AH = HC$$

$$\therefore HC = CO$$

$$\therefore AH = HC = CO = \frac{1}{2} AO \quad (\text{Q.E.D.1})$$

In  $\triangle DHO$

∴ C is the midpoint of  $\overline{HO}$  ∴  $\overline{CX} \parallel \overline{DH}$

∴ X is the midpoint of  $\overline{DO}$

$$\therefore OX = XD \quad (\text{Q.E.D.2})$$



## Geometry and Measurement

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In  $\triangle BCO$ • D is the midpoint of  $\overline{CB}$ ,  $\overline{DZ} \parallel \overline{OC}$ • Z is the midpoint of  $\overline{BO}$ •  $OZ = ZB$ 

(1)

• In  $\triangle AZD$ : H is the midpoint of  $\overline{AD}$ ,  $\overline{HO} \parallel \overline{DZ}$ • O is the midpoint of  $\overline{AZ}$ •  $AO = OZ$ 

(2)

From (1) & (2)  $\therefore AO = OZ = ZB$ 

(Q.E.D.)

24

• ABCD is a parallelogram

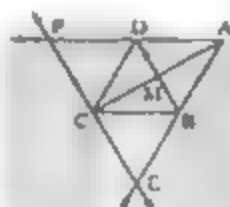
• M is the point of intersection of its diagonals

M is the midpoint of  $\overline{AC}$ • In  $\triangle AEC$ :• M is the midpoint of  $\overline{AC}$ ,  $\overline{MB} \parallel \overline{EC}$ B is the midpoint of  $\overline{AE}$  $AB = BE$ 

(Q.E.D. 1)

• In  $\triangle ACF$ :• M is the midpoint of  $\overline{AC}$ ,  $\overline{MD} \parallel \overline{CF}$ D is the midpoint of  $\overline{AF}$ •  $AD = DF$ 

(Q.E.D. 2)



25

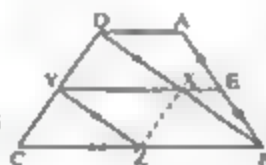
•  $\overline{AD} \parallel \overline{BC}$ ,  $\overline{HX} \parallel \overline{BC}$  $\overline{AD} \parallel \overline{BC} \parallel \overline{HX}$ ,  $\therefore AH = HB$  $\therefore DX = XC$ • X is the midpoint of  $\overline{DC}$ • In  $\triangle DBC$ :• X is the midpoint of  $\overline{DC}$ ,  $\overline{XY} \parallel \overline{BD}$ Y is the midpoint of  $\overline{BC}$ 

(Q.E.D.)

26

Construction: Draw  $\overline{XZ}$ 

Proof:

•  $\overline{AD} \parallel \overline{EY} \parallel \overline{BC}$ ,  $AE = EB$  $\therefore DX = XB$ ,  $DY = YC$  $\therefore$  In  $\triangle DBC$ • Y is the midpoint of  $\overline{DC}$ ,  $\overline{YZ} \parallel \overline{BD}$ • Z is the midpoint of  $\overline{BC}$ • X is the midpoint of  $\overline{BD}$  $\therefore \overline{XZ} \parallel \overline{CD}$ ,  $\overline{XD} \parallel \overline{ZY}$  $\therefore XDYZ$  is a parallelogram $\therefore XD = YZ$ 

(Q.E.D.)

27

Construction: Draw  $\overline{AF} \parallel \overline{DX}$ 

Proof:

•  $\overline{AF} \parallel \overline{DX} \parallel \overline{EY} \parallel \overline{BC}$ •  $AD = DE = EB = 3$  cm $\therefore AX = XY = YC = \frac{3}{2}$ From  $\triangle AEY$ :  $\therefore$  D is the midpoint of  $\overline{AE}$ • X is the midpoint of  $\overline{AY}$  $\therefore DX = \frac{1}{2} EY$   $\therefore EY = 2 DX = 8$  cm

The perimeter of the shape DEYX

 $= 4 + 3 + 8 + \frac{3}{2} = 15 + 2\frac{1}{2} = 17\frac{1}{2}$  cm. (The req.)

28

Construction: Draw  $\overline{AC}$ Proof: In  $\triangle ABC$ X is the midpoint of  $\overline{AB}$ • Y is the midpoint of  $\overline{BC}$  $\overline{XY} \parallel \overline{AC}$ ,  $XY = \frac{1}{2} AC$ 

(1)

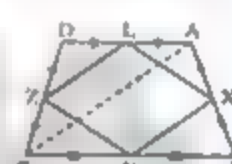
• In  $\triangle ADC$ L is the midpoint of  $\overline{AD}$ • Z is the midpoint of  $\overline{CD}$  $\overline{LZ} \parallel \overline{AC}$ ,  $LZ = \frac{1}{2} AC$ 

(2)

From (1) and (2):

•  $\overline{XY} \parallel \overline{LZ}$ ,  $XY = LZ$  $\therefore XYZL$  is a parallelogram

(Q.E.D.)

In  $\triangle ABC$ X is the midpoint of  $\overline{AB}$ • Y is the midpoint of  $\overline{BC}$  $\therefore XY = \frac{1}{2} AC = AZ$ 

(1)

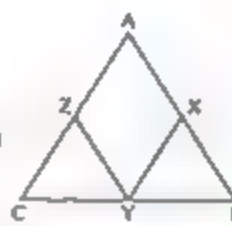
•  $\overline{XY} \parallel \overline{AC}$ •  $\overline{XY} \parallel \overline{AZ}$ 

(2)

From (1) &amp; (2)

•  $AXYZ$  is a parallelogram•  $AX = \frac{1}{2} AB$ ,  $AZ = \frac{1}{2} AC$ •  $AC = AB$  $\therefore AZ = AX$ •  $AXYZ$  is a rhombus

(Q.E.D.)

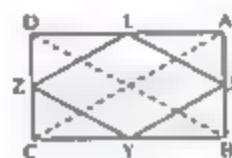


30

Construction :

Draw  $\overline{BD}$  and  $\overline{AC}$ Proof : In  $\triangle ABD$  $\because$  X is the midpoint of  $\overline{AB}$  $\therefore$  L is the midpoint of  $\overline{AD}$  $\therefore \overline{XL} \parallel \overline{BD}$  ,  $XL = \frac{1}{2} BD$  $\therefore$  In  $\triangle DBC$  : $\therefore$  Y is the midpoint of  $\overline{BC}$  , Z is the midpoint of  $\overline{DC}$  $\therefore \overline{YZ} \parallel \overline{BD}$  ,  $YZ = \frac{1}{2} BD$ From (1) & (2) :  $\overline{XL} \parallel \overline{YZ}$  ,  $XL = YZ$  $\therefore$  XYZL is a parallelogram  $\therefore$  From  $\triangle ABC$  : $\therefore$  X is the midpoint of  $\overline{AB}$  $\therefore$  Y is the midpoint of  $\overline{BC}$  $\therefore XY = \frac{1}{2} AC$ 

From (1) &amp; (3)

 $\therefore$  The two diagonals of the rectangle are equal in length $\therefore LX = XY$  $\therefore$  XYZL is a rhombus $\therefore$  the perimeter of the rhombus $= 4 XL = 4 \times \frac{1}{2} BD = 2 BD$ 

(1)

(2)

(3)

(Q.E.D. 1)

(Q.E.D. 2)

31

 $\therefore$  ABCD is a parallelogram  $\therefore$  its diagonals intersect at M $\therefore$  M is the midpoint of each of  $\overline{AC}$  and  $\overline{DB}$  $\therefore$  DBCH is a parallelogram  $\therefore$  its diagonals intersect at L $\therefore$  L is the midpoint of each of  $\overline{DC}$  and  $\overline{BH}$ In  $\triangle DBC$  : $\therefore$  M is the midpoint of  $\overline{DB}$  $\therefore$  L is the midpoint of  $\overline{DC}$  $\therefore \overline{ML} \parallel \overline{BC}$  $\therefore ML = \frac{1}{2} BC$  $\therefore$  ABCD is a parallelogram $\therefore$  DBCH is a parallelogram $\therefore AH = AD + DH = 2 BC$  $\therefore$  From (1) : $\therefore ML = \frac{1}{2} (\frac{1}{2} AH) = \frac{1}{4} AH$ 

(Q.E.D. 1)

(1)

 $\therefore BC = AD$  $\therefore BC = DH$  $\therefore BC = \frac{1}{2} AH$ 

(Q.E.D. 2)

32

In  $\triangle ABC$  $\therefore$  F is the midpoint of  $\overline{AB}$  , E is the midpoint of  $\overline{BC}$  $\therefore FE = \frac{1}{2} AC$  $\therefore 2x = 3x - 6$  $\therefore AB = 2 \times 6 + 1 = 13 \text{ cm}$  $\therefore$  D is the midpoint of  $\overline{AC}$  , E is the midpoint of  $\overline{BC}$  $\therefore DE = \frac{1}{2} AB$  $y = 6.5 \text{ cm}$  $\therefore x = \frac{1}{2} (3x - 6)$  $\therefore x = 6 \text{ cm}$  (First req) $\therefore y = \frac{1}{2} AB$ 

(Second req)

33

In  $\triangle ABD$  : $\therefore$  X is the midpoint of  $\overline{AB}$  , $\therefore$  L is the midpoint of  $\overline{AD}$  $\therefore XL = \frac{1}{2} BD = 32 \text{ cm}$  $\therefore$  In  $\triangle ADC$  : $\therefore$  L is the midpoint of  $\overline{AD}$  , $\therefore$  Z is the midpoint of  $\overline{CD}$  $\therefore LZ = \frac{1}{2} AC = 45 \text{ cm}$ In  $\triangle BCD$  $\therefore$  Y is the midpoint of  $\overline{BC}$  , Z is the midpoint of  $\overline{CD}$  $\therefore YZ = \frac{1}{2} BD = 32 \text{ cm}$ In  $\triangle ABC$  : $\therefore$  X is the midpoint of  $\overline{AB}$  , Y is the midpoint of  $\overline{BC}$  $\therefore XY = \frac{1}{2} AC = 45 \text{ cm}$  $\therefore$  The length of the stripe  $= 32 + 45 + 32 + 45 = 154 \text{ cm}$ 

(The req)



34

 $\therefore \angle BAO$  is an exterior angle of  $\triangle ABC$  $\therefore m(\angle BAO) = m(\angle ABC) + m(\angle ACB)$  $\therefore m(\angle ABC) = m(\angle ACB)$  $\therefore m(\angle BAO) = 2m(\angle ABC)$  $\therefore \frac{1}{2} m(\angle BAO) = m(\angle ABC)$  $\therefore m(\angle HAB) = m(\angle ABC)$  and they are alternate angles $\therefore \overline{BC} \parallel \overline{AH}$  $\therefore$  In  $\triangle AHD$  :  $\therefore$  C is the midpoint of  $\overline{AD}$  ,  $\overline{CB} \parallel \overline{AH}$  $\therefore$  B is the midpoint of  $\overline{DH}$  $\therefore DB = BH$ 

(Q.E.D.)

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## Geometry and Measurement

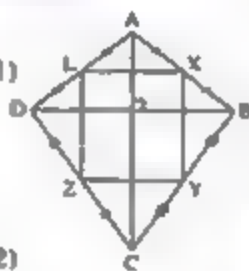
24

In  $\triangle ABC$ :  $\because X$  is the midpoint of  $\overline{AB}$  $\therefore Y$  is the midpoint of  $\overline{BC}$  $\therefore \overline{XY} \parallel \overline{AC}, XY = \frac{1}{2} AC$  (1) $\therefore$  From  $\triangle ADC$ : $\therefore L$  is the midpoint of  $\overline{AD}$  $\therefore Z$  is the midpoint of  $\overline{DC}$  $\therefore \overline{LZ} \parallel \overline{AC}, LZ = \frac{1}{2} AC$  (2)From (1) & (2):  $\therefore \overline{XY} \parallel \overline{LZ}, XY = LZ$  $\therefore XYZL$  is a parallelogram. $\therefore$  In  $\triangle ABD$ : $\therefore X$  is the midpoint of  $\overline{AB}$  &  $L$  is the midpoint of  $\overline{AD}$  $\therefore \overline{XL} \parallel \overline{BD}, XL = \frac{1}{2} BD$  $\therefore \because \overline{AC} \perp \overline{BD}$  are perpendicular. $\therefore \overline{XY} \parallel \overline{AC}, \overline{XL} \parallel \overline{BD}$  $\therefore \overline{XY}$  and  $\overline{XL}$  are perpendicular $\therefore XYZL$  is a rectangle $\therefore$  The area of the rectangle

$$= XL \times XY = \frac{1}{2} BD \times \frac{1}{2} AC$$

$$= \frac{1}{4} AC \times BD$$

(Q.E.D.)



## Answers of Exercise 7

1

Fig. (1): In  $\triangle ABC$ :  $\because m(\angle A) = 90^\circ$ 

$$\therefore (BC)^2 = (AB)^2 + (AC)^2 = 64 + 36 = 100$$

$$\therefore BC = \sqrt{100} = 10 \text{ cm.}$$

Fig. (2): In  $\triangle ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$= 144 + 25 = 169$$

$$\therefore AC = \sqrt{169} = 13 \text{ cm.}$$

Fig. (3): In  $\triangle ABC$ :  $\because m(\angle C) = 90^\circ$ 

$$\therefore (AB)^2 = (AC)^2 + (BC)^2$$

$$= 256 + 144 = 400$$

$$\therefore AB = \sqrt{400} = 20 \text{ cm}$$

Fig. (4): In  $\triangle ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (BC)^2 = (AC)^2 - (AB)^2$$

$$= 225 - 81 = 144$$

$$\therefore BC = \sqrt{144} = 12 \text{ cm.}$$

Fig. (5): In  $\triangle ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (BC)^2 = (AC)^2 - (AB)^2$$

$$= 625 - 225 = 400$$

$$\therefore BC = \sqrt{400} = 20 \text{ cm.}$$

Fig. (6): In  $\triangle ABC$ :  $\because m(\angle A) = 90^\circ$ 

$$\therefore (AB)^2 = (BC)^2 - (AC)^2$$

$$= 625 - 576 = 49$$

$$\therefore AB = \sqrt{49} = 7 \text{ cm}$$

2

 $\because ABCD$  is a square. $\therefore DC = 4 \text{ cm.} \therefore m(\angle DCB) = 90^\circ$  $\therefore$  In  $\triangle DCE$ :  $(DE)^2 = (DC)^2 + (CE)^2$ 

$$= (4)^2 + (3)^2 = 16 + 9 = 25$$

 $\therefore DE = 5 \text{ cm.}$ 

(The req.)

3

In  $\triangle ADC$ :  $\because m(\angle ADC) = 90^\circ$ 

$$\therefore (AD)^2 = (AC)^2 - (DC)^2 = 400 - 256 = 144$$

$$AD = \sqrt{144} = 12 \text{ cm}$$

(First req.)

 $\therefore$  In  $\triangle ABD$ :  $\because m(\angle ADB) = 90^\circ$ 

$$\therefore (AB)^2 = (AD)^2 + (BD)^2 = 144 + 81 = 225$$

$$\therefore AB = \sqrt{225} = 15 \text{ cm.}$$

(Second req.)

 $\therefore$  The area of  $\triangle ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 25 \times 12$ 

$$= 150 \text{ cm}^2$$

(Third req.)

4

In  $\triangle ABD$ :  $\because m(\angle ADB) = 90^\circ$ 

$$\therefore (BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$$

$$\therefore BD = \sqrt{100} = 10 \text{ cm}$$

 $\therefore$  In  $\triangle ADC$ :  $\because m(\angle ADC) = 90^\circ$ 

$$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$$

$$\therefore CD = \sqrt{324} = 18 \text{ cm.}$$

$$\therefore BC = BD + DC = 10 + 18 = 28 \text{ cm.}$$

(First req.)

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 28 \times 24$$

$$= 336 \text{ cm}^2$$

(Second req.)



5

In  $\Delta XYZ$ :  $\because m(\angle Y) = 90^\circ$ 

$$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$$

$$\therefore XZ = \sqrt{625} = 25 \text{ cm.} \quad (\text{First req.})$$

In  $\Delta XLZ$ :  $\because m(\angle L) = 90^\circ$ 

$$\therefore (LZ)^2 = (XZ)^2 - (XL)^2 = 625 - 225 = 400$$

$$\therefore LZ = \sqrt{400} = 20 \text{ cm.} \quad (\text{Second req.})$$

6

In  $\Delta ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 81 + 144 = 225$$

$$\therefore AC = \sqrt{225} = 15 \text{ cm.} \quad (\text{First req.})$$

In  $\Delta ACD$ :  $\because m(\angle ACD) = 90^\circ$ 

$$\therefore (AD)^2 = (AC)^2 + (CD)^2 = 225 + 400 = 625$$

$$\therefore AD = \sqrt{625} = 25 \text{ cm.} \quad (\text{Second req.})$$

The perimeter of the figure ABCD

$$= 9 + 12 + 20 + 25 = 66 \text{ cm.} \quad (\text{Third req.})$$

The area of the figure ABCD

= The area of  $\Delta ABC$  + The area of  $\Delta ACD$ 

$$= \frac{1}{2} \times 12 \times 9 + \frac{1}{2} \times 20 \times 15 = 54 + 150 = 204 \text{ cm}^2 \quad (\text{Fourth req.})$$

7

In  $\Delta ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (BC)^2 = (AC)^2 - (AB)^2 = 100 - 64 = 36$$

$$\therefore BC = \sqrt{36} = 6 \text{ cm.} \quad (\text{First req.})$$

In  $\Delta ABD$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 289 - 64 = 225$$

$$\therefore BD = \sqrt{225} = 15 \text{ cm.} \quad (\text{Second req.})$$

$$\therefore CD = BD - BC$$

$$\therefore CD = 15 - 6 = 9 \text{ cm.} \quad (\text{Third req.})$$

8

In  $\Delta ABD$ :  $\because m(\angle ABD) = 90^\circ$ 

$$\therefore (BD)^2 = (AD)^2 - (AB)^2 = 225 - 81 = 144$$

$$\therefore BD = \sqrt{144} = 12 \text{ cm.}$$

 $\because \overline{AB} \parallel \overline{CD}$ ,  $\overline{BD}$  is a transversal

$$\therefore m(\angle BDC) = m(\angle ABD) = 90^\circ$$

In  $\Delta BDC$ :  $m(\angle BDC) = 90^\circ$ 

$$\therefore (BC)^2 = (BD)^2 + (DC)^2 = 144 + 25 = 169$$

$$\therefore BC = \sqrt{169} = 13 \text{ cm.} \quad (\text{The req.})$$

9

 $\because \overline{AD} \parallel \overline{EC}$ ,  $m(\angle AEB) = m(\angle C) = 90^\circ$ 

and they are corresponding angles

 $\therefore \overline{AE} \parallel \overline{DC}$   $\therefore$  The figure AECD is a parallelogram

$$\therefore m(\angle C) = 90^\circ$$

The figure AECD is a rectangle

$$\therefore EC = AD = 9 \text{ cm.}$$

$$\therefore BE = 17 - 9 = 8 \text{ cm.}$$

In  $\Delta AEB$ :  $m(\angle AEB) = 90^\circ$ 

$$\therefore (AE)^2 = (AB)^2 - (BE)^2 = 289 - 64 = 225$$

$$\therefore AE = \sqrt{225} = 15 \text{ cm.}$$

$$\therefore DC = AE = 15 \text{ cm.}$$

(First req.)

The area of the trapezium ABCD

= The area of  $\Delta ABE$  + The area of the rectangle AECD

$$= \frac{1}{2} \times 8 \times 15 + 9 \times 15 = 195 \text{ cm}^2 \quad (\text{Second req.})$$

10

In  $\Delta ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 36 + 64 = 100$$

$$\therefore AC = \sqrt{100} = 10 \text{ cm.}$$

 $\because \overline{AB} \parallel \overline{DE}$ ,  $\overline{BD}$  is a transversal

$$\therefore m(\angle D) = m(\angle B) \quad (\text{alternate angles})$$

In  $\Delta ABC$  &  $\Delta EDC$ 

$$\begin{cases} m(\angle D) = m(\angle B) \\ m(\angle ACB) = m(\angle ECD) \\ BC = DC \end{cases} \quad (\text{V.O.A.})$$

$$\therefore \Delta ABC \cong \Delta EDC$$

$$\therefore CE = CA = 10 \text{ cm.} \quad (\text{The req.})$$

11

In  $\Delta ABC$ :  $\because m(\angle B) = 90^\circ$ 

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = 16 + 9 = 25$$

$$\therefore AC = \sqrt{25} = 5 \text{ cm.}$$

$$\therefore \text{The area of } \Delta ABC = \frac{1}{2} \times BC \times AB$$

$$= \frac{1}{2} \times AC \times BD$$

$$\therefore \frac{1}{2} \times 3 \times 4 = \frac{1}{2} \times 5 \times BD \quad \therefore 6 = \frac{5}{2} \times BD$$

$$\therefore BD = 6 \div \frac{5}{2} = 2.4 \text{ cm.} \quad (\text{The req.})$$



## Geometry and Measurement

- 12 The sum of areas of the squares on the sides of the right angle

1 15      2 15      3 144      4 10  
5 13      6 7      7 169      8 4

- 13 1 (d)      2 (b)      3 (c)      4 (c)

14

Constr. : Draw  $\overline{AC}$ Proof : In  $\triangle ABC$ 

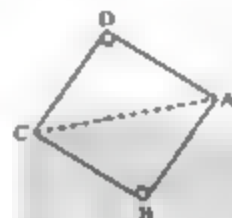
$$\therefore m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 \quad (1)$$

$$\text{In } \triangle ADC \quad \therefore m(\angle D) = 90^\circ$$

$$\therefore (AC)^2 = (AD)^2 + (DC)^2 \quad (2)$$

$$\text{From (1) \& (2) } \therefore (AB)^2 + (BC)^2 = (AD)^2 + (DC)^2 \quad (\text{Q.E.D.})$$



15

$$\text{In } \triangle ABC : \therefore m(\angle B) = 90^\circ$$

$$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$$

$$\therefore AB = \sqrt{256} = 16 \text{ cm.}$$

$$\therefore BD = 9 \text{ cm} \quad \therefore AD = 16 - 9 = 7 \text{ cm} \quad (\text{First req.})$$

$$\therefore AE = 2 BC = 2 \times 12 = 24 \text{ cm}$$

$$\therefore \overline{EA} \parallel \overline{BC}, \overline{AB} \text{ is a transversal}$$

$$\therefore m(\angle EAB) = m(\angle B) = 90^\circ \quad (\text{alternate angles})$$

$$\therefore \text{In } \triangle EAD \quad \therefore m(\angle EAD) = 90^\circ$$

$$\therefore (ED)^2 = (EA)^2 + (AD)^2 = 576 + 49 = 625$$

$$\therefore ED = \sqrt{625} = 25 \text{ cm} \quad (\text{Second req.})$$

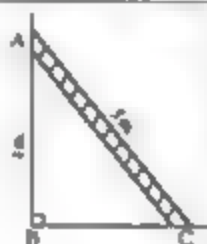
16

$$\text{In } \triangle ABC : \therefore m(\angle B) = 90^\circ$$

$$\therefore (BC)^2 = (AC)^2 - (AB)^2 \\ = 25 - 16 = 9$$

$$\therefore BC = \sqrt{9} = 3 \text{ m.}$$

$$\therefore \text{The distance} = 3 \text{ m} \quad (\text{The req.})$$



17

M is the midpoint of  $\overline{AB}$ 

$$\therefore AM = \frac{15}{2} = 7.5 \text{ m.}$$



$$\text{In } \triangle ACM : \therefore m(\angle A) = 90^\circ$$

$$\therefore (MC)^2 = (AC)^2 + (AM)^2 = 16 + 56.25 = 72.25$$

$$\therefore MC = \sqrt{72.25} = 8.5 \text{ m} \quad (\text{The req.})$$

18

$$\text{In } \triangle ABC : \therefore m(\angle B) = 90^\circ$$

$$\therefore (AB)^2 = (AC)^2 - (BC)^2$$

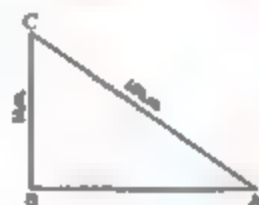
$$= 100 - 36 = 64$$

$$\therefore AB = \sqrt{64} = 8 \text{ km.}$$

$\therefore$  If Mina takes the two roads (way to school and way to airport), the distance will be  $= 6 + 8 = 14 \text{ km.}$

If he takes the main road, the distance will be  $= 10 \text{ km.}$

$$\therefore \text{The distance he saves} = 14 - 10 = 4 \text{ km} \quad (\text{The req.})$$



19

$$\text{In } \triangle ABC \quad m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$= (AB)^2 + (2BD)^2$$

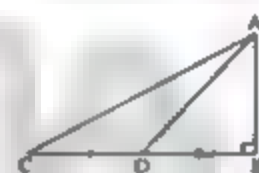
$$= (AB)^2 + 4(BD)^2 \quad (1)$$

$$\text{In } \triangle ABD \quad m(\angle B) = 90^\circ$$

$$\therefore (AD)^2 = (AB)^2 + (BD)^2 \quad (2)$$

By subtracting (2) from (1)

$$\therefore (AC)^2 - (AD)^2 = 3(BD)^2 \quad (\text{Q.E.D.})$$



20

$$\text{Let } r_1 = \frac{1}{2} AC, r_2 = \frac{1}{2} AB, r_3 = \frac{1}{2} BC$$

$$\text{In } \triangle ABC : \therefore m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$\therefore \left(\frac{1}{2} AC\right)^2 = \left(\frac{1}{2} AB\right)^2 + \left(\frac{1}{2} BC\right)^2$$

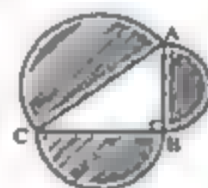
$$\therefore r_1^2 = r_2^2 + r_3^2 \text{ multiplying by } \frac{1}{2} \pi$$

$$\therefore \frac{1}{2} \pi r_1^2 = \frac{1}{2} \pi r_2^2 + \frac{1}{2} \pi r_3^2$$

$$\therefore \text{The area of the semicircle drawn on the hypotenuse}$$

equals the sum of areas of the two semicircles drawn

on the two sides of the right angle  $(\text{Q.E.D.})$



### Answers of exams on the second part of unit three

#### Model 1

- 1 (b) 2 (d) 3 (a)  
4 (c) 5 (c) 6 (b)

- 2 1 the sum of measures of its non adjacent interior angles.  
2 acute 3 parallel  
4 the sum of areas of squares on the sides of the right angle.  
5 bisects the third side

- 3 [a]  $m(\angle A) = 75^\circ$ ,  $m(\angle B) = 55^\circ$ ,  $m(\angle C) = 50^\circ$   
[b] The perimeter of  $\triangle DEP = 11 \frac{1}{2}$  cm.

- 4 [a]  $XZ = 25$  cm,  $LZ = 20$  cm.  
[b]  $m(\angle A) = 40^\circ$ ,  $m(\angle B) = 60^\circ$ ,  $m(\angle C) = 80^\circ$

- 5 [a] Prove by yourself  
[b] 1  $BC = 28$  cm.  
2 The area of  $\triangle ABC = 336$  cm<sup>2</sup>

#### Model 2

- 1 (c) 2 (d) 3 (d) 4 (b) 5 (a) 6 (c)

- 2 1 obtuse-angled 2  $(AC)^2 - (BC)^2$   
3 17 4  $70^\circ$  5 3

- 3 [a]  $YZ = 4$  cm  
[b]  $AB = 20$  cm,  $AD = 12$  cm.

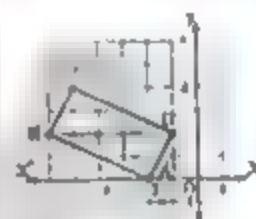
- 4 [a]  $m(\angle C) = 40^\circ$   
 $m(\angle ABC) = 45^\circ$ ,  $m(\angle BAC) = 95^\circ$   
 $m(\angle ABD) = 135^\circ$   
[b] Prove by yourself

- 5 [a] 1 The perimeter of ABCD = 32 cm  
2 The area of ABCD = 36 cm<sup>2</sup>  
[b] 1  $m(\angle A) = 67^\circ$   
2  $m(\angle BCE) = 120^\circ$

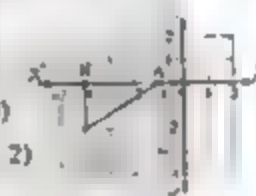
#### Answers of Exercise 8

- 1 1 translation 2 rotation 3 reflection  
2 1 rotation 2 rotation 3 translation  
3 1 reflection 2 rotation 3 translation  
4 1 translation 2 rotation 3 reflection  
5 1 translation 2 rotation 3 reflection  
6 1 translation 2 reflection 3 rotation  
4 rotation 5 reflection

- 7 1  $(X, y) \rightarrow (-X, y)$   
 $A(2, 0) \rightarrow \hat{A}(-2, 0)$   
 $B(6, 2) \rightarrow \hat{B}(-6, 2)$   
 $C(5, 4) \rightarrow \hat{C}(-5, 4)$   
 $D(1, 2) \rightarrow \hat{D}(-1, 2)$   
(The type is reflection)



- 2  $(X, y) \rightarrow (-X, -y)$   
 $A(1, 0) \rightarrow \hat{A}(-1, 0)$   
 $B(4, 0) \rightarrow \hat{B}(-4, 0)$   
 $C(4, 2) \rightarrow \hat{C}(-4, 2)$   
(The type is rotation)



- 3  $(X, y) \rightarrow (X + 2, y + 3)$   
 $A(-1, 1) \rightarrow \hat{A}(1, 4)$   
 $B(1, -1) \rightarrow \hat{B}(3, 2)$   
 $C(3, 1) \rightarrow \hat{C}(5, 4)$   
 $D(1, 3) \rightarrow \hat{D}(3, 6)$   
(The type is translation)



- 4  $(X, y) \rightarrow (X, y - 3)$   
 $A(2, 5) \rightarrow \hat{A}(2, 2)$   
 $B(4, 3) \rightarrow \hat{B}(4, 0)$   
 $C(4, 0) \rightarrow \hat{C}(4, -3)$   
 $O(0, 0) \rightarrow \hat{O}(0, -3)$   
 $E(0, 3) \rightarrow \hat{O}(0, 0)$   
(The type is translation)



## Geometry and Measurement

$$8 \quad (X, y) \longrightarrow (y, -x)$$

$$\therefore A(-4, 0) \longrightarrow C(0, -4)$$

$$\therefore B(-3, 6) \longrightarrow \hat{B}(6, 3)$$

$$\therefore C(0, 4) \longrightarrow \hat{C}(4, 0)$$

$$\therefore O(0, 0) \longrightarrow O(0, 0)$$

(The type is rotation)



$$9 \quad (X, y) \longrightarrow (-X, y)$$

$$\therefore A(4, 1) \longrightarrow \hat{A}(-4, 1)$$

$$\therefore B(3, 2) \longrightarrow \hat{B}(-3, 2)$$

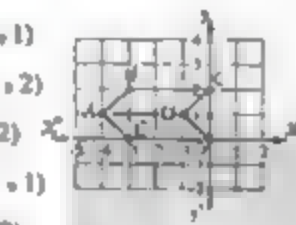
$$\therefore C(0, 2) \longrightarrow \hat{C}(0, 2)$$

$$\therefore D(1, 1) \longrightarrow \hat{D}(-1, 1)$$

$$\therefore O(0, 0) \longrightarrow O(0, 0)$$

$$\therefore L(3, 0) \longrightarrow \hat{L}(-3, 0)$$

(The type is reflection)



10

$$1 \quad (X, y) \longrightarrow (-X, y)$$

$$\therefore A(4, -3) \longrightarrow \hat{A}(-4, -3)$$

$$\therefore B(4, 1) \longrightarrow \hat{B}(-4, 1)$$

$$\therefore C(1, -3) \longrightarrow \hat{C}(-1, -3)$$

(The type is reflection)



$$2 \quad (X, y) \longrightarrow (-X, -y)$$

$$\therefore A(4, -3) \longrightarrow \hat{A}(-4, 3)$$

$$\therefore B(4, 1) \longrightarrow \hat{B}(-4, -1)$$

$$\therefore C(1, -3) \longrightarrow \hat{C}(-1, 3)$$

(The type is rotation)



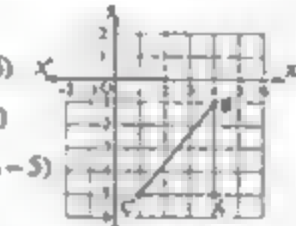
$$3 \quad (X, y) \longrightarrow (X, y - 2)$$

$$\therefore A(4, -3) \longrightarrow \hat{A}(4, -5)$$

$$\therefore B(4, 1) \longrightarrow \hat{B}(4, -1)$$

$$\therefore C(1, -3) \longrightarrow \hat{C}(1, -5)$$

(The type is translation)



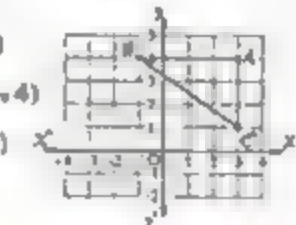
$$4 \quad (X, y) \longrightarrow (-y, X)$$

$$\therefore A(4, -3) \longrightarrow \hat{A}(3, 4)$$

$$\therefore B(4, 1) \longrightarrow \hat{B}(-1, 4)$$

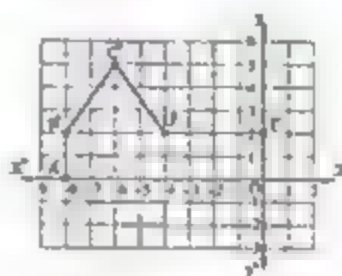
$$\therefore C(1, -3) \longrightarrow \hat{C}(3, 1)$$

(The type is rotation)



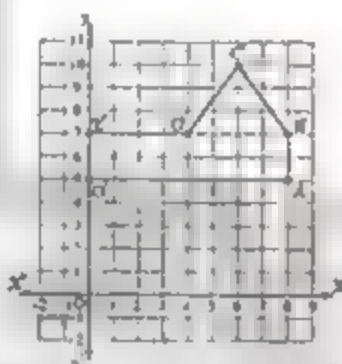
9

$$1 \quad (X, y) \longrightarrow (-X, y)$$



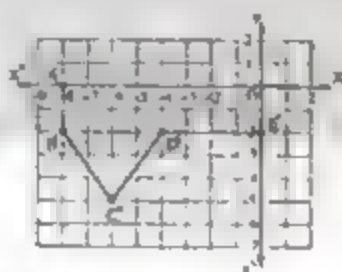
The type is reflection

$$2 \quad (X, y) \longrightarrow (X, y + 5)$$



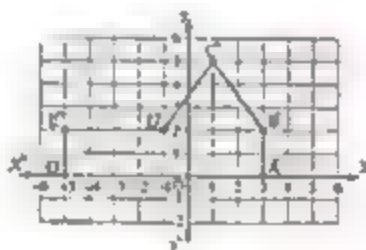
The type is translation

$$3 \quad (X, y) \longrightarrow (-X, -y)$$



The type is rotation

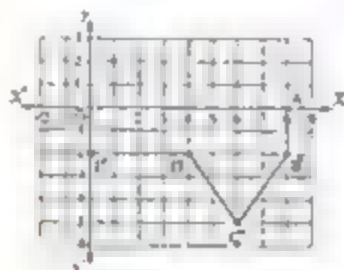
$$4 \quad (X, y) \longrightarrow (X - 5, y)$$



The type is translation



9)  $(x, y) \rightarrow (x, -y)$



The type is reflection

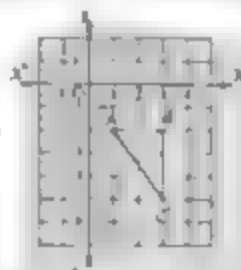
10

1)  $(x, y) \rightarrow (x, -y)$

$\therefore A(1, 2) \rightarrow \bar{A}(1, -2)$

$\therefore B(3, 2) \rightarrow \bar{B}(3, -2)$

$\therefore C(3, 5) \rightarrow \bar{C}(3, -5)$



2)  $(x, y) \rightarrow (x+1, y-3)$

$\therefore A(1, 2) \rightarrow \bar{A}(2, -1)$

$\therefore B(3, 2) \rightarrow \bar{B}(4, -1)$

$\therefore C(3, 5) \rightarrow \bar{C}(4, 2)$



3)  $(x, y) \rightarrow (-y, x)$

$\therefore A(1, 2) \rightarrow \bar{A}(-2, 1)$

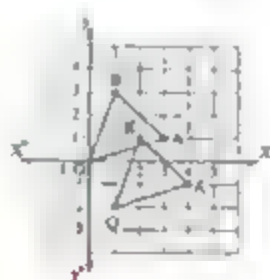
$\therefore B(3, 2) \rightarrow \bar{B}(-2, 3)$

$\therefore C(3, 5) \rightarrow \bar{C}(-5, 3)$



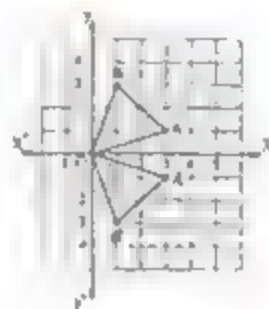
11

1)  $(x, y) \rightarrow (x+1, y-2)$



The type is translation

2)  $(x, y) \rightarrow (x, -y)$



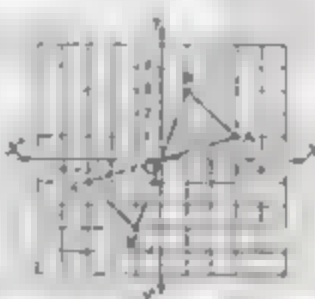
The type is reflection

3)  $(x, y) \rightarrow (-y, x)$



The type is rotation

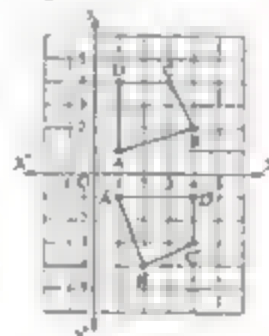
4)  $(x, y) \rightarrow (-x, -y)$



The type is rotation

12

1)  $(x, y) \rightarrow (y, -x)$

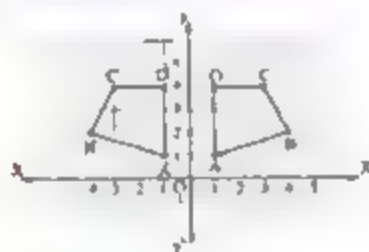


The type is rotation



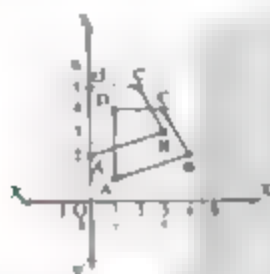
# Geometry and Measurement

$$(X, y) \longrightarrow (-X, y)$$



The type is reflection

$$(X, y) \longrightarrow (X-1, y+1)$$



The type is translation.

13

$$(X, y) \longrightarrow (y, X)$$

$$\therefore A(X, y) \longrightarrow \hat{A}(1, -1)$$

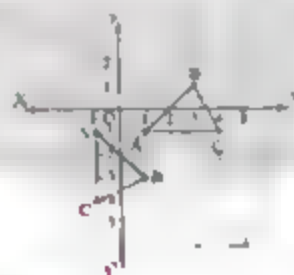
$$\therefore A(-1, -1)$$

$$\therefore B(X, y) \longrightarrow \hat{B}(3, 1)$$

$$\therefore B(1, -3)$$

$$\therefore C(X, y) \longrightarrow \hat{C}(4, -4)$$

$$\therefore C(-1, -)$$



The type is rotation

## Answers of Exercise 9

First : Problems on reflection in the plane :

1



Fig. (1)

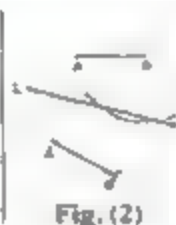


Fig. (2)

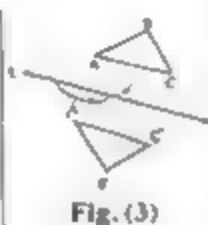


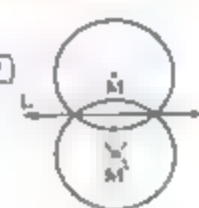
Fig. (3)

50

2



2



3



1



2



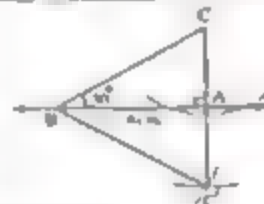
3



4

(4) , (5) has no axis of symmetry

4



5



6



7



8

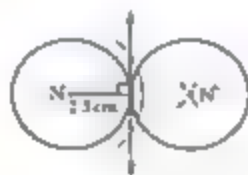
The resulting figure is a rectangle and its perimeter =  $2 \times (12 + 4) = 32$  cm.



9



10



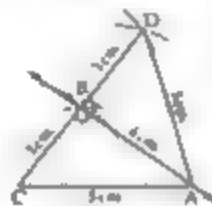
11

1 The perimeter of  $\triangle ACD$ 

$$= 16 \text{ cm.}$$

2 The area of  $\triangle ACD$ 

$$= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$$



12

1 The point D

2 BM

3  $\triangle DLM$ 4  $\triangle BYM$ 5  $\triangle AXM$ 6  $\triangle DMC$ 7  $\triangle BMA$ 

8 The square DZML, the square ALMX

9 The square DCBA

10 XZ

11 BD

13

1  $\overline{AE} + \overline{BF} + \overline{CD}$  2  $\overline{AE}$  3  $\overline{CF} + \overline{BD}$ 4  $\triangle AMF + \triangle AMF$  measures of the angles5  $\triangle AMC$ 6  $\triangle AMC + \triangle BMA$  lengths of the line segments

14

- 1 (a) the lengths of line segments  
(b) the measures of angles.  
(c) parallelism (d) betweenness

2 the axis of symmetry of the figure.

- 3 (a) 3 (b) 1 (c) zero  
(d) zero (e) 2 (f) 2  
(g) 4 (h) zero  
(i) 1 (j) an infinite number

4  $20^\circ$ 

15

the straight line L is the axis of symmetry of the figure ABCDE

$$\therefore m(\angle A) = m(\angle E) = 130^\circ$$

$$+ m(\angle D) = m(\angle B) = 110^\circ$$

from the pentagon ABCDE

$$m(\angle BCD) = 540^\circ - (130^\circ + 130^\circ + 110^\circ + 110^\circ)$$

$$= 60^\circ$$

(The req)

16

A is the image of itself

by reflection in  $\overline{AB}$ 

B is the image of itself

by reflection in  $\overline{AB}$ C is the image of C by reflection in  $\overline{AB}$ A is the image of A by reflection in  $\overline{AB}$ m( $\angle CAB$ ) = m( $\angle CAB$ )

$$m(\angle CAC) = 2m(\angle CAB) \quad (\text{Q.E.D. 1}) \quad (1)$$

similarly we can prove that

$$m(\angle ACA) = 2m(\angle ACD) \quad (2)$$

ABCD is a rectangle  $\overline{AB} \parallel \overline{CD}$ 

$$m(\angle BAC) = m(\angle ACD) \text{ (alternate angles)} \quad (3)$$

from (1) + (2) + (3)

$$\therefore m(\angle CAC) = m(\angle ACA)$$

but they are alternate angles

$$\therefore \overline{AC} \parallel \overline{AC} \quad (\text{Q.E.D. 2})$$

### Second : Problems on reflection in the Cartesian plane :

1

$$1 \quad A(2, 4) \xrightarrow[\text{X-axis}]{\text{its image in}} \hat{A}(2, -4)$$

$$B(4, 0) \xrightarrow[\text{X-axis}]{\text{its image in}} \hat{B}(4, 0)$$

$$C(0, 2) \xrightarrow[\text{X-axis}]{\text{its image in}} \hat{C}(0, -2)$$

$$D(-2, 1) \xrightarrow[\text{X-axis}]{\text{its image in}} \hat{D}(-2, -1)$$

$$E(-3, -2) \xrightarrow[\text{X-axis}]{\text{its image in}} \hat{E}(-3, 2)$$

$$2 \quad A(2, 4) \xrightarrow[\text{Y-axis}]{\text{its image in}} \hat{A}(-2, 4)$$

$$B(4, 0) \xrightarrow[\text{Y-axis}]{\text{its image in}} \hat{B}(-4, 0)$$

Geometry and Measurement

$$C(0, -2) \xrightarrow[\text{y-axis}]{\text{its image in}} \hat{C}(0, +2)$$

$$D(-2, 1) \xrightarrow[\text{y-axis}]{\text{its image in}} \hat{D}(2, 1)$$

$$E(-3, -2) \xrightarrow[\text{y-axis}]{\text{its image in}} \hat{E}(3, -2)$$

2

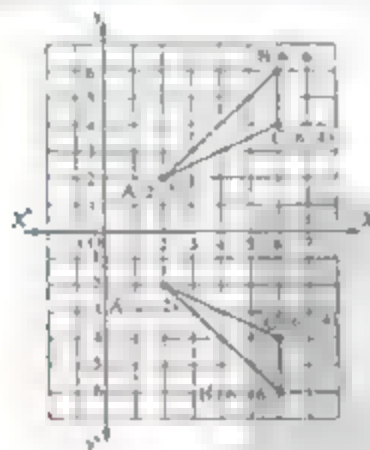


Fig. (1)



Fig. (2)



Fig. (3)

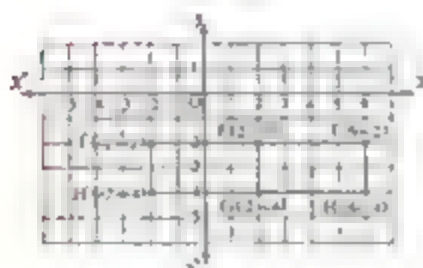
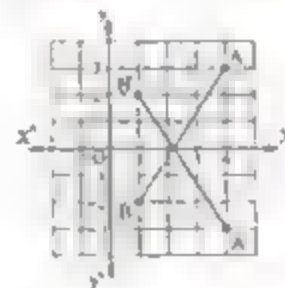


Fig. (4)

3

$$1) A(4, 3) \xrightarrow[\text{x-axis}]{\text{its image in}} \hat{A}(4, -3)$$

$$B(1, -2) \xrightarrow[\text{x-axis}]{\text{its image in}} \hat{B}(1, 2)$$



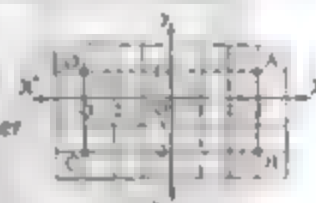
$$2) A(4, 3) \xrightarrow[\text{y-axis}]{\text{its image in}} \hat{A}(-4, 3)$$

$$B(1, -2) \xrightarrow[\text{y-axis}]{\text{its image in}} \hat{B}(-1, -2)$$

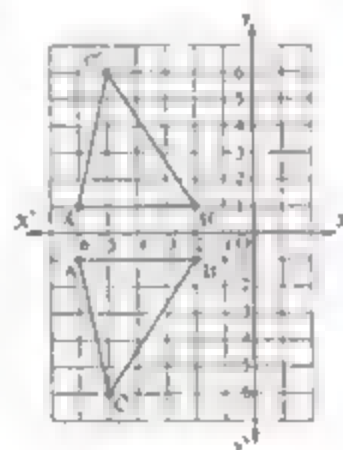


4

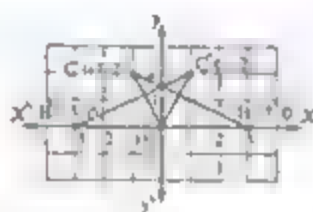
The figure ABCD is  
a rectangle and its perimeter  
=  $(6 + 3) \times 2$   
= 18 length units



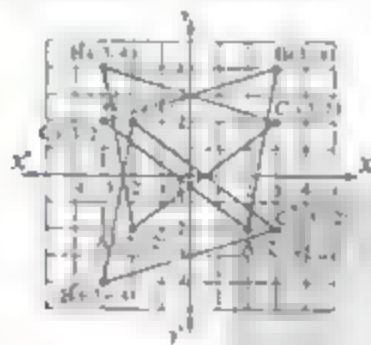
5



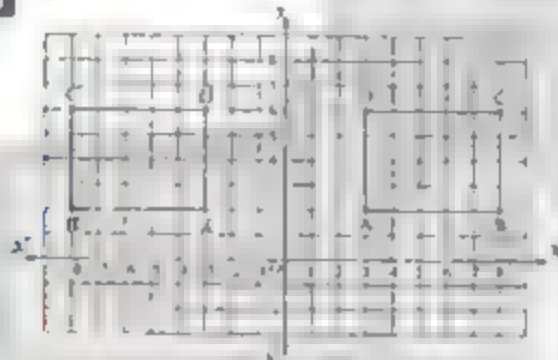
10



7

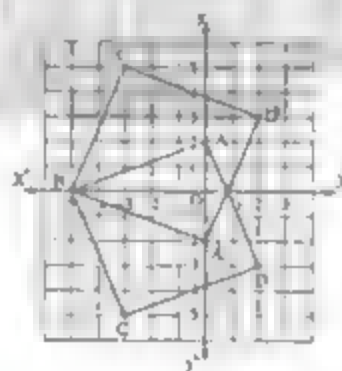


8

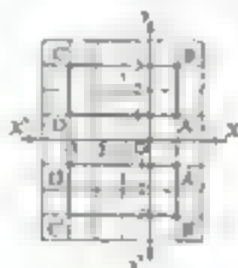


9

We notice that  
 $AB = \hat{A}B$ ,  $BC = \hat{B}C$   
 $CD = \hat{C}D$   
 $AD = \hat{A}D$   
 The area of the  
 square ABCD  
 = The area of the  
 square  $\hat{A}\hat{B}\hat{C}\hat{D}$



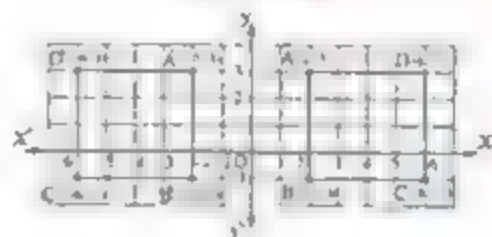
10 D = (-3, 1)



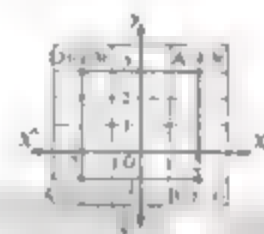
11

There are two cases to draw the square.

The first case



The second case

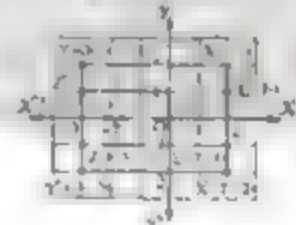


In the second case we notice that the image of the square ABCD by reflection in y-axis is DCBA  
 i.e. The same square.

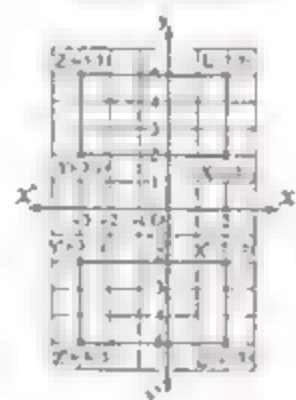
12

There are two cases to draw the rectangle

The first case



The second case





## Geometry and Measurement

13

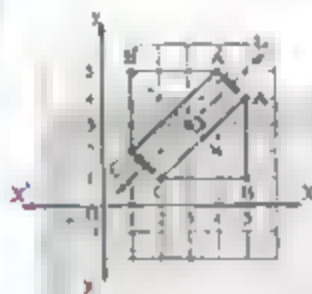
The point	Its image by reflection in the X-axis	Its image by reflection in the y-axis
$(3, -2)$	$(3, 2)$	$(-3, -2)$
$(1, -2)$	$(1, 2)$	$(-1, -2)$
$(2, 4)$	$(2, -4)$	$(-2, 4)$
$(0, 5)$	$(0, -5)$	$(0, 5)$
$(3, 0)$	$(3, 0)$	$(-3, 0)$
$(0, 0)$	$(0, 0)$	$(0, 0)$

14

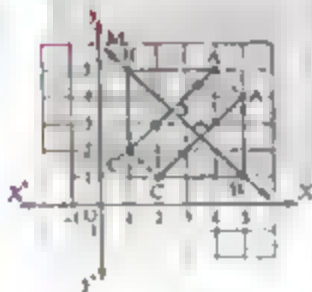
- 1  $(1, -3)$       2  $(2, 5)$       3 X-axis  
 4 y-axis      5 y-axis      6 X-axis  
 7  $(-2, -1)$       8  $(-2, 3)$       9 A  $(2, 3)$

15

1

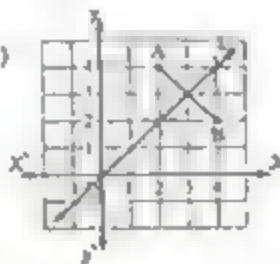


2



16

By taking  $A(2, 4)$  &  $B(4, 2)$   
 • then we draw  $\overline{AB}$   
 • then the axis of symmetry of  $\overline{AB}$  is the straight line L  
 "try to solve this question by another way"



## Answers of Exercise 10

First : Problems on reflection in the plane :

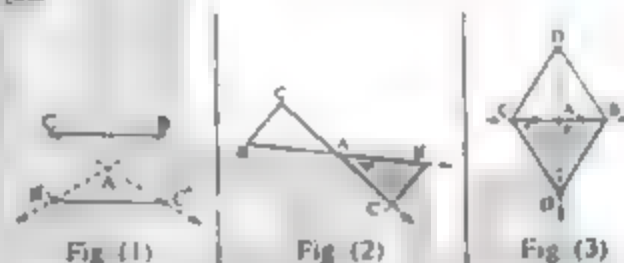
1

- 1 (c)      2 (b)      3 (d)      4 (c)

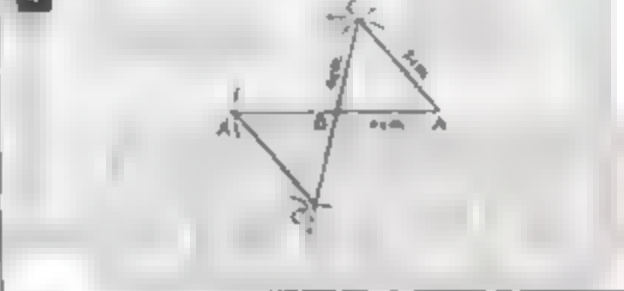
2

- 1 the point C      2 the point Z      3  $\overline{CY}$   
 4  $\overline{MX}$       5  $\overline{DM}$       6  $\overline{BX}$   
 7  $\triangle CYM$       8  $\triangle DZM$       9  $\triangle CMD$   
 10 the square CZMY

3



4



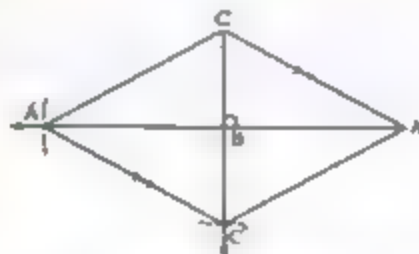
5

Fig. (3)



$\hat{A}$  is the image of A by reflection in the point B  
 $\hat{C}$  is the image of C by reflection in the point B  
 $\overline{AC}$  is the image of  $\overline{AC}$  by reflection in the point B  
 $\therefore \overline{AC} \parallel \overline{\hat{A}\hat{C}}$  &  $AC = \hat{A}\hat{C}$   
 $\therefore$  the quadrilateral  $AC\hat{A}\hat{C}$  is a parallelogram.

Fig. (2)

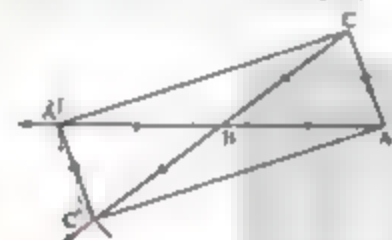


As we said previously the quadrilateral  $ACAC$  is a parallelogram

$$\overline{AA} \perp \overline{CC}$$

the quadrilateral  $ACAC$  is a rhombus.

Fig. (3)



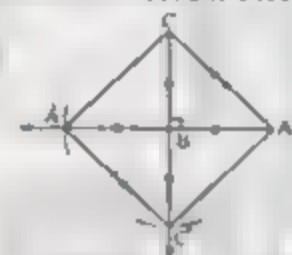
As we mentioned before the quadrilateral  $ACAC$  is a parallelogram

$$\therefore AB = \hat{A}B, CB = \hat{C}B, AB = BC$$

$$\therefore \overline{AA} = \overline{CC}$$

$\therefore$  the quadrilateral  $ACAC$  is a rectangle

Fig. (4)



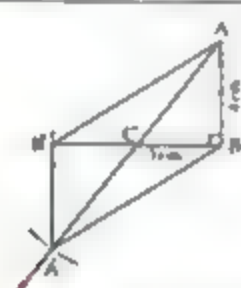
As we mentioned before  $ACAC$  is a parallelogram

$$\therefore AB = \hat{A}B, CB = \hat{C}B, AB = BC$$

$$\therefore \overline{AA} = \overline{CC}, \overline{AA} \perp \overline{CC}$$

$\therefore$  the quadrilateral  $ACAC$  is a square.

6

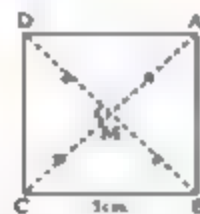


$\therefore \hat{A}$  is the image of A by reflection in C  
 $\therefore \hat{B}$  is the image of B by reflection in C

$$\therefore AB = \hat{A}\hat{B}, \overline{AB} \parallel \overline{\hat{A}\hat{B}}$$

$\therefore$  the figure  $AB\hat{A}\hat{B}$  is a parallelogram. (Q.E.D.)

7



The image of the square ABCD by reflection in the point M is the square CDAB

We notice that we got the same square.

8

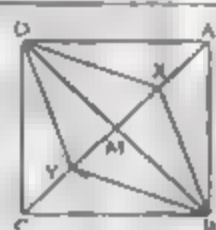


The figure ABCD is a parallelogram.

1 A right-angled triangle at B

2 An isosceles triangle ( $AB = BC$ )

9



ABCD is a square.  $\therefore AM = MC$  (1)

$\therefore Y$  is the image of X by reflection in M

$$\therefore XM = YM$$
 (2)

Subtracting (2) from (1) :

$$\therefore AX = CY$$

$\Delta \Delta DAX, BCY$  in them

$\left\{ \begin{array}{l} AD = CB \text{ (properties of square)} \\ m(\angle DAX) = m(\angle BCY) \text{ (alternate angles)} \\ AX = CY \end{array} \right.$

$$\therefore \Delta DAX = \Delta BCY$$

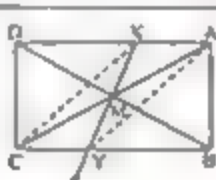
$$\therefore BM = MD, XM = YM$$

$$\therefore \Delta DAX = \Delta BCY \quad (\text{Q.E.D. 1})$$

$$\therefore BM = MD, XM = YM$$

$$\therefore \text{the figure } DXBY \text{ is a parallelogram} \quad (\text{Q.E.D. 2})$$

10



$\therefore$  ABCD is a rectangle

$$\overline{AD} \parallel \overline{BC}$$

## Geometry and Measurement

$$\therefore m(\angle XAM) = m(\angle YCM) \quad (\text{alternate angles})$$

$\Delta \Delta AMX$  &  $CMY$  is them :

$$\begin{cases} m(\angle XAM) = m(\angle YCM) \\ m(\angle AMX) = m(\angle CMY) \\ AM = CM \end{cases} \quad (\text{V.O.A.})$$

$$\therefore \Delta AMX = \Delta CMY \text{ then we deduce that } XM = MY$$

$$\therefore Y \in XM$$

$\therefore Y$  is the image of  $X$  by reflection in the point  $M$   
(Q.E.D. 1)

$$\therefore AM = CM, MX = MY$$

$\therefore$  the figure  $AXCY$  is a parallelogram. (Q.E.D. 2)

11



$\Delta \Delta ABX$  &  $CDY$  is them

$$\begin{cases} AB = CD \\ m(\angle BAX) = m(\angle DCY) \quad (\text{alternate angles}) \\ m(\angle ABX) = m(\angle CDY) \end{cases}$$

$$\therefore \Delta ABX = \Delta CDY \text{ then we deduce that } AX = CY$$

$$\therefore AM = CM$$

$$\therefore AM - AX = CM - CY$$

$$\therefore XM = YM$$

$\therefore Y$  is the image of  $X$  by reflection in  $M$

$$\therefore AM = CM$$

$\therefore C$  is the image of  $A$  by reflection in  $M$

$$\therefore BM = DM$$

$\therefore D$  is the image of  $B$  by reflection in  $M$

$\therefore \Delta ABX$  is the image of  $\Delta CDY$  by reflection in  $M$

(Q.E.D. 1)

$$\therefore XM = YM, BM = DM$$

$\therefore$  the figure  $XB YD$  is a parallelogram. (Q.E.D. 2)

**Second : Problems on reflection in the Cartesian plane :**

1

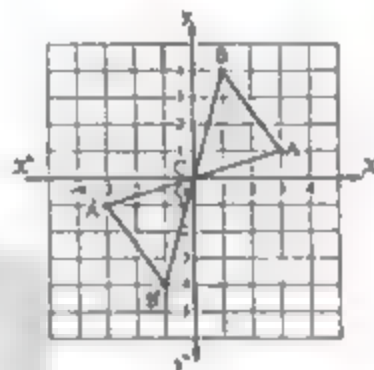
1 (c)

2 (c)

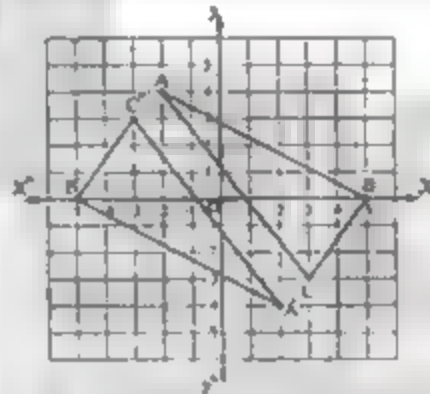
3 (c)

4 (b)

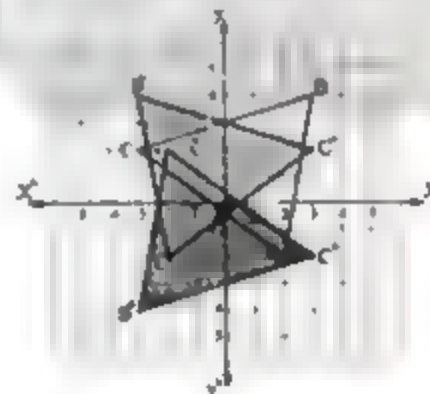
2



3

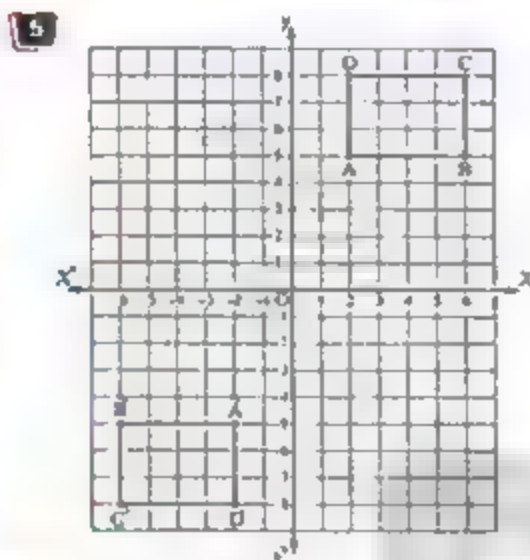


4



$\Delta \Delta A'B'C'$  is the image of  $\Delta ABC$  by reflection in the origin point.

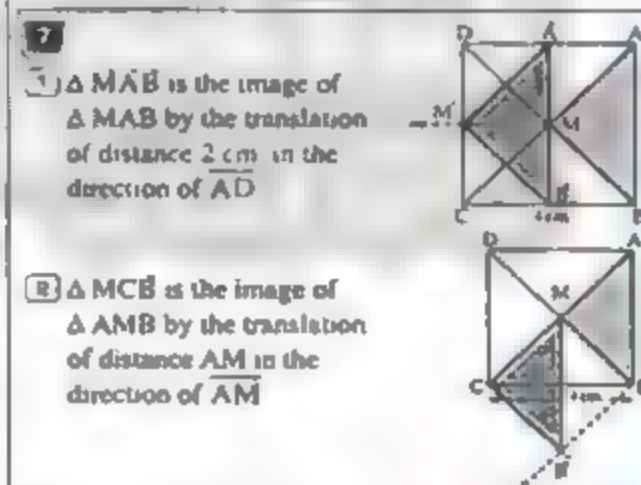
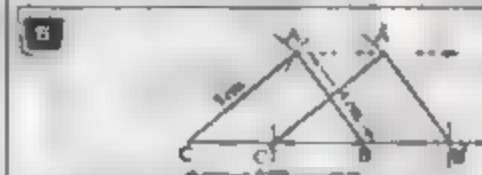
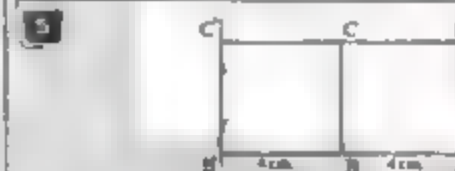
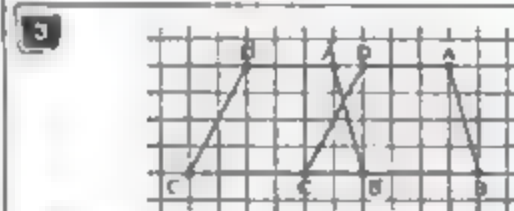
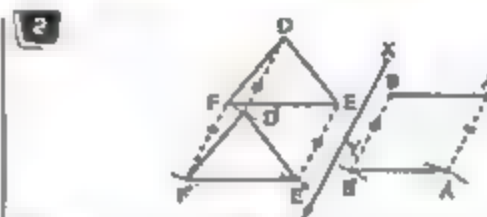
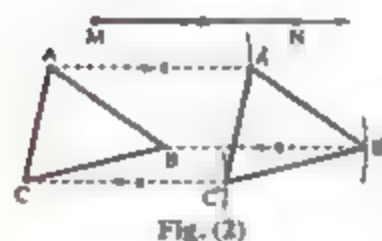
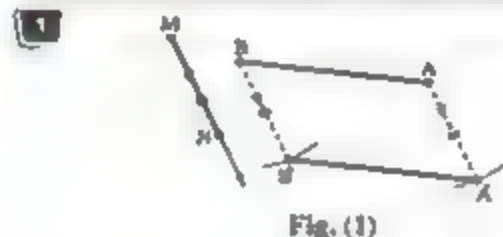
We deduce that the reflection in the origin point is equivalent to reflection in y-axis followed by reflection in x-axis.



- 6
- $\therefore \overline{CD}$  is the image of  $\overline{BA}$  by reflection in M
  - $\therefore BA = CD \quad \therefore 2x + 5 = x + 9 \quad \therefore x = 4$
  - $\therefore$  the length of  $\overline{CD} = 4 + 9 = 13$  cm (First req)
  - $\therefore \overline{CD}$  is the image of  $\overline{BA}$  by reflection in M
  - $\therefore BM = MC \quad \therefore AM = MD$
  - $\therefore ACDB$  is a parallelogram.
  - $\therefore m(\angle BAD) = m(\angle CDA)$  (alternate angles)
  - $2y = 60^\circ \quad \therefore y = 30^\circ$  (Second req)

### Answers of Exercise 11

First : Problems on translation in the plane :



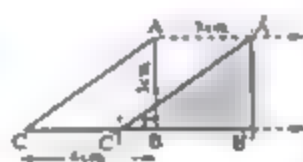
- 8
- 1)  $\triangle DBE$
  - 2)  $\overline{BE}, \overline{BE}$
- 9
- 1) M
  - 2)  $\overline{BM}$
  - 3)  $\triangle ABM$
  - 4)  $\overline{AM}$  in the direction  $\overline{AM}$
- 10
- 1)  $\overline{MN}$
  - 2) The square  $FBLG$
  - 3) 4 cm,  $\overline{CY}$



# Geometry and Measurement

11

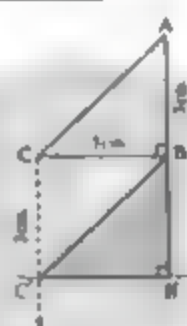
$\triangle A'B'C'$  is the image of  $\triangle ABC$  by the translation of distance 3 cm in the direction of  $\overrightarrow{CB}$



$A'$  is the image of  $A$ ,  $C'$  is the image of  $C$   
 $\therefore \overline{A'C'}$  is the image of  $\overline{AC}$  by the translation of distance 3 cm in the direction of  $\overrightarrow{CB}$   
 $\therefore \overline{A'C'} \parallel \overline{AC}$ ,  $A'C' = AC$   
 $\therefore AA'C'C$  is a parallelogram (Q.E.D.)

12

$\triangle B'B'C'$  is the image of  $\triangle ABC$  by the translation of distance 3 cm in the direction of  $\overrightarrow{AB}$



$B'$  is the image of  $B$ ,  $C'$  is the image of  $C$   
 $\therefore \overline{B'C'}$  is the image of  $\overline{BC}$  by the translation of distance 3 cm in the direction of  $\overrightarrow{AB}$   
 $\therefore \overline{B'C'} \parallel \overline{BC}$ ,  $B'C' = BC$   
 $\therefore BB'C'C$  is a parallelogram.  
 $m(\angle ABC) = 90^\circ \therefore m(\angle B'B'C') = 90^\circ$   
 "the translation reserves the measures of angles"  
 $\therefore BB'C'C$  is a rectangle.  $\therefore AB = BC = 3$  cm  
 $\therefore$  the length of the image of  $\overline{AB}$  = the length of the image of  $\overline{BC} = 3$  cm  
 "Translation reserves the lengths of line segments"  
 $\therefore BB' = B'C'$   
 $\therefore BB'C'C$  is a square. (Q.E.D.)

13

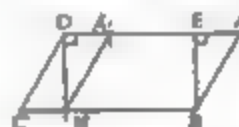
$\triangle DCE$  is the image of  $\triangle ABE$  by the translation of distance  $\overline{AD}$  in the direction of  $\overrightarrow{AD}$



$\therefore \overline{CE}$  is the image of  $\overline{BE}$  by translation of distance  $\overline{AD}$  in the direction of  $\overrightarrow{AD}$   
 $\therefore \overline{CE} \parallel \overline{BE}$ ,  $CE = BE$   
 $\therefore$  The figure  $BCDE$  is a parallelogram. (Q.E.D.)

14

$\triangle A'DE$  is the image of  $\triangle ABE$  by the translation of distance  $\overline{ED}$  in the direction of  $\overrightarrow{AD}$



$\therefore \overline{BD}$  is the image of  $\overline{BE}$  by translation of distance  $\overline{ED}$  in the direction of  $\overrightarrow{AD}$   
 $\therefore \overline{BD} \parallel \overline{BE}$ ,  $BD = BE$

$\therefore EBB'D$  is a parallelogram

$\therefore m(\angle ADB) = m(\angle AEB) = 90^\circ$

"The translation reserves the measures of angles"

The figure  $EBB'D$  is a rectangle (Q.E.D.)

## Second : Problems on translation in the Cartesian plane :

1

- 1 (4, 6)      2 (6, 0)      3 (-1, -1)  
 4 (-4, 5)      5 (3, 1)

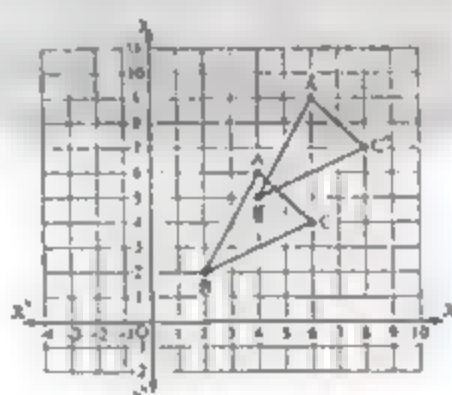
- 2 1 (b) 2 (a) 3 (b) 4 (d) 5 (c) 6 (a)

3

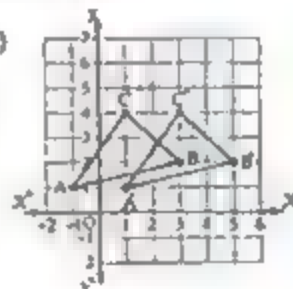
- 1  $A(5, 2) \rightarrow \hat{A}(2, 2)$   
 $B(2, 5) \rightarrow \hat{B}(-1, 1)$



- 2  $A(4, 6) \rightarrow \hat{A}(6, 9)$   
 $B(2, 2) \rightarrow \hat{B}(4, 5)$   
 $C(6, 4) \rightarrow \hat{C}(8, 7)$



- 3  $A(-1, 1) \rightarrow \hat{A}(1, 1)$   
 $B(3, 2) \rightarrow \hat{B}(5, 2)$   
 $C(1, 4) \rightarrow \hat{C}(3, 4)$



4.  $A(1, 1) \rightarrow A'(5, 0)$   
 $B(4, 2) \rightarrow B'(-1, 1)$   
 $C(5, 4) \rightarrow C'(0, 2)$   
 $D(3, 3) \rightarrow D'(5, 1)$



5. Translation  $(X, y) \rightarrow (X + 2, y - 3)$  followed  
 by rotation  $(X, y) \rightarrow (X - 3, y + 1)$   
 equivalent to  $(X, y) \rightarrow (X + 2 - 3, y - 3 + 1)$   
 and this equivalent to  $(X - 1, y - 2)$

1. The image of  $A(4, 2) \rightarrow A'(3, 4)$   
 2. The image of  $B(5, 3) \rightarrow B'(-1, 1)$   
 3. The image of  $C(0, 2) \rightarrow C'(-1, 0)$

6



7



8

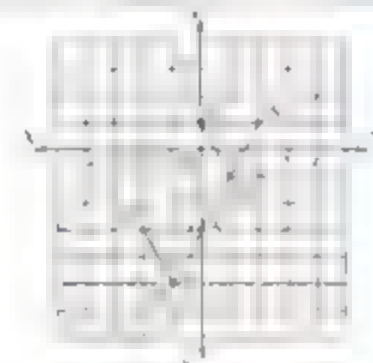


9

- $(X, y) \rightarrow (X + 3, y + 2)$   
 1. The image of  $B(-2, 3)$  by translation LM  
 is  $B'(1, 5)$   
 2. The image of  $C(5, 4)$  by translation LM  
 is  $C'(8, 6)$   
 3. The image of  $D(3, 0)$  by translation LM  
 is  $D'(6, 2)$

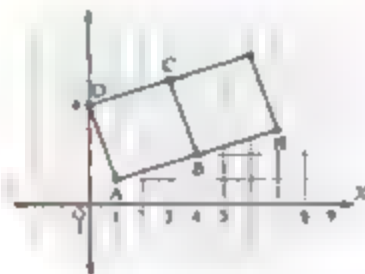


10



## Geometry and Measurement

13



• The mapping rule is  $(X, y) \longrightarrow (X + 3, y + 1)$

13 (0, 0)

14

•  $\hat{A}(2, 2)$  is the image of the point  $A(1, 1)$

• The mapping rule of this translation is  $(X, y) \longrightarrow (X + 1, y + 1)$

• The image of  $O(0, 0)$  is  $\hat{O}(1, 1)$

• The image of  $B(-1, 3)$  is  $\hat{B}(0, 4)$

• The image of  $C(-3, 5)$  is  $\hat{C}(-2, 6)$

14  $(X, y) \longrightarrow (X + 4, y - 3)$ 

15

$(X, y) \longrightarrow (X + 2, y - 1)$

• The image of  $C(1, -1)$  is  $\hat{C}(3, -2)$

• Let  $D(X, y)$   $\therefore X + 2 = 2$

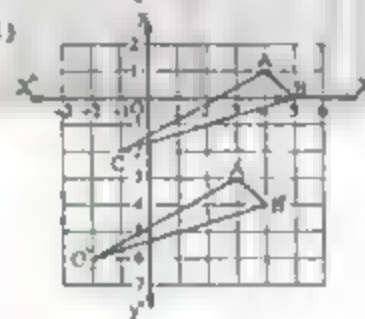
$\therefore X = 0, y - 1 = 1 \therefore y = 2 \therefore D(0, 2)$

16

Let  $A(X, y)$   $\therefore X - 1 = 3$

$\therefore X = 4$   $y - 4 = -3 \therefore y = 1$

$\therefore A(4, 1)$



17



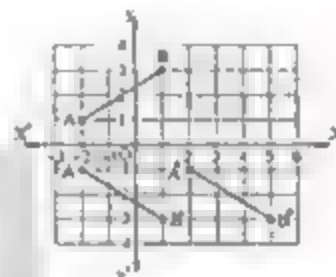
18

$A(-2, 1) \xrightarrow[\text{in } X\text{-axis}]{\text{by reflection}} \hat{A}(-2, -1)$

by translation  $\hat{A}(2, -1)$

$B(1, 3) \xrightarrow[\text{in } X\text{-axis}]{\text{by reflection}} \hat{B}(1, -3)$

by translation  $\hat{B}(5, -3)$



19

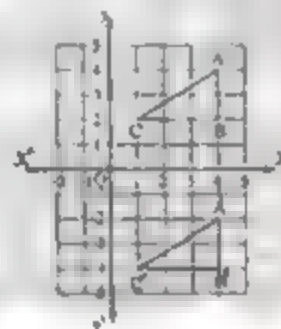
Fig. (1) Translation  $(X, y) \longrightarrow (X - 5, y - 3)$

Fig. (2) Reflection and the reflection axis is X-axis

Fig. (3) Reflection and the reflection axis is y-axis

Fig. (4) Translation  $(X, y) \longrightarrow (X - 5, y + 3)$

20



21

•  $A(2, 1)$  is the image of  $B$  by reflection

in X-axis followed by reflection in y-axis

$\therefore B(-2, -1)$

• The mapping rule of translation that makes the point  $A(2, 1)$  the image of the point  $B(-2, -1)$  is  $(X, y) \longrightarrow (X + 4, y + 2)$

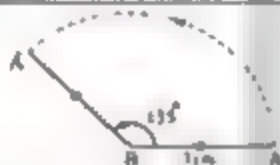
## Answers of Exercise 12

First : Problems on rotation in the plane :

1

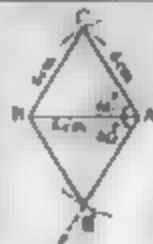


2



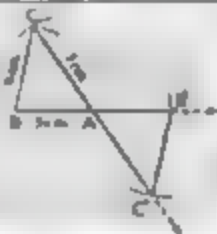
3

$\triangle ABB$  is the image of  $\triangle ABC$  by the rotation  $R(A, 60^\circ)$



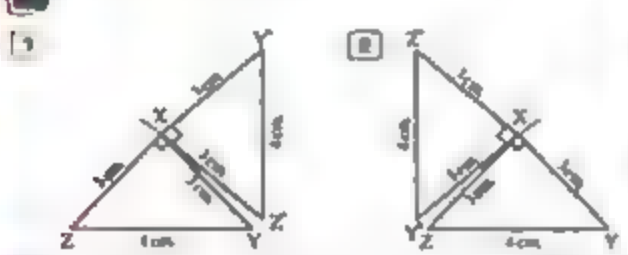
4

1  $\triangle ABC$  is the image of  $\triangle ABC$  by rotation  $R(A, 180^\circ)$



2 The image of  $\triangle ABC$  by rotation  $R(A, 360^\circ)$  is itself

5

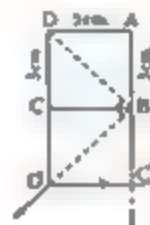


6

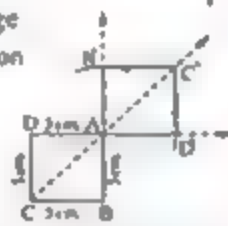


7

1 The square  $CBCD$  is the image of the square  $ABCD$  by rotation  $R(B, 90^\circ)$

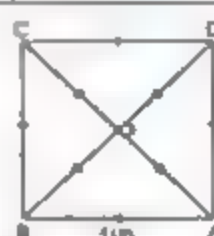


2 The square  $ABCD$  is the image of the square  $ABCD$  by rotation  $R(A, 180^\circ)$



8

According to the shown figure, the square  $DABC$  is the image of the square  $ABCD$  by rotation about its centre with an angle of measure  $90^\circ$

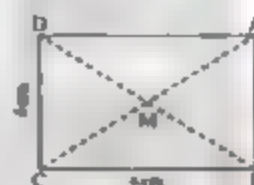


9

1 The rectangle  $ABCD$  is the image of the rectangle  $ABCD$  by the rotation  $R(A, 90^\circ)$



2 The rectangle  $CDAB$  is the image of the rectangle  $ABCD$  by the rotation  $R(M, 180^\circ)$



10

- 1 (d) 2 (b) 3 (c)  
4 (c) 5 (c) 6 (b)

11

- 1  $D, A, \overline{DA}, \overline{DA}$  2  $\overline{BC}, \overline{BC}, \overline{BC}$   
3  $C, D, \overline{CD}$  4  $\overline{CD}$

12

- 1 The point C 2  $\overline{DC}$  3  $\overline{EF}$   
4  $\triangle MDE$  5  $M, 120^\circ$  6  $\triangle FMA$

13

- 1 (b) 2 (d) 3 (c) 4 (a)

14

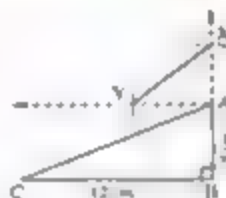
- 1  $\triangle DLO$  2  $\triangle DLO$



## Geometry and Measurement

15

From the figure  $XY = 6.4$  cm  
(by measure)



16

$\overline{XZ} \parallel \overline{YZ}$ ,  $\overline{XY}$  is a transversal to them

$\therefore m(\angle ZXY) = m(\angle Y) = 30^\circ$  (alternate angles)

$$m(\angle YXY) = m(\angle YXZ) + m(\angle ZXY) \\ = 90^\circ + 30^\circ = 120^\circ$$

The measure of the angle of rotation =  $120^\circ$

(First req)

Z is the image of Z and Y is the image of Y by the same rotation and X is the image of itself.

$\Delta YZX$  is the image of  $\Delta YZX$  by rotation

$R(X, 120^\circ)$

$$ZX = ZX = 5 \text{ cm}$$

(Second req)

**Second :** Problems on rotation in the Cartesian plane

1

1.  $(3, 2) \rightarrow (-2, 3)$

2.  $90^\circ$

3.  $(-5, 2)$

4.  $(1, 1)$

5.  $\pm 360$

6.  $(0, -1) \rightarrow (-1, 0)$

7.  $(4, 1)$

8.  $(7, -3)$

9.  $(y, x)$

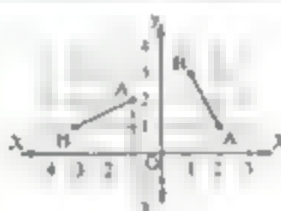
10. zero

2

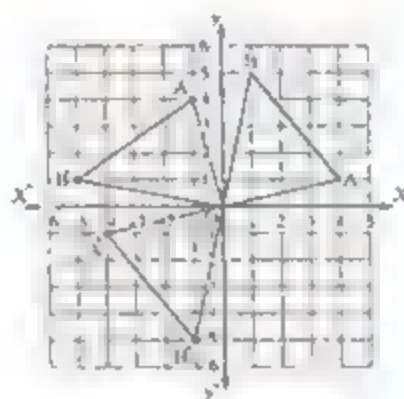
$$(2, 1) \rightarrow (-2, -1) \rightarrow (-2, 1) \rightarrow (-1, 3) \rightarrow$$

$$(1, 2) \rightarrow (-1, -2) \rightarrow (-2, 1) \rightarrow (2, -1)$$

3



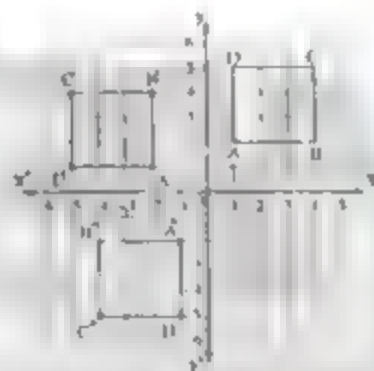
4



1.  $\Delta A'B'O$  is the image of  $\Delta ABO$  by rotation about O with an angle of measure  $90^\circ$

2.  $\Delta A'B'O$  is the image of  $\Delta ABO$  by rotation about O with an angle of measure  $180^\circ$

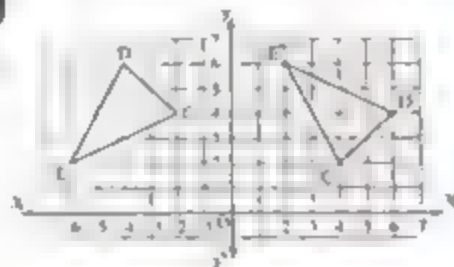
5



1. The square  $A'B'C'D'$  is the image of  $ABCD$  by rotation about O with an angle of measure  $90^\circ$

2. The square  $A'B'C'D'$  is the image of the square  $ABCD$  by rotation about O with an angle of measure  $180^\circ$

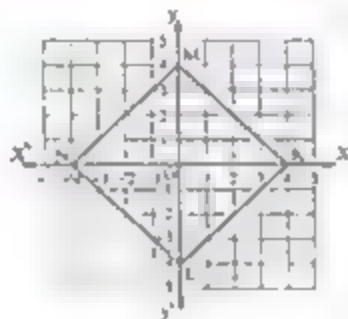
6



$\Delta D'C'E'$  is the image of  $\Delta DCE$  by rotation about O with an angle of measure  $-90^\circ$

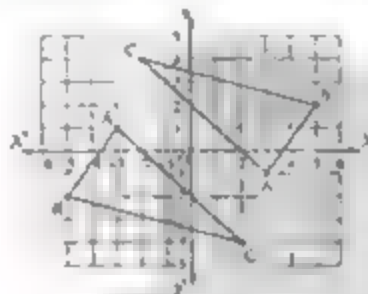
2

The square MNLK is the image of the square KMNL by rotation about O with an angle of measure  $90^\circ$



7

$\triangle \hat{A}BC$  is the image of  $\triangle ABC$  by rotation about the origin point with an angle of measure  $180^\circ$

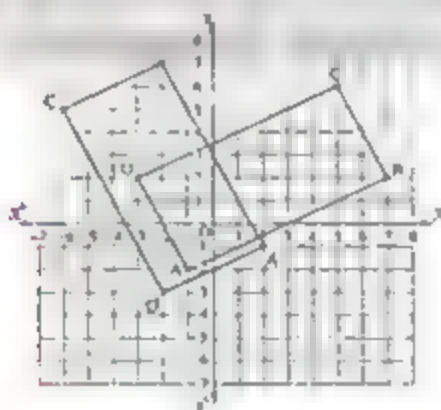


8



$\triangle \hat{A}OB$  is the image of  $\triangle AOB$  by rotation about O with an angle of measure  $90^\circ$

9



The rectangle  $\hat{A}B\hat{C}\hat{D}$  is the image of the rectangle ABCD by rotation about the origin where:  $(x, y) \rightarrow (-y, x)$

10



1 The figure  $\hat{A}B\hat{C}\hat{D}$  is the image of the figure ABCD by rotation about the origin with an angle of measure  $90^\circ$

2 The figure  $\hat{A}B\hat{C}\hat{D}$  is the image of the figure ABCD by rotation about the origin with an angle of measure  $-180^\circ$

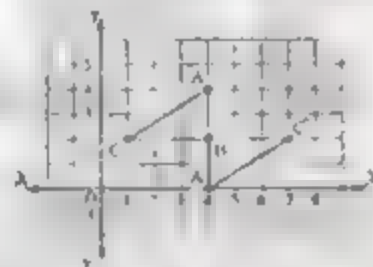
11  $C = (3, 4)$  ,  $\hat{C} = (-3, -4)$

12

The square  $\hat{A}D\hat{C}\hat{D}$  is the image of the square ABCD by rotation about A with an angle of measure  $90^\circ$

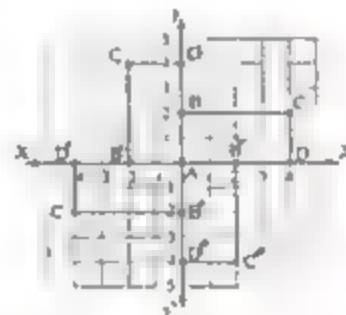


13



$\triangle \hat{A}BC$  is the image of  $\triangle ABC$  by rotation about B with an angle of measure  $180^\circ$

14 (a)



1 The rectangle  $\hat{A}B\hat{C}\hat{D}$  is the image of the rectangle ABCD by rotation about the origin with an angle of measure  $90^\circ$

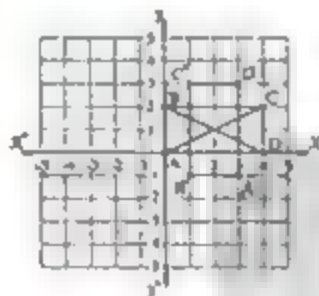
## Geometry and Measurement

- 2 The rectangle  $A'B'C'D'$  is the image of the rectangle  $ABCD$  by rotation about the origin with an angle of measure  $180^\circ$
- 3 The rectangle  $A'B'C'D'$  is the image of the rectangle  $ABCD$  by rotation about the origin with an angle of measure  $270^\circ$

(b)  $(2, 1)$ 

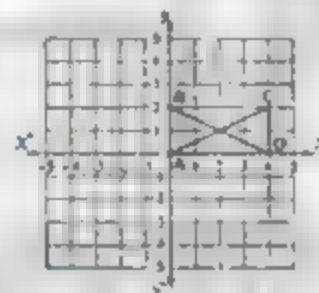
(c)

1



The rectangle  $A'B'C'D'$  is the image of the rectangle  $ABCD$  by rotation about the centre of the rectangle with an angle of measure  $90^\circ$

2



The rectangle  $CDAB$  is the image of the rectangle  $ABCD$  by rotation about the centre of the rectangle with an angle of measure  $180^\circ$

3



The rectangle  $A'B'C'D'$  is the image of the rectangle  $ABCD$  by rotation about the centre of the rectangle with an angle of measure  $270^\circ$

15

$\Delta ABC$  is a right-angled triangle at  $A$  from Pythagoras  $(AC)^2 = (BC)^2 - (AB)^2 = 25 - 9 = 16$

$$\therefore AC = 4 \text{ cm}$$

$\therefore \Delta CA'B'$  is the image of  $\Delta CAB$  by the given rotation

$$\therefore \Delta CA'B' \cong \Delta CAB$$

$$\therefore \hat{A}C = AC = 4 \text{ cm}, \text{ then } \hat{A}\hat{A} = 8 \text{ cm}$$

$$\hat{A}\hat{B} = AB = 3 \text{ cm}$$

$\therefore$  the area of  $\Delta \hat{A}\hat{A}\hat{B}$

$$= \frac{1}{2} \hat{A}\hat{A} \times \hat{A}\hat{B} = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2 \quad (\text{The req.})$$

### Answers of exams on the third part of unit three

#### ? Model 1

- 1 1 (c) 2 (c) 3 (d)  
4 (b) 5 (b) 6 (d)

2 1 2, zero

2 lengths of line segments, measures of angles, parallelism. (There are other solutions)

- 3 (-7, -3) 4 (5, 2) 5 (2, -4)

3 [a] Draw by yourself

- [b] 1  $\Delta AMB$  2  $\Delta DMC$   
3  $\Delta MDE$  4  $\Delta EMD$

4 [a] Draw by yourself

[b] Draw by yourself

5 [a] Draw by yourself

[b] Draw by yourself

#### ? Model 2

- 1 1 (c) 2 (d) 3 (a)  
4 (b) 5 (a) 6 (c)

2

1 Lengths of line segments + measures of angles  
+ parallelism (There are other solutions)

2 1 3  $360^\circ$

4 The magnitude of translation + the direction of translation

5 y-axis

3 (a) Draw by yourself

(b) 1  $\triangle CMD$  2  $\overline{BC}$

4 (a) 1 The translation is :  $(0 + 2)$

2  $(3 + 3)$

(b) Draw by yourself

5 (a)  $(0 + 4)$

(b) Draw by yourself

### Answers of accumulative basic skills

1 3 (b) 2 (c) 3 (d) 4 (d)

5 (b) 6 (c) 7 (d) 8 (a)

9 (c) 10 (b) 11 (d) 12 (b)

13 1 125 2 2 3 an acute 4  $60^\circ$

5 115.5 6 50 7 6 8  $360^\circ$

9 18 10 37.5 % 11 16 12 28



## Guide Answers

# of The Notebook

- Quizzes.
- Final Examinations.



### Answers of the quizzes on Algebra and Statistics

#### Quiz ①

1

1 d

2 c

3 a

2

(a)  $-\frac{7}{9}$ (b)  $\frac{8}{10}$ 

#### Quiz ②

1

1 b

2 c

3 c

2

(a)  $1 \frac{27}{125}$ (2)  $x^4$ (3)  $\frac{64}{125}$ (b)  $8 \frac{1}{2}$ 

#### Quiz ③

1

1 c

2 c

3 d

2

(a) 1

(2)  $\frac{7}{9}$ (3)  $\frac{y^4}{x^5}$ (b)  $\frac{8}{49}$ 

#### Quiz ④

1

1 b

(2) b

(3) c

2

(a) (1)  $5.832 \times 10^7$ (2)  $1.31 \times 10^{-3}$  $3.3 \times 10^8$ (b) (1)  $\frac{1}{3}$ 

(2) 1

#### Quiz ⑤

1

1 c

2 b

3 a

2

(a) 1 = 3

(2) 22

(3) 3

(b) 49

#### Quiz ⑥

1

1 b

(2) b

(3) c

2

(a) 1 37 cm.

(2)  $-\frac{1}{9}$ 

(b) 8

#### Quiz ⑦

1

1 a

(2) d

3 c

2

(a) (1) {3}

(2) 5 + 10

(b) zero

#### Quiz ⑧

1

1 b

2 c

3 b

2

(a) 1 {x : x ∈ Z, x &gt; 2}

(2) { $\frac{7}{3}$ }

(b) 3

#### Quiz ⑨

1

(1) c

(2) b

3 d

2

(a) { $\frac{4}{3}$ }

(b) S = {1, 2, 3, 4, 5, 6, 7, 8}

1  $\frac{1}{2}$ (2)  $\frac{1}{4}$

## Algebra and Statistics

Answers of school book models  
on Algebra and Statistics

## Model 1

1

1 -2    2 2    3  $\frac{7}{12}$     4  $3.5 \times 10^{-3}$     5 1

2

1 (b)    2 (a)    3 (a)    4 (a)    5 (a)    6 (a)

3

[a]  $1 \times \frac{4}{25} \times \sqrt{\frac{25}{4}} = 1 \times \frac{4}{25} \times \frac{5}{2} = \frac{2}{5}$

[b]  $3ab + 8a + (4b)$

$= 3 \times 4 \times (-2) + 8 \times 4 + (4 \times -2)$

$= -24 + 32 + (-8)$

$= -24 - 4 = -28$

4

[a]  $\therefore 3x + 1 = 25$

$\therefore x = 8$

$\therefore 3x = 24$

The S.S. = {8}

[b]  $\frac{8 \times 8^{-2}}{8} = 8^1 - 2 + 1 = 8^0 = 64$

5

[a] 1 The probability of change it before travelled  
50 thousand km. =  $\frac{80}{800} = \frac{1}{10}$

2 The probability of change it after travelled more  
than 100 thousand km. =  $\frac{600}{800} = \frac{3}{4}$

[b]  $\therefore 2x + 5 < 16$

$\therefore 2x < 11$

$\therefore x < \frac{11}{2}$

$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x < \frac{11}{2}\}$

## Model 2

1

1 1    2  $\frac{4}{7}$     3 zero

4  $13 \div 21$     5 510 students

2

1 (a)    2 (d)    3 (d)    4 (b)    5 (c)    6 (d)

3

[a]  $\therefore 5x - 2x = 30 \quad \therefore 3x = 30 \quad \therefore x = 10$

$\therefore \text{The two numbers are : } 20, 50$

[b]  $\frac{5^4}{5^3} = 5^{-4+7-3} = 5^0 = 1$

4

[a] 1  $\therefore 3x + 7 = 13 \quad \therefore 3x = 6$

$\therefore x = 2$

$\therefore \text{The S.S.} = \{2\}$

2  $\therefore 2x + 15 < 19$

$\therefore 2x < 4$

$\therefore x < 2$

$\therefore \text{The S.S.} = \{x : x \in \mathbb{Q}, x < 2\}$

[b]  $\frac{1}{5} + \frac{8}{5} - 1 = 1 - 1 = \text{zero}$

5

[a] 1 The probability of getting a prime even  
number =  $\frac{1}{6}$

2 The probability of getting an odd number less  
than 4 =  $\frac{2}{6} = \frac{1}{3}$

[b]  $\left(\frac{y}{x^2}\right)^{-2} = \left[\frac{\frac{-3}{4}}{\left(\frac{-1}{2}\right)^2}\right]^{-2} = \left(\frac{\frac{-3}{4}}{\frac{1}{4}}\right)^2 = (-3)^2$   
 $= \frac{1}{(-3)^2} = \frac{1}{9}$

Model examination for the  
merge students

1

1 a

2 b

3 a

4 b

5 b

2

1 4

2  $\frac{1}{2}$

3 zero

4  $\frac{2}{3}$

5 42

3

1  $12 \times 2^2 + 24 + 3^2 = 12 \times 4 + 24 + 9$

$= 48 + 24 + 9 = 2 + 9 = 11$

2  $\frac{8+20}{8-4} = \frac{28-4}{4} = \frac{24}{4} = 6$

4

1 ✓

2 x

3 ✓

4 x

5 x

5

1  $\{8 \div 6 \div 4 \div 2\}$

2  $\frac{1}{2}$

3  $\{8 \div 7\}$

4 1

5  $\frac{1}{8}$

### Answers of schools examinations on Algebra and Statistics

#### 1 Cairo

- 1 a b d c  
4 a c b

2 a  $\left| \frac{5xy}{6} \right|$  b  $\{-13\}$  c  $-\frac{4}{23}$   
4 1 b -3

3  
[a]  $x^2 y^3 z^2 = \left(\frac{-3}{2}\right)^2 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{-4}{3}\right)^2$   
 $= \frac{9}{4} \times \frac{1}{8} \times \frac{16}{9} = 1$

[b]  $\therefore 5x + 8 = 13 - 2x \quad \therefore 5x + 2x = 13 - 8$   
 $\therefore 7x = 5 \quad \therefore x = \frac{5}{7}$   
 $\therefore$  The S.S. =  $\left\{ \frac{5}{7} \right\}$

4  
[a]  $\therefore 3x - 1 \leq 5$   
 $\therefore 3x \leq 6 \quad \therefore x \leq 2$   
 $\therefore$  The S.S. =  $\{x: x \in \mathbb{Q}, x \leq 2\}$

[b]  $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} \cdot \left(\frac{1}{3}\right)^0 = \frac{1}{9} + \frac{8}{9} = 1 = 0$

5  
[a] ① The probability that the ball is red =  $\frac{5}{13} = \frac{1}{3}$   
2 The probability that the ball is black =  $\frac{0}{13} = 0$   
3 The probability that the ball is white or blue  
 $= \frac{4+6}{13} = \frac{10}{13} = \frac{2}{3}$

[b] Let the two numbers be:  $x$  and  $2x$   
 $\therefore x + 2x = 24 \quad \therefore 3x = 24 \quad \therefore x = 8$   
 $\therefore$  The two numbers are:  $8 + 16$

#### 2 Cairo

- 1 a c b b  
4 a b d c

2 a  $\frac{1}{4}$  b  $2.5 \times 10^{-4}$  c  $\frac{4}{3}$   
4 1 b  $\frac{11}{36}$

3  
[a]  $\left(\frac{7^2 \times 7^3}{7^4}\right)^2 = (7^{2+3-4})^2 = (7^1)^2 = (1)^2 = 1$   
[b]  $\left(\frac{-3}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{2}{3}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$   
[c]  $12 \times 2^2 + 24 \div 3^2 = 12 \times 4 \div 24 \div 9$   
 $= 48 \div 24 \div 9 = 2 \div 9 = 11$

4  
[a] ①  $\sqrt{100 - (-8)^2} = \sqrt{100 - 64} = \sqrt{36} = 6$   
2  $(x^2 y^2)^3 = \left[\left(\frac{1}{2}\right)^2 \times \left(\frac{2}{3}\right)^2\right]^3$   
 $= \left[\frac{1}{4} \times \frac{4}{9}\right]^3 = \left(\frac{1}{9}\right)^3 = (9)^3 = 729$   
[b]  $\therefore 3x + 6 > 3 \quad \therefore 3x > -3 \quad \therefore x > -1$   
 $\therefore$  The S.S. =  $\{x: x \in \mathbb{Q}, x > -1\}$

5  
[a]  $\therefore 4x - 5 = 27 \quad \therefore 4x = 32$   
 $\therefore x = 8$   
 $\therefore$  The S.S. =  $\{8\}$   
[b] ① The probability of appearance of an even number =  $\frac{3}{6} = \frac{1}{2}$   
2 The probability of appearance of a number greater than 4 =  $\frac{2}{6} = \frac{1}{3}$

#### 3 Cairo

- 1 a b d c  
4 a b c a

2 a -6 b  $\frac{1}{2}$  c  $\frac{1}{6}$   
4 10 b 32

3  
[a]  $\left(\frac{5^3 \times 5^{-2}}{5^1}\right)^{-1} = (5^{3-2-1})^{-1} = (5^{-1})^{-1} = 5^1 = 25$   
[b]  $\therefore -2 < 4x + 1 < 6 \quad \therefore -3 < 4x < 5$   
 $\therefore -\frac{3}{4} < x < \frac{5}{4}$   
 $\therefore$  The S.S. =  $\{x: x \in \mathbb{Q}, -\frac{3}{4} < x < \frac{5}{4}\}$



## Algebra and Statistics

4

$$(a) \left(\frac{-2}{7}\right)^2 \times \sqrt{\frac{16}{49}} \times \left(\frac{-1}{7}\right)^0 = \frac{49}{4} \times \frac{4}{7} \times 1 = 7$$

(b) Let the length of the rectangle be :  $X$  cm.

$$\therefore \text{The width} = (X - 5) \text{ cm}$$

$$\therefore 2(X + X - 5) = 42$$

$$\therefore 2X - 5 = 21 \quad \therefore 2X = 26 \quad \therefore X = 13$$

$$\therefore \text{The length} = 13 \text{ cm}$$

$$\therefore \text{the width} = 8 \text{ cm.}$$

$$\therefore \text{The area of rectangle} = 13 \times 8 = 104 \text{ cm}^2$$

5

$$(a) \frac{(4X^3Y^2)^3}{(2XY^2)^3} = \frac{16X^9Y^6}{8X^3Y^6} = \frac{2X^6}{Y^0}$$

$$\text{At } X = \frac{1}{2} \text{ and } Y = 3$$

$$\begin{aligned} \text{The result} &= 2 \times \left(\frac{1}{2}\right)^6 \times 3^6 = 2 \times \frac{1}{64} \times 729 \\ &= \frac{1}{32} \times \frac{1}{2} = \frac{1}{64} \end{aligned}$$

$$(b) \therefore \text{The probability of its loss} = 1 - (0.2 + 0.7) = 0.1$$

$$\therefore \text{The number of loss matches} = 0.1 \times 30 = 3 \text{ matches}$$

## 4 Giza

3

$$1. d$$

$$2. b$$

$$3. a$$

$$4. b$$

$$5. c$$

$$6. c$$

2

$$1. 6$$

$$2. \frac{4}{9}$$

$$3. 5$$

$$4. 16$$

$$5. 1$$

3

$$(a) \left(\frac{-2}{7}\right)^0 \times \left(\frac{-2}{7}\right)^2 \times \sqrt{\frac{16}{9}} = 1 \times \frac{4}{7} \times \frac{4}{3} = 3$$

$$(b) (4.4 \times 10^3) \times (3 \times 10^3) = (4.4 \times 3) \times (10^3 \times 10^3) = 13.2 \times 10^6 = 1.32 \times 10^7$$

4

$$(a) \therefore 3X + 1 = 25 \quad \therefore 3X = 24$$

$$\therefore X = 8$$

$$\therefore \text{The S.S.} = \{8\}$$

$$(b) \frac{3^4 \times 3^7}{3^5} = 3^{4+7-5} = 3^6 = 1$$

5

$$(a) 2X + 5 < 9 \quad \therefore 2X < 4$$

$$\therefore X < 2$$

$$\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X < 2\}$$

$$(b) S = \{1, 2, 3, 4, 5, 6\}$$

1. The probability of getting a number greater than 6 =  $\frac{0}{6} = 0$

2. The probability of getting a number satisfies the inequality  $1 < X < 6$  is  $\frac{4}{6} = \frac{2}{3}$

3. The probability of getting a number divisible by 3 =  $\frac{2}{6} = \frac{1}{3}$

## 5 Giza

1

$$1. a$$

$$2. c$$

$$3. b$$

$$4. b$$

$$5. d$$

$$6. c$$

2

$$1. zero$$

$$2. \frac{1}{9}$$

$$3. 5$$

$$4. \frac{25}{4}$$

$$5. \frac{5}{6}$$

3

$$(a) 1. \frac{9^1 \times 9^5}{9^1} = 9^{1+5-1} = 9^5 = 1$$

$$2. \left(\frac{-1}{2}\right)^2 \times \sqrt{\frac{64}{9}} \times \left(\frac{4}{3}\right)^0 = \frac{9}{4} \times \frac{8}{3} \times 1 = 6$$

$$(b) \therefore 4X + 1 = 21$$

$$\therefore 4X = 20$$

$$\therefore X = 5$$

$$\therefore \text{The S.S.} = \{5\}$$

4

$$(a) (X \cdot Y \cdot Z)^2 = \left(\frac{1}{2} \times \frac{2}{3} \times \frac{-1}{3}\right)^2 = \left(\frac{-1}{3}\right)^2 = \frac{1}{9}$$

$$(b) \therefore 3X - 1 \leq 2X + 3 \quad \therefore 3X - 2X \leq 3 + 1$$

$$\therefore X \leq 4$$

$$\therefore \text{The S.S.} = \{X : X \in \mathbb{Q}, X \leq 4\}$$

5

$$(a) \text{Let the numbers be : } X, X + 1, X + 2$$

$$\therefore X + X + 1 + X + 2 = 24$$

$$\therefore 3X + 3 = 24 \quad \therefore 3X = 21$$

$$\therefore X = 7$$

$$\therefore \text{The numbers are : } 7, 8, 9$$

6

(b) 1. The probability of appearance of an even number =  $\frac{3}{6} = \frac{1}{2}$

2. The probability of appearance of a number greater than 5 =  $\frac{1}{6}$

## 6 Alexandria

- 1 1 a 2 5 3 5  
4 0.2 5 X

- 2 1 a 2 c 3 a  
4 d 5 a 6 b

3  
[a]  $\frac{x^3 + x^{-2}}{x^{-2} + x} = x^{3-2+2-1} = x^2$  at  $x = -2$   
 $\therefore x^2 = (-2)^2 = -32$

[b] 1  $\therefore 3x + 5 > 1 \quad \therefore 3x > -4$   
 $\therefore x > -\frac{4}{3}$   
 $\therefore$  The S.S. =  $\{x : x \in \mathbb{Q}, x > -\frac{4}{3}\}$   
2  $3x + 6 = 30 - 5x$   
 $\therefore 3x + 5x = 30 - 6$   
 $\therefore 8x = 24 \quad \therefore x = 3$   
 $\therefore$  The S.S. =  $\{3\}$

4  
[a] 1 The probability of getting an even number  
 $= \frac{3}{6} = \frac{1}{2}$   
2 The probability of getting a number is a factor  
of 6 =  $\frac{4}{6} = \frac{2}{3}$   
[b]  $\sqrt{6 \cdot \frac{1}{4} \cdot \left(\frac{2}{3}\right)^{-1} \cdot \left(\frac{-2}{5}\right)^2} = \sqrt{\frac{25}{4} \times 1 \times \frac{4}{25}}$   
 $= \frac{5}{2} \times 1 \times \frac{4}{25} = \frac{2}{5}$

5  
[a] Let the width be  $x$  cm. and the length be  $(x + 3)$  cm  
 $2(x + 3 + x) = 26$   
 $\therefore 2x + 3 = 13 \quad \therefore 2x = 10$   
 $\therefore x = 5 \quad \therefore$  The length = 8 cm  
and the width = 5 cm  
 $\therefore$  The area =  $8 \times 5 = 40 \text{ cm}^2$   
[b]  $x^3 y^2 = \left(\frac{1}{2}\right)^3 \times \left(\frac{4}{3}\right)^2 = \frac{1}{8} \times \frac{16}{9} = \frac{2}{9}$

## 7 Alexandria

- 1 1 b 2 b 3 c  
4 c 5 a 6 c

- 2 1 1 2 -4 3  $-\frac{4}{9}$   
4 1 5  $2^{11}$

3  
[a] 1  $\therefore 3x + 1 = 16 \quad \therefore 3x = 15$   
 $\therefore x = 5 \quad \therefore$  The S.S. =  $\{5\}$   
2  $\therefore 7x - 1 < 13 \quad \therefore 7x < 14$   
 $\therefore x < 2$   
 $\therefore$  The S.S. =  $\{x : x \in \mathbb{Q}, x < 2\}$   
[b]  $5^2 + [3 \times 8 + 2^2 - 2 \times 3] = 25 + [24 + 4 - 6]$   
 $= 25 + [6 - 6]$   
 $= 25 + 0 = 25$

4  
[a]  $(x + y)^{-1} = \left(\frac{1}{3} + \frac{1}{6}\right)^{-1} = \left(\frac{2+1}{6}\right)^{-1}$   
 $= \left(\frac{3}{6}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$   
[b] Let the number be  $x$   
 $\therefore x + 3x = 24 \quad \therefore 4x = 24$   
 $\therefore x = 6$   
The number is 6

5  
[a]  $\frac{2^3 \times 3^0}{2^3} = \frac{8 \times 1}{8} = 1$   
[b] 1 The probability of getting the ball is blue  
 $= \frac{6}{13} = \frac{6}{13}$   
2 The probability of getting the ball is white or red  
 $\frac{4+3}{13} = \frac{7}{13} = \frac{7}{13}$

## 8 El-Kalyoubia

- 1 1 c 2 c 3 b  
4 c 5 b 6 b  
2 1  $\frac{16}{81}$  2  $\frac{1}{36}$  3 8  
4 5 5  $\frac{2}{3}$

3  
[a]  $\left(\frac{1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{5}{6}\right)^0 = \frac{1}{9} + \frac{8}{9} - 1 = 0$   
[b]  $\left(\frac{y}{x}\right)^2 = \left[\left(\frac{-3}{4}\right) + \left(\frac{-1}{2}\right)\right]^2$   
 $= \left[\frac{-3}{4} + \frac{-2}{4}\right]^2 = \left[\frac{-5}{4}\right]^2 = \left[\frac{5}{4}\right]^2 = \frac{25}{16}$

## Algebra and Statistics

4

$$1 \cdot 3x + 1 = 25 \quad \therefore 3x = 24 \quad \therefore x = 8$$

 $\therefore \text{The S.S.} = \{8\}$ 

$$E \cdot 15 + 2x < 1 \quad \therefore 2x < -14 \quad \therefore x < -7$$

 $\therefore \text{The S.S.} = \{x: x \in \mathbb{Q}, x < -7\}$ 

5

$$[a] (2 \times \sqrt{36 - 2^4}) + 4 = (2 \times 6 - 16) + 4 \\ = (12 - 16) + 4 = -4 + 4 = -1$$

[b] 1 The probability of getting a prime even number =  $\frac{1}{6}$

2 The probability of getting an odd number less than 4 =  $\frac{2}{6} = \frac{1}{3}$

## El-Sharkia

$$1 \cdot 1 \quad 2 \cdot 1 \quad 3 \cdot x^2 | b |$$

$$4 \cdot 2.3 \times 10^{-4} \quad 5 \cdot 42$$

$$2 \cdot 1 \cdot c \quad 2 \cdot b \quad 3 \cdot d$$

$$4 \cdot a \quad 5 \cdot b \quad 6 \cdot c$$

3

$$[a] \cdot 2x + 1 = 9 \quad \therefore 2x = 8$$

$$\therefore x = 4$$

 $\therefore \text{The S.S.} = \{4\}$ 

$$[b] \left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} + \left(\frac{3}{7}\right)^0 = \frac{1}{9} + \frac{8}{9} + 1 = 2$$

4

$$[a] \frac{3^4 \times 3^{-2}}{3^2} = 3^{4-2-2} = 3^0 = 1$$

$$[b] \frac{7}{x^2} = \left(\frac{3}{4}\right) + \left(\frac{-1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

5

$$[a] \cdot 3x - 2 \leq 7 \quad \therefore 3x \leq 9 \quad \therefore x \leq 3$$

 $\therefore \text{The S.S.} = \{x: x \in \mathbb{Q}, x \leq 3\}$ 

[b] 1 The probability of the ball is white =  $\frac{3}{14}$

2 The probability of the ball is not red =  $\frac{5+3}{14} = \frac{8}{14} = \frac{4}{7}$

## 10 El-Monofia

$$1 \cdot 1 \cdot c \quad 2 \cdot d \quad 3 \cdot c$$

$$4 \cdot d \quad 5 \cdot c \quad 6 \cdot d$$

$$2 \cdot 1 \cdot 13 \quad 2 \cdot \emptyset \quad 3 \cdot 3$$

$$4 \cdot \frac{2}{3} \quad 5 \cdot \text{zero}$$

3

$$[a] 1 \cdot 5x - 2 = 8 \quad \therefore 5x = 10 \quad \therefore x = 2$$

 $\therefore \text{The S.S.} = \{2\}$ 

$$2 \cdot 2x + 3 > 4 \quad \therefore 2x > 1 \quad \therefore x > \frac{1}{2}$$

 $\therefore \text{The S.S.} = \{x: x \in \mathbb{Q}, x > \frac{1}{2}\}$ 

$$[b] 2[(3^2 + 1) - (4^2 - 1)] = 2[(25 + 1) - (16 - 1)] \\ = 2[26 - 15] = 2 \times 11 = 22$$

4

$$[a] \left(\frac{-2}{3}\right)^2 \times \sqrt{\frac{81}{64}} = \left(\frac{1}{3}\right)^{\text{num}} = \frac{-8}{27} \times \frac{9}{8} \times 1 = \frac{-1}{3}$$

[b] 1 The probability of the drawn ball is yellow =  $\frac{4}{12} = \frac{1}{3}$

2 The probability of the drawn ball is not green =  $\frac{1+4}{12} = \frac{5}{12}$

5

$$[a] \frac{7^2 \times 7^2}{7^3} = 7^{2+2-3} = 7^1 = 7$$

$$[b] (4x^3 - y)^2 - \left[4\left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2\right] = \left[4 \times \frac{1}{4} - \frac{1}{9}\right]^2 \\ = \left[1 - \frac{1}{9}\right]^2 = \left[\frac{8}{9}\right]^2 = \left[\frac{2}{3}\right]^2 = \frac{4}{9}$$

## El-Dakshila

$$1 \cdot 1 \cdot b \quad 2 \cdot c \quad 3 \cdot b$$

$$4 \cdot d \quad 5 \cdot c \quad 6 \cdot d$$

$$2 \cdot 1 \cdot 3 \quad 2 \cdot 13 | K | \quad 3 \cdot 144$$

$$4 \cdot \text{Event} \quad 5 \cdot \{5, 6\}$$

3

$$[a] (25ab + 5a) + 5a = 5b + 1 \text{ at } a = 2, b = -1$$

$$\therefore 5b + 1 = 5(-1) + 1 = -5 + 1 = -4$$

$$[b] \left(\frac{-49}{25}\right)^0 \times \left(\frac{-2}{7}\right)^2 \times \sqrt{12 \frac{1}{4}} = 1 \times \frac{4}{49} \times \sqrt{\frac{49}{4}} \\ = 1 \times \frac{4}{49} \times \frac{7}{2} = \frac{2}{7}$$

## Answers of Final Examinations

4

(a) ①  $2x + 7 < 15 \quad \therefore 2x < 8 \quad \therefore x < 4$

$\therefore$  The S.S. =  $\{x : x \in \mathbb{Q}, x < 4\}$

②  $\therefore 6x + 6 = 6 \quad \therefore 6x = 0 \quad \therefore x = 0$

$\therefore$  The S.S. =  $\{0\}$

(b)  $\frac{(3)^{-6} \times (3)^{11}}{(3)^7} = 3^{-6+11-7} = 3^2 = 9$

5

(a)  $\therefore 5x - 2 \geq 3 \quad \therefore 5x \geq 5 \quad \therefore x \geq 1$

$\therefore$  The S.S. =  $\{1, 2, 3, 4, 5, \dots\}$



(b) ① The probability of getting a black ball =  $\frac{0}{15} = 0$

② The probability of getting a red ball =  $\frac{5}{15} = \frac{1}{3}$

## 12 Elfarafle

1

① b

② c

③ a

④ d

⑤ b

⑥ a

2

① 30

②  $15y^4$ ③  $\frac{5x}{y^2}$ 

④ 11

⑤ 4

3

(a)  $\left(\frac{-1}{3}\right)^2 + \sqrt{\frac{64}{81}} - \left(\frac{3}{7}\right)^0 = \frac{1}{9} + \frac{8}{9} - 1 = 0$

(b)  $\frac{8^7 \times 6^3}{8^7 \times 6^4} = 8^7 \div 8^7 \times 6^3 \div 6^4 = 6^{-1}$

at  $a = -3 \quad \therefore a^2 = (-3)^2 = 9$

4

(a)  $\therefore 3x + 4 < 25 \quad \therefore 3x < 21 \quad \therefore x < 7$

$\therefore$  The S.S. =  $\{x : x \in \mathbb{Q}, x < 7\}$

(b)  $x^2 y^2 z = \left(\frac{1}{2}\right)^2 \times \left(\frac{-2}{3}\right)^2 \times \left(\frac{3}{4}\right) = \frac{1}{4} \times \frac{4}{9} \times \frac{3}{4} = \frac{4}{27}$

5

(a) ① The probability of getting an even number =  $\frac{4}{8} = \frac{1}{2}$

② The probability of getting a prime number =  $\frac{4}{8} = \frac{1}{2}$

③ The probability of getting a number more than 7 =  $\frac{1}{8}$

(b)  $\therefore 6x - 8 = 22 \quad \therefore 6x = 30 \quad \therefore x = 5$

$\therefore$  The S.S. =  $\{5\}$

## 13 Damietta

1

① b

② a

③ b

④ c

⑤ c

⑥ b

2

① 1

②  $5.3 \times 10^{-2}$ ③  $\frac{4}{5}$ ④  $\frac{4}{9}$ 

⑤ 13

3

(a)  $\left(\frac{-2}{3}\right)^2 \times \left(\frac{-4}{9}\right)^0 \times \sqrt{3 \frac{6}{25}} = \frac{25}{9} \times 1 \times \sqrt{\frac{81}{25}}$   
 $= \frac{25}{9} \times 1 \times \frac{9}{5} = 5$

(b) ①  $\therefore 3x + 1 > 25 \quad \therefore 3x > 24$

$\therefore x > 8$

$\therefore$  The S.S. =  $\{x : x \in \mathbb{Q}, x > 8\}$

②  $5x + 8 = 15 - 2x$

$\therefore 5x + 2x = 15 - 8$

$\therefore 7x = 7 \quad \therefore x = 1$

$\therefore$  The S.S. =  $\{1\}$

4

(a)  $\frac{x^3 \times x^2}{x^3 \times x^2} = x^{3+2-3-2} = x^0$

at  $x = -3 \quad \therefore x^2 = (-3)^2 = 9$

(b) Let the numbers be :  $x, x + 2, x + 4$

$\therefore x + x + 2 + x + 4 = 60$

$\therefore 3x + 6 = 60 \quad \therefore 3x = 54$

$\therefore x = 18$

$\therefore$  The numbers are :  $18, 20, 22$

5

(a)  $\sqrt{x^2 + y^2} = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

(b) ① The probability of getting a blue ball =  $\frac{6}{15} = \frac{2}{5}$

② The probability of getting a white or red ball =  $\frac{4+5}{15} = \frac{9}{15} = \frac{3}{5}$

③ The probability of getting a green ball =  $\frac{0}{15} = 0$



## Algebra and Statistics

## 14 El-Fayoum

- 1 1) d 2) a 3) b  
4) b 5) c 6) b

- 2 1) 25 2) 5 3)  $\frac{3}{2}$   
4)  $\frac{7}{16} + \frac{9}{32}$  5)  $4.5 \times 10^6$

3  
[a] 1)  $\frac{10^{-3} \times 10^6}{10^2} = 10^{-3+6-2} = 10$   
2)  $\frac{\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{-1}}{\frac{1}{2}} = \left(\frac{1}{2}\right)^{2-1-1} = \left(\frac{1}{2}\right)^{-4} = (2)^4 = 16$

[b] 1)  $8 + 2x = 14$       $2x = 6$   
 $x = 3$       $\therefore$  The S.S. =  $\{3\}$   
2)  $3x - 1 = -10$       $3x = -9$   
 $x = -3$   
 $\therefore$  The S.S. =  $\{-3\}$

4  
[a] 1)  $\therefore 2(x-3) = -x+12$   
 $\therefore 2x-6 = -x+12$   
 $\therefore 2x+x = 12+6$       $\therefore 3x = 18$   
 $\therefore x = 6$       $\therefore$  The S.S. =  $\{6\}$   
2)  $\therefore 5x-1 = 29$       $5x = 30$   
 $\therefore x = 6$       $\therefore$  The S.S. =  $\{6\}$   
[b] 1)  $\frac{9-b}{a^2} = \frac{9-5}{2^2} = \frac{4}{8} = \frac{1}{2}$   
2)  $\frac{6^2}{a+1} = \frac{6^2}{2+1} = \frac{36}{3} = 12$

5  
[a] 1)  $\therefore 3x-4 \geq -10$       $\therefore 3x \geq -6$       $\therefore x \geq -2$   
 $\therefore$  The S.S. =  $\{-2, -1, 0, 1, 2, \dots\}$   
2)  $\therefore x+2 \geq 2$       $\therefore x \geq 2-2$       $\therefore x \geq 0$   
 $\therefore$  The S.S. =  $\{0, 1, 2, 3, \dots\}$   
[b] 1) The probability of the drawn card carries an odd prime number =  $\frac{5}{13} = \frac{1}{3}$   
2) The probability of the drawn card carries a number less than or equal to 1 =  $\frac{1}{13}$

3 The probability of the drawn card carries a number more than 15 =  $\frac{0}{13} = 0$

4 The probability of the drawn card carries the number 15 =  $\frac{1}{13}$

## 15 Qena

- 1 1) b 2) a 3) b  
4) b 5) c 6) b

- 2 1)  $x+5$  2)  $\frac{4}{9}$  3) zero  
4) 2 5) 39

3  
[a]  $\therefore \frac{3}{2}x + 4 < 28$       $\therefore \frac{3}{2}x < 24$   
 $\therefore 3x < 48$       $\therefore x < 16$   
 $\therefore$  The S.S. =  $\{x: x \in \mathbb{Q}, x < 16\}$   
[b]  $a^3 b^2 = \left(-\frac{2}{3}\right)^3 \times \left(\frac{3}{4}\right)^2 = -\frac{8}{27} \times \frac{27}{64} = -\frac{1}{8}$

4  
[a]  $\left(\frac{4}{9}\right)^{-2} \times \left(\frac{1}{9}\right)^0 = \left(\frac{4}{9}\right)^{-2+0} = \left(\frac{4}{9}\right)^{-2} = \frac{81}{64}$   
[b] Let the numbers be:  $x, x+2, x+4$   
 $\therefore x + x+2 + x+4 = 156$   
 $\therefore 3x+6 = 156$       $\therefore 3x = 150$   
 $\therefore x = 50$   
 $\therefore$  The numbers are: 50, 52, 54

5  
[a]  $3^{x+y} = 3^x \times 3^y = 7 \times 5 = 35$   
[b] 1) The probability of appearance of an even number =  $\frac{3}{6} = \frac{1}{2}$   
2) The probability of appearance of a number greater than 3 =  $\frac{3}{6} = \frac{1}{2}$   
3) The probability of appearance of the number 5 =  $\frac{1}{6}$

### Answers of the quizzes on Geometry and Measurement

#### Quiz 1

1

1 equal in measure    2  $360^\circ$     3 parallel

2

[a] Prove by yourself

[b]  $X = 37.5^\circ$ 

#### Quiz 2

1

1  $900^\circ$     2  $360^\circ$     3 6

2

[a] Prove by yourself

[b] 1  $120^\circ$     2  $60^\circ$ 

#### Quiz 3

1

1  $125^\circ$     2 trapezium    3  $120^\circ$ 

2

[a] Prove by yourself

[b]  $X = 125^\circ$ 

#### Quiz 4

1

1 square    2  $45^\circ$     3 concave

2

[a]  $m(\angle D) = 116^\circ$ [b] 12 sides  $\therefore 1800^\circ$ 

#### Quiz 5

1

1  $180^\circ$     2  $120^\circ$     3  $120^\circ$ 

2

[a]  $m(\angle ABC) = 55^\circ$  ,  $m(\angle ACB) = 40^\circ$   
 $\therefore m(\angle BAC) = 85^\circ$ [b] 1 Prove by yourself    2  $CE = 4$  cm.

#### Quiz 6

1

1 bisects the third side

2 obtuse-angled triangle    3 half

2

[a]  $LM = 5$  cm.[b]  $m(\angle A) = 100^\circ$ 

#### Quiz 7

1

1 5    2 right

3 The sum of areas of the two squares drawn on the right sides

2

[a]  $LX = 15$  cm

[b] Prove by yourself

#### Quiz 8

1

1  $(1 + -2)$     2  $(XY)^2 \cdot (YZ)^2$     3  $80^\circ$ 

2

[a]  $150^\circ$ 

[b] Draw by yourself

#### Quiz 9

1

1  $(3 + 5)$     2 acute    3  $(-2 + 3)$ 

2

[a] Prove by yourself    [b] Draw by yourself

## Geometry and Measurement

## Quiz (10)



- 1  $(-2 + 4)$       2 CMB  
3 axis of symmetry



- [a]  $BD = 5 \text{ cm}$ ,  $AC = 20 \text{ cm}$ .  
[b] Draw by yourself

## Quiz (11)



- 1 measures of angles + parallelism  
2 direction + magnitude      3  $(-1 + 7)$



- 3 Draw by yourself

## Quiz (12)



- 1  $90^\circ$       2  $(2 + -4)$       3  $\overline{NL}$



- [a] 1  $\triangle DBE$       [a]  $\triangle EDF$       2  $\triangle FDE$   
[b] Draw by yourself

### Answers of school book models on Geometry and Measurement

#### Model 1

1 (c) 2 (b) 3 (d) 4 (b) 5 (b) 6 (b)

2

1 (2 + -1) 2 80° 3 4  
4 160° 5 180°

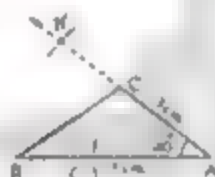
3

[a]  $\because \angle ACD$  is an exterior angle of  $\triangle ABC$

$$\therefore m(\angle ACD) = m(\angle A) + m(\angle B) = 25^\circ + 25^\circ = 50^\circ$$

(The req.)

[b]



4

[a]  $\because M \in \overline{AC}$

$$\therefore m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$$

In  $\triangle BMC$

$$m(\angle MCB) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$$

$$\therefore m(\angle MCB) = m(\angle MAD)$$

and they are alternate angles

$$\overline{AD} \parallel \overline{BC} \quad \therefore \overline{AB} \parallel \overline{DC}$$

$\therefore ABCD$  is a parallelogram

(Q.E.D.)

[b] The point (0, 0)

5

[a] In  $\triangle ABD$  :  $\because m(\angle ADB) = 90^\circ$

$$(BD)^2 = (AB)^2 - (AD)^2 = 676 - 576 = 100$$

$$\therefore BD = \sqrt{100} = 10 \text{ cm}$$

In  $\triangle ADC$  :  $\because m(\angle ADC) = 90^\circ$

$$\therefore (CD)^2 = (AC)^2 - (AD)^2 = 900 - 576 = 324$$

$$\therefore CD = \sqrt{324} = 18 \text{ cm}$$

$$\therefore BC = 10 + 18 = 28 \text{ cm.} \quad (\text{First req.})$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} BC \times AD$$

$$= \frac{1}{2} \times 28 \times 24 = 336 \text{ cm}^2$$

(Second req.)

[b]  $ABCD$  is a square

$$\overline{AD} \parallel \overline{BC}$$

$$\because E \in \overline{BC}$$

$$\therefore \overline{AD} \parallel \overline{EC}$$

$$\because \overline{AC} \parallel \overline{DE}$$

$\therefore ACED$  is a parallelogram

(Q.E.D.)

#### Model 2

1

1 (a) 2 (c) 3 (b) 4 (c) 5 (c) 6 (d)

2

1 44 2 (5 + 5) 3 1728000

4 bisects the third side. 5 ZYC

3

[a] In  $\triangle XYZ$  :  $\because m(\angle Y) = 90^\circ$

$$\therefore (XZ)^2 = (XY)^2 + (YZ)^2 = 49 + 576 = 625$$

$$XZ = \sqrt{625} = 25 \text{ cm.}$$

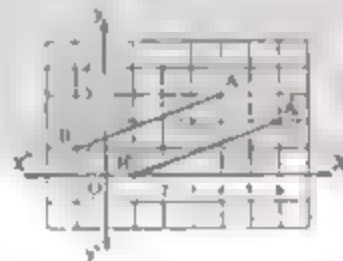
In  $\triangle LXZ$  :  $\because m(\angle L) = 90^\circ$

$$\therefore (LZ)^2 = (XZ)^2 - (LX)^2 = 625 - 225 = 400$$

$$\therefore LZ = \sqrt{400} = 20 \text{ cm.}$$

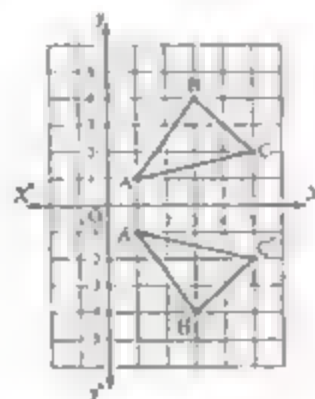
(The req.)

[b]



4

[a]





## Geometry and Measurement

- (b) In  $\triangle ABC$ ,  $m(\angle ACB) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$   
 $\therefore \overline{BD} \cap \overline{AO} = \{C\}$   
 $\therefore m(\angle ACB) = m(\angle OCD) = 60^\circ$  (V.O.A.)  
 $\therefore m(\angle E) = 360^\circ - (60^\circ + 120^\circ + 90^\circ) = 90^\circ$   
 (The req.)

5

- (a)  $\therefore \overline{EO} \parallel \overline{CD}$ ,  $\overline{EB}$  is a transversal  
 $\therefore m(\angle CBA) = m(\angle E) = 50^\circ$  (alternate angles)  
 From  $\triangle ABC$   
 $m(\angle BAC) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$   
 $\therefore \angle ABD$  is an exterior angle of  $\triangle ABC$   
 $\therefore m(\angle ABD) = 30^\circ + 100^\circ = 130^\circ$  (The req.)

- (b)  $\therefore \overline{AD} \parallel \overline{XY} \parallel \overline{BC}$ ,  $\therefore AX = XB$   
 $\therefore DY = YC$   
 $\therefore Y$  is the midpoint of  $\overline{CD}$   
 In  $\triangle CDE$ :  $\therefore \overline{ZY} \parallel \overline{DE}$   
 $\therefore Y$  is the midpoint of  $\overline{CD}$   
 $\therefore Z$  is the midpoint of  $\overline{CE}$   
 $\therefore CZ = ZE$  (Q.E.D.)

## Model examination for the merge students

- |   |   |               |          |
|---|---|---------------|----------|
| 1 | 1 c   | 2 b           | 3 b      |
|   | 4 a   | 5 b           |          |
| 2 | 1 half                                      | 2 right       | 3 6      |
|   | 4 (3, 2)                                    | 5 $70^\circ$  |          |
| 3 | 1 x   | 2 x           | 3 ✓      |
|   | 4 x   | 5 ✓           |          |
| 4 | 1 $360^\circ$                               | 2 $120^\circ$ | 3 (4, 0) |
|   | 4 (1, -3)                                   | 5 $45^\circ$  |          |
| 5 | Fig (1): $x = 8$<br>Fig (2): $x = 90^\circ$ |               |          |

## Answers of Final Examinations

Answers of schools examinations  
on Geometry and Measurement

## 1 Cairo

1

- 1  $360^\circ$       2 equal in measure      3  $90^\circ$   
4  $(6 \div 1)$       5 half

2

- 1 a      2 d      3 b      4 d      5 d      6 c

3

[a]  $\because \overline{GH} \parallel \overline{AB}$ ,  $\overline{AG}$  is a transversal to them

$$\therefore m(\angle A) + m(\angle G) = 180^\circ$$

(two interior angles in the same side of the transversal)

$$\therefore m(\angle A) = 180^\circ - 135^\circ = 45^\circ$$

$\because \overline{DC} \parallel \overline{AB}$ ,  $\overline{BD}$  is a transversal to them

$$\therefore m(\angle B) + m(\angle D) = 180^\circ$$

(two interior angles in the same side of the transversal)

$$\therefore m(\angle B) = 180^\circ - 145^\circ = 35^\circ$$

$\therefore$  In  $\triangle ABE$

$$m(\angle BEA) = 180^\circ - (45^\circ + 35^\circ) = 100^\circ$$

$$\therefore \overline{AG} \cap \overline{BD} = \{E\}$$

$$\therefore m(\angle DEG) = m(\angle BEA) = 100^\circ \quad (\text{The req.})$$

[b] In  $\triangle ABC$ :

$\therefore E$  is the midpoint of  $\overline{AB}$

$\therefore D$  is the midpoint of  $\overline{AC}$

$$\therefore DE = \frac{1}{2} BC$$

$$\therefore \text{similarly, } ME = \frac{1}{2} AC$$

$$\therefore DE = MC$$

$$\therefore ME = CD$$

$$\therefore DEMC \text{ is a parallelogram} \quad (\text{Q.E.D.})$$

4

[a] In  $\triangle ABC$ :

$$\because m(\angle ACB) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 - (BC)^2 = (13)^2 - (12)^2 = 25$$

$$\therefore AC = \sqrt{25} = 5 \text{ cm.} \quad (\text{First req.})$$

$\therefore$  the perimeter of the figure ABCD

$$= 13 + 12 + 4 + 3 = 32 \text{ cm.} \quad (\text{Second req.})$$

[b] In  $\triangle ABC$ 

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore F$  is the midpoint of  $\overline{BC}$

$$\therefore AC = 2 DF = 2 \times 4 = 8 \text{ cm}$$

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore E$  is the midpoint of  $\overline{AC}$

$$\therefore BC = 2 DE = 2 \times 6 = 12 \text{ cm.}$$

$\therefore F$  is the midpoint of  $\overline{BC}$

$\therefore E$  is the midpoint of  $\overline{AC}$

$$\therefore AB = 2 FE = 2 \times 3 = 6 \text{ cm}$$

$$\therefore \text{The perimeter of } \triangle ABC = 8 + 12 + 6 = 26 \text{ cm.}$$

(The req.)

5

[a]

[b] In  $\triangle ABC$ :

$$m(\angle ACB) = 180^\circ - (30^\circ + 90^\circ) = 60^\circ$$

$$\therefore m(\angle DCF) = m(\angle ACB) = 60^\circ \quad (\text{V.O.A.})$$

$\therefore$  From the quadrilateral CDEF

$$m(\angle E) = 360^\circ - (120^\circ + 60^\circ + 90^\circ) = 90^\circ$$

(The req.)

## 2 Cairo

1

- 1 b      2 d      3 a      4 s      5 d      6 c

2

1 a trapezium

2  $135^\circ$

3  $(3 \div 1)$

4 9

5  $130^\circ$

Geometry and Measurement

3

[a] In  $\triangle ABC$ :

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore E$  is the midpoint of  $\overline{AC}$

$\therefore \overline{DE} \parallel \overline{BC}$   $\therefore \angle FECB$

$\therefore \overline{DE} \parallel \overline{BF}$  (1)

$\therefore DE = \frac{1}{2} BC$   $\therefore BF = \frac{1}{2} BC$

$\therefore DE = BF$  (2)

From (1) and (2)

$\therefore DFBE$  is a parallelogram. (Q.E.D.)

[b]



4

[a] In  $\triangle ABC$ :

$\therefore m(\angle B) = 90^\circ$

$(AC)^2 = (AB)^2 + (BC)^2$

$= (9)^2 + (12)^2 = 81 + 144 = 225$

$\therefore AC = \sqrt{225} = 15 \text{ cm.}$  (First req.)

In  $\triangle ACD$

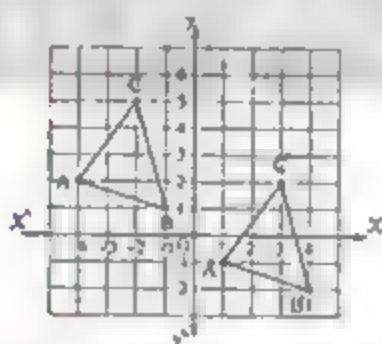
$\therefore m(\angle ACD) = 90^\circ$

$(AD)^2 = (AC)^2 + (CD)^2$

$= (15)^2 + (20)^2 = 225 + 400 = 625$

$\therefore AD = \sqrt{625} = 25 \text{ cm}$  (Second req.)

[b]



5

[a] In  $\triangle COE$

$m(\angle COE) = 180^\circ - (50^\circ + 35^\circ) = 95^\circ$

$\therefore m(\angle BOD) = m(\angle COE) = 95^\circ$  (V.O.A.)

from the quadrilateral ABOD

$\therefore m(\angle B) = 360^\circ - (85^\circ + 100^\circ + 95^\circ) = 80^\circ$

(The req.)

[b]  $\therefore \overline{AC} \cap \overline{DE} = \{O\}$

$\therefore m(\angle CBE) = m(\angle ABD) = 40^\circ$  (V.O.A.)

$\therefore \overline{BE}$  bisects  $\angle CBF$

$\therefore m(\angle EBF) = m(\angle CBE) = 40^\circ$

$\therefore m(\angle ABF) = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$

(The req.)

3

Cairo

1

1 c

2 b

3 c

4 a

5 b

6 a

2

1 the third side

2  $(-2, -4)$

3 2

4  $120^\circ$

5 216

3

[a] In  $\triangle ABC$ :

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore E$  is the midpoint of  $\overline{BC}$

$\therefore DE = \frac{1}{2} AC$   $\therefore DE = 4 \text{ cm}$

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore F$  is the midpoint of  $\overline{AC}$

$\therefore DF = \frac{1}{2} BC$   $\therefore DF = 3.5 \text{ cm.}$

$\therefore F$  is the midpoint of  $\overline{AC}$

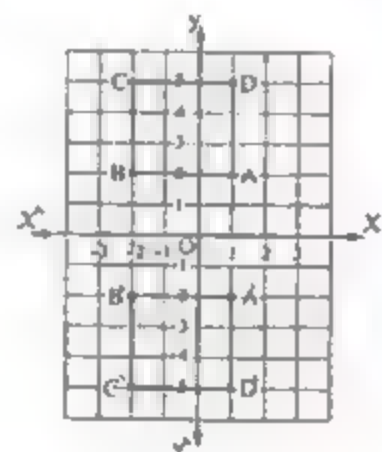
$\therefore E$  is the midpoint of  $\overline{BC}$

$\therefore FE = \frac{1}{2} AB$   $\therefore FE = 2.5 \text{ cm.}$

$\therefore$  The perimeter of  $\triangle DEF = 4 + 3.5 + 2.5 = 10 \text{ cm.}$

(The req.)

[b]



## Answers of Final Examinations

4

[a] In  $\triangle ABC$ :

$$\therefore m(\angle B) = 90^\circ$$

$$(AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$$

$$\therefore AC = \sqrt{625} = 25 \text{ cm.} \quad (\text{First req.})$$

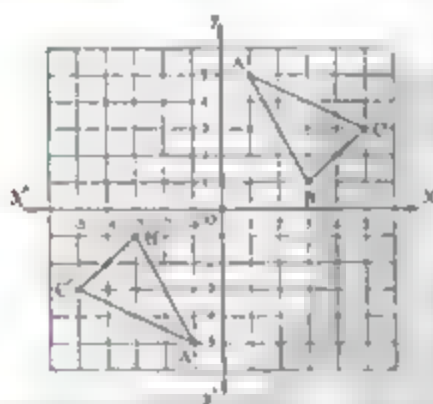
In  $\triangle ADC$ 

$$m(\angle D) = 90^\circ$$

$$\therefore (DC)^2 = (AC)^2 - (AD)^2 = (25)^2 - (15)^2 = 400$$

$$DC = \sqrt{400} = 20 \text{ cm} \quad (\text{Second req.})$$

(b)



5

[a] In  $\triangle ABC$ 

$$m(\angle ABC) = 180^\circ - (50^\circ + 70^\circ) = 60^\circ$$

$$\therefore m(\angle DBE) = m(\angle ABC) = 60^\circ \quad (\text{V.O.A.})$$

from the quadrilateral BDFE,

$$m(\angle E) = 360^\circ - (60^\circ + 130^\circ + 90^\circ) = 80^\circ$$

(The req.)

(b) ABCD is a rhombus

BD is a diagonal

$$\therefore m(\angle B) = 2 \times 62^\circ = 124^\circ$$

$$\therefore m(\angle A) = 180^\circ - 124^\circ = 56^\circ \quad (\text{The req.})$$

4 Giz

1

1 c 2 c 3 a 4 b 5 a 6 b

2

1  $180^\circ$  2 equal in measure  
3 half 4  $(3 \div 5)$  5  $(3 \div 0)$

3

[a] The perimeter of a regular hexagon

$$= 6 \times 15 = 90 \text{ cm.} \quad (\text{First req.})$$

$$\begin{aligned} \text{the measure of each angle} &= \frac{(6-2) \times 180^\circ}{6} \\ &= 120^\circ \end{aligned}$$

(Second req.)

(b)  $\therefore ABCD$  is a parallelogram  $\therefore m(\angle B) = 120^\circ$ 

$$\therefore m(\angle C) = 180^\circ - 120^\circ = 60^\circ$$

$$\therefore m(\angle D) = m(\angle B) = 120^\circ$$

$$\therefore AD = BC = 7 \text{ cm}$$

$$\therefore DC = AB = 5 \text{ cm} \quad (\text{The req.})$$

4

[a]  $\therefore \overline{DE} \parallel \overline{BC}$ ,  $\overline{BD}$  is a transversal

$$\therefore m(\angle B) = m(\angle D) = 50^\circ \quad (\text{alternate angles})$$

In  $\triangle ABC$ 

$$m(\angle BAC) = 180^\circ - (50^\circ + 30^\circ) = 100^\circ$$

(The req.)

(b)  $\therefore X$  is the midpoint of  $\overline{AB}$  $\therefore Y$  is the midpoint of  $\overline{BC}$ 

$$XY = \frac{1}{2} AC \quad \therefore XY = 3 \text{ cm}$$

 $\therefore X$  is the midpoint of  $\overline{AB}$  $\therefore Z$  is the midpoint of  $\overline{AC}$ 

$$XZ = \frac{1}{2} BC \quad \therefore XZ = 5 \text{ cm}$$

 $\therefore Y$  is the midpoint of  $\overline{BC}$  $\therefore Z$  is the midpoint of  $\overline{AC}$ 

$$\therefore YZ = \frac{1}{2} AB \quad \therefore YZ = 4 \text{ cm.}$$

$$\text{The perimeter of } \triangle XYZ = 3 + 5 + 4 = 12 \text{ cm}$$

(The req.)

5

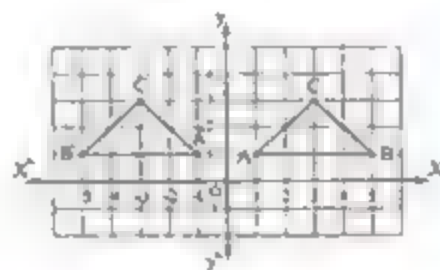
[a] In  $\triangle ABC$ 

$$m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$$

$$\therefore AC = \sqrt{625} = 25 \text{ cm} \quad (\text{The req.})$$

(b)





## Geometry and Measurement

## 5 Giza

1

- 1) (3, 5)      **B** bisects  
 2) perpendicular and bisect each other  
 3)  $360^\circ$       **B** a right-angled triangle

2

- 1 d    2 d    3 a    4 d    5 a    6 c

3

[a] In  $\triangle ABC$ :

$$\because m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (4)^2 + (3)^2 = 25$$

$$AC = \sqrt{25} = 5 \text{ cm} \quad (\text{First req.})$$

In  $\triangle ACD$ :

$$\because m(\angle C) = 90^\circ$$

$$\therefore (CD)^2 = (AD)^2 - (AC)^2 = (13)^2 - (5)^2 = 144$$

$$\therefore CD = \sqrt{144} = 12 \text{ cm.} \quad (\text{Second req.})$$

[b]  $\because$  ABCD is a square

$$\therefore BC = AD$$

$$\therefore BC = CE \quad \therefore AD = CE \quad (1)$$

$$\therefore \overline{BC} \parallel \overline{AD} \text{ and } E \in \overline{BC}$$

$$\therefore \overline{CE} \parallel \overline{AD} \quad (2)$$

From (1) and (2):

$$\therefore AD = CE \text{ and } \overline{AD} \parallel \overline{CE}$$

$$\therefore ACED \text{ is a parallelogram} \quad (\text{First req.})$$

 $\overline{AC}$  is a diagonal in square ABCD

$$\therefore m(\angle ACB) = 45^\circ$$

$$\therefore m(\angle E) = m(\angle ACB) = 45^\circ$$

$$(\text{corresponding angles}) \quad (\text{Second req.})$$

4

[a] In  $\triangle ABC$ :X is the midpoint of  $\overline{AB}$ Y is the midpoint of  $\overline{BC}$ 

$$\therefore AC = 2XY = 2 \times 4 = 8 \text{ cm}$$

X is the midpoint of  $\overline{AB}$ Z is the midpoint of  $\overline{CA}$ 

$$\therefore BC = 2XZ = 2 \times 5 = 10 \text{ cm}$$

Z is the midpoint of  $\overline{CA}$ Y is the midpoint of  $\overline{BC}$ 

$$\therefore AB = 2YZ = 2 \times 6 = 12 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle ABC = 8 + 10 + 12 = 30 \text{ cm.}$$

(The req.)

[b] In parallelogram ABCD

$$m(\angle BCD) = m(\angle A) = 60^\circ \quad (1)$$

In parallelogram EBCF

$$m(\angle EBC) = 180^\circ - 130^\circ = 50^\circ \quad (2)$$

In  $\triangle XBC$ :

From (1) &amp; (2):

$$m(\angle BXC) = 180^\circ - (60^\circ + 50^\circ) = 70^\circ \quad (\text{The req.})$$

5

[a] In  $\triangle CEF$ :

$$m(\angle ECF) = m(\angle E) = m(\angle F) = 60^\circ$$

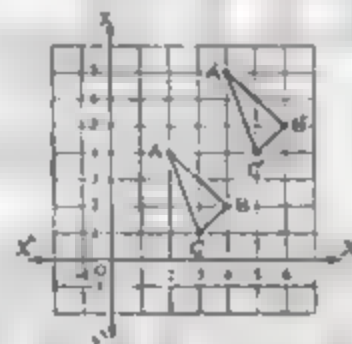
$$\therefore m(\angle BCD) = m(\angle ECF) = 60^\circ \quad (\text{V.O.A.})$$

from the quadrilateral ABCD

$$\therefore m(\angle D) = 360^\circ - (80^\circ + 120^\circ + 60^\circ) = 100^\circ$$

(The req.)

[b]



## 6 Alexandria

1

- 1 a cube  
 2 half the length of the third side  
 3) 125      4)  $(2x-1)$       5)  $120^\circ$

2

- 1 d    2 b    3 a    4 c    5 a    6 d

3

[a]  $\overline{AB} \parallel \overline{CD}$ ,  $\overline{AC}$  is a transversal

$$\therefore m(\angle ACD) = m(\angle A) = 50^\circ \quad (\text{alternate angles})$$

$$\therefore \angle ACE \text{ is a right angle}$$

## Answers of Final Examinations

$$\therefore m(\angle ECD) = 90^\circ - 50^\circ = 40^\circ$$

$$\therefore m(\angle E) = m(\angle ECD) = 40^\circ$$

and they are alternate angles

$$\therefore \overline{CD} \parallel \overline{EF}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad \therefore \overline{AB} \parallel \overline{EF} \quad (\text{Q.E.D.})$$

[b] (0, 0)

4

[a] In  $\triangle ABC$ ,

$$\therefore m(\angle B) = 90^\circ$$

$$\therefore (AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$$

$$\therefore AB = \sqrt{256} = 16 \text{ cm.}$$

$$\therefore BD = 9 \text{ cm}$$

$$\therefore AD = 16 - 9 = 7 \text{ cm}$$

(First req.)

$$\therefore AE = 2BC = 2 \times 12 = 24 \text{ cm}$$

$$\therefore \overline{AE} \parallel \overline{BC}, \overline{AB} \text{ is a transversal}$$

$$\therefore m(\angle EAB) = m(\angle B) = 90^\circ \text{ (alternate angles)}$$

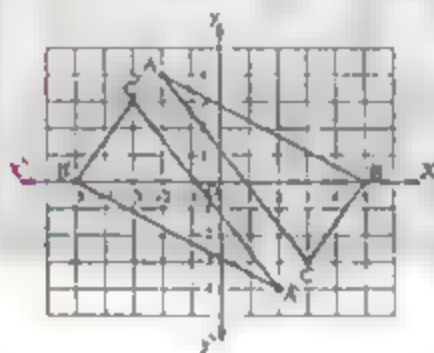
$$\therefore \text{In } \triangle EAD$$

$$\therefore m(\angle EAD) = 90^\circ$$

$$\therefore (ED)^2 = (AE)^2 + (AD)^2 = (24)^2 + (7)^2 = 625$$

$$\therefore ED = \sqrt{625} = 25 \text{ cm.} \quad (\text{Second req.})$$

[b]



5

[a]  $\therefore ME \parallel AC$

$$\therefore m(\angle BMC) = 180^\circ - 70^\circ = 110^\circ$$

$\therefore \text{In } \triangle BMC$ :

$$m(\angle MCB) = 180^\circ - (110^\circ + 40^\circ) = 30^\circ$$

$$\therefore m(\angle MCB) = m(\angle MAD)$$

and they are alternate angles.

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore \overline{AB} \parallel \overline{DC}$$

$$\therefore ABCD \text{ is a parallelogram.} \quad (\text{Q.E.D.})$$

[b] In  $\triangle ABC$

$$\therefore D \text{ is the midpoint of } \overline{AB}$$

$$\therefore E \text{ is the midpoint of } \overline{BC}$$

$$\therefore DE = \frac{1}{2} AC \quad \therefore DE = 3.5 \text{ cm}$$

$$\therefore D \text{ is the midpoint of } \overline{AB}$$

$$\therefore F \text{ is the midpoint of } \overline{AC}$$

$$\therefore DF = \frac{1}{2} BC \quad \therefore DF = 4 \text{ cm.}$$

$$\therefore E \text{ is the midpoint of } \overline{BC}$$

$$\therefore F \text{ is the midpoint of } \overline{AC}$$

$$\therefore EF = \frac{1}{2} AB \quad \therefore EF = 2.5 \text{ cm}$$

$$\therefore \text{The perimeter of } \triangle DEF = 3.5 + 4 + 2.5 = 10 \text{ cm} \quad (\text{The req.})$$

## 7 Alexandria

1

$$\text{1 } 180^\circ$$

$$\text{2 } (4, 2)$$

$$\text{3 } \text{bisects the third side.}$$

$$\text{4 } (2, -1)$$

$$\text{5 } \text{half the length of the third side}$$

$$\text{6 } \text{parallelogram}$$

2

$$\text{1 } c$$

$$\text{2 } d$$

$$\text{3 } b$$

$$\text{4 } d$$

$$\text{5 } c$$

3

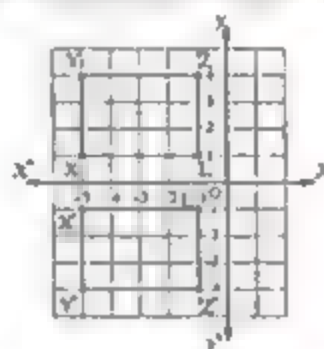
[a] In  $\triangle ABC$ :

$$\therefore m(\angle A) = 90^\circ$$

$$\therefore (AC)^2 = (CB)^2 - (AB)^2 = 100 - 36 = 64$$

$$\therefore AC = \sqrt{64} = 8 \text{ cm.} \quad (\text{The req.})$$

[b]



4

[a] In  $\triangle ABC$

$$\therefore D \text{ is the midpoint of } \overline{AB}$$

$$\therefore E \text{ is the midpoint of } \overline{BC}$$

## Geometry and Measurement

$$\therefore DE = \frac{1}{2} AC \quad \therefore DE = 4$$

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore F$  is the midpoint of  $\overline{AC}$

$$\therefore DF = \frac{1}{2} BC \quad DF = 5$$

$\therefore E$  is the midpoint of  $\overline{BC}$

$\therefore F$  is the midpoint of  $\overline{AC}$

$$EF = \frac{1}{2} AB \quad EF = 3.5 \text{ cm}$$

The perimeter of  $\triangle FDE$

$$= 4 + 5 + 3.5 = 12.5 \text{ cm} \quad (\text{The req.})$$

[b] Let the measures of the interior angles of the quadrilateral be  $2x, 2x, 3x, 5x$

the sum of the measures of the interior angles of the quadrilateral =  $360^\circ$

$$2x + 2x + 3x + 5x = 360^\circ$$

$$12x = 360^\circ \quad x = \frac{360^\circ}{12} = 30^\circ$$

$$\therefore \text{The measure of the biggest angle} = 5 \times 30^\circ = 150^\circ$$

(The req.)

3

[a] (3)  $\rightarrow$  B

[b] In  $\triangle ABC$

$\therefore \angle ACD$  is an exterior angle of  $\triangle ABC$

$$m(\angle ACD) = m(\angle A) + m(\angle B)$$

$$= 30^\circ + 30^\circ = 60^\circ \quad (\text{The req.})$$

### 8 El-Kalyoubia

1

1 b    2 b    3 d    4 c    5 d    6 b

2

1 2    2 8000    (3)  $(-3) + (-2)$

4  $140^\circ$     5 120

3

[a] In  $\triangle ABC$

$$m(\angle ABC) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ$$

$$m(\angle DBF) = m(\angle ABC) = 60^\circ \quad (\text{V.O.A.})$$

from the quadrilateral BDEF

$$m(\angle F) = 360^\circ - (60^\circ + 140^\circ + 90^\circ) = 70^\circ$$

(The req.)

[b]  $D$  is the midpoint of  $\overline{AB}$

$\therefore F$  is the midpoint of  $\overline{AC}$

$$\therefore DF = \frac{1}{2} BC = 6 \text{ cm}$$

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore E$  is the midpoint of  $\overline{BC}$

$$\therefore DE = \frac{1}{2} AC = 5 \text{ cm}$$

$$\therefore CE = \frac{1}{2} BC = 6 \text{ cm}, \quad CP = \frac{1}{2} AC = 5 \text{ cm}$$

$\therefore$  The perimeter of  $\triangle DCF$

$$= 6 + 5 + 6 + 5 = 22 \text{ cm} \quad (\text{The req.})$$

4

[a]  $\therefore ABCD$  is a square

$$\therefore \overline{AD} \parallel \overline{BC}$$

$\therefore B \in \overline{BC}$

$$\therefore \overline{AD} \parallel \overline{CE}$$

$\therefore \overline{AC} \parallel \overline{ED}$

$ACED$  is a parallelogram (First req.)

$\therefore \overline{AC}$  is a diagonal in square  $ABCD$

$$m(\angle ACB) = 45^\circ$$

$$m(\angle ACE) = 180^\circ - 45^\circ = 135^\circ \quad (\text{Second req.})$$

[b] In  $\triangle ADC$ ,

$$\therefore m(\angle ADC) = 90^\circ$$

$$\therefore (CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$$

$$CD = \sqrt{324} = 18 \text{ cm}$$

In  $\triangle ADB$

$$m(\angle ADB) = 90^\circ$$

$$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$$

$$\therefore BD = \sqrt{100} = 10 \text{ cm}$$

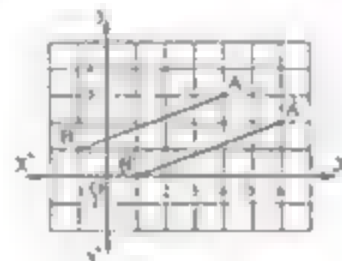
$$\therefore BC = 18 + 10 = 28 \text{ cm} \quad (\text{First req.})$$

The area of  $\triangle ABC$

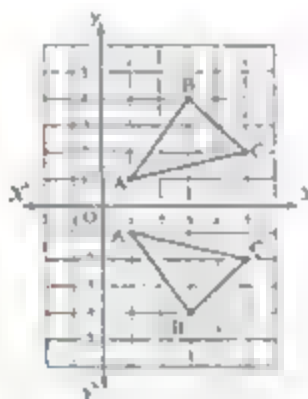
$$= \frac{1}{2} \times 28 \times 24 = 336 \text{ cm}^2 \quad (\text{Second req.})$$

5

[a]



[b]



## 9 El-Gharbia

- 1 d 2 d 3 b 4 c 5 c 6 c

2

1. parallel to the third side

2. 120

3.  $(-3, -2)$ 

4. equal in measure

5. a right angle

3

[a]  $\therefore \overline{AC} \cap \overline{DE} = \{B\}$ 
 $\therefore m(\angle CBE) = m(\angle ABD) = 40^\circ$  (V.O.A.)

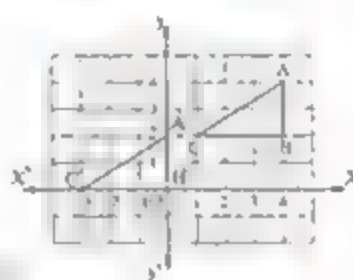
 $\overline{BE}$  bisects  $\angle CBF$ 
 $m(\angle FBE) + m(\angle EBC) = 80^\circ$ 
 $\therefore m(\angle FBC) = 80^\circ$ 
 $B \in \overline{AC}$ 
 $\therefore m(\angle ABF) = 180^\circ - 80^\circ = 100^\circ$  (The req.)
[b]  $\therefore \overline{DE} \parallel \overline{CB}$   $\therefore \overline{BD}$  is a transversal
 $\therefore m(\angle B) = m(\angle D) = 50^\circ$  (alternate angles)
In  $\triangle ABC$ 
 $m(\angle BAC) = 180^\circ - (35^\circ + 50^\circ) = 95^\circ$  (The req.)

4

[a] In  $\triangle XYZ$ 
 $D$  is the midpoint of  $\overline{XY}$ 
 $O$  is the midpoint of  $\overline{YZ}$ 
 $DO = \frac{1}{2} \overline{XZ} = 3 \text{ cm}$ 
 $\therefore D$  is the midpoint of  $\overline{XY}$ 
 $E$  is the midpoint of  $\overline{XZ}$ 
 $\therefore DE = \frac{1}{2} \overline{YZ} = 3.5 \text{ cm}$ 
 $\therefore O$  is the midpoint of  $\overline{YZ}$ 
 $E$  is the midpoint of  $\overline{XZ}$ 
 $\therefore OE = \frac{1}{2} \overline{XY} = 2.5 \text{ cm}$ 
 $\therefore$  The perimeter of  $\triangle DOE$ 
 $= 3 + 3.5 + 2.5 = 9 \text{ cm}$ 

(The req.)

[b]

[a] In  $\triangle ABC$ 
 $\therefore m(\angle B) = 90^\circ$ 
 $\therefore (AC)^2 = (AB)^2 + (BC)^2 = (12)^2 + (9)^2 = 225$ 
 $\therefore AC = \sqrt{225} = 15 \text{ cm}$ 
In  $\triangle ACD$ 
 $\therefore m(\angle ACD) = 90^\circ$ 
 $(AD)^2 = (AC)^2 + (CD)^2 = (15)^2 + (120)^2 = 625$ 
 $\therefore AD = \sqrt{625} = 25 \text{ cm}$ 

(The req.)

[b] ABCD is a square

 $\overline{AD} \parallel \overline{BC}$ 
 $\therefore E \in \overline{BC}$ 
 $\overline{AD} \parallel \overline{CE}$ 
 $\therefore \overline{AC} \parallel \overline{DE}$ 
 $\therefore ACED$  is a parallelogram

(Q.E.D.)

## 10 El-Dakahlia

1

1 d

2 c

3 d

4 c

5 a

6 c

2

1 120°

2 35°

3 9 cm

4 (9, -6)

5 a right angle

3

[a] In  $\triangle ADC$ 
 $\therefore m(\angle ADC) = 90^\circ$ 
 $(CD)^2 = (AC)^2 - (AD)^2 = (30)^2 - (24)^2 = 324$ 
 $\therefore CD = \sqrt{324} = 18 \text{ cm}$ 
In  $\triangle ADB$ 
 $\therefore m(\angle ADB) = 90^\circ$



## Geometry and Measurement

$$\therefore (BD)^2 = (AB)^2 - (AD)^2 = (26)^2 - (24)^2 = 100$$

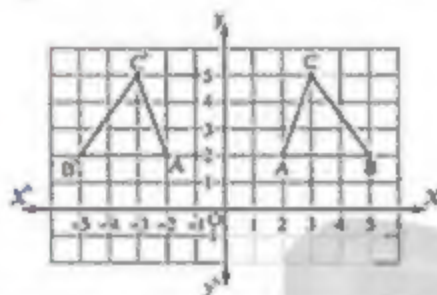
$$\therefore BD = \sqrt{100} = 10 \text{ cm.}$$

$$\therefore BC = 18 + 10 = 28 \text{ cm.} \quad (\text{First req.})$$

$\therefore$  The area of  $\triangle ABC$

$$= \frac{1}{2} \times 28 \times 24 = 336 \text{ cm}^2. \quad (\text{Second req.})$$

[b]



4

[a] In  $\triangle DEF$ :

$$\therefore m(\angle DFE) = 60^\circ$$

$$\therefore m(\angle CFG) = m(\angle DFE) = 60^\circ \quad (\text{V.O.A.})$$

$\therefore$  the sum of measures of interior angles of the pentagon  $= (5 - 2) \times 180^\circ = 540^\circ$

$$\therefore m(\angle C) + m(\angle G) = 540^\circ - (100^\circ + 90^\circ + 60^\circ) = 290^\circ$$

$$\therefore m(\angle C) = m(\angle G)$$

$$\therefore m(\angle C) = \frac{290^\circ}{2} = 145^\circ \quad (\text{The req.})$$

[b] In  $\triangle ABC$ :

$\therefore D$  is the midpoint of  $\overline{AB}$

$\therefore E$  is the midpoint of  $\overline{AC}$

$$\therefore \overline{DE} \parallel \overline{BC}, DE = \frac{1}{2} BC = 6 \text{ cm.}$$

$\therefore$  In  $\triangle FDE$ :

$\therefore X$  is the midpoint of  $\overline{DF}$

$\therefore \overline{XY} \parallel \overline{DE}$

$\therefore Y$  is the midpoint of  $\overline{EF}$

$$\therefore XY = \frac{1}{2} DE = 3 \text{ cm.} \quad (\text{The req.})$$

5

[a]  $\therefore ABCD$  is a square

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore E \in \overline{BC} \therefore \overline{AD} \parallel \overline{CE}$$

$$\therefore \overline{AC} \parallel \overline{ED}$$

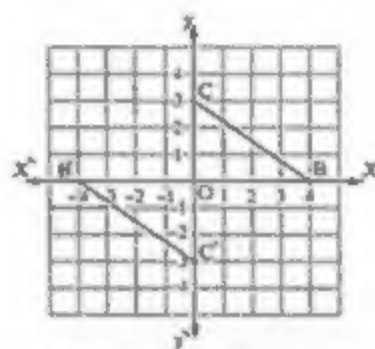
$\therefore ACED$  is a parallelogram. (First req.)

$\therefore \overline{AC}$  is a diagonal in square  $ABCD$

$$\therefore m(\angle ACB) = 45^\circ$$

$$\therefore m(\angle ACE) = 180^\circ - 45^\circ = 135^\circ \quad (\text{Second req.})$$

[b]



11 Suez

1

- 1 b    2 b    3 d    4 b    5 d    6 c

2

- 1  $140^\circ$     2 bisects the third side.  
3  $(1, 2)$     4  $540^\circ$     5  $\overline{AC} \perp \overline{BD}$

3

$$[a] m(\angle ABE) = 180^\circ - 116^\circ = 64^\circ$$

$\therefore \overline{BD}$  bisects  $\angle ABE$

$$\therefore m(\angle ABD) = \frac{1}{2} m(\angle ABE) = 32^\circ \quad (\text{The req.})$$

$$[b] m(\angle DBE) = 360^\circ - (110^\circ + 35^\circ + 140^\circ) = 75^\circ \quad (\text{The req.})$$

4

[a]  $\therefore \overline{AB} \parallel \overline{CD}, \overline{AC}$  is a transversal

$$\therefore m(\angle ACD) = m(\angle A) = 50^\circ \quad (1) \quad (\text{alternate angles})$$

$\therefore \overline{CD} \parallel \overline{EF}, \overline{CE}$  is a transversal

$$\therefore m(\angle ECD) = m(\angle E) = 40^\circ \quad (2) \quad (\text{alternate angles})$$

From (1) + (2):

$$\therefore m(\angle ACE) = m(\angle ACD) + m(\angle ECD) = 50^\circ + 40^\circ = 90^\circ \quad (\text{The req.})$$

[b] In  $\triangle ABC$ :

$$\therefore m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (6)^2 + (8)^2 = 36 + 64 = 100$$

$$\therefore AC = \sqrt{100} = 10 \text{ cm.} \quad (\text{The req.})$$

5

- [a] 1  $(-2, 3)$     2  $(-3, -2)$   
3  $(-3, -2)$     4 3

80



[b] In  $\triangle ABC$ :

$$m(\angle ACB) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

$$\therefore m(\angle DCF) = m(\angle ACB) = 30^\circ \quad (\text{V.O.A.})$$

 $\therefore$  In the polygon CDEF:

$$m(\angle E) = 360^\circ - (30^\circ + 120^\circ + 90^\circ) = 120^\circ$$

(The req.)

## 12 Port Said

1

- 1 b    2 c    3 b    4 b    5 b    6 d

2

- 1  $\frac{1}{2}$     2  $(3, -2)$     3  $360^\circ$   
4  $120^\circ$     5 6

3

[a] In  $\triangle ABC$ :

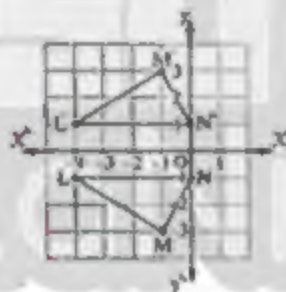
$$\therefore m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$= (3)^2 + (4)^2 = 9 + 16 = 25$$

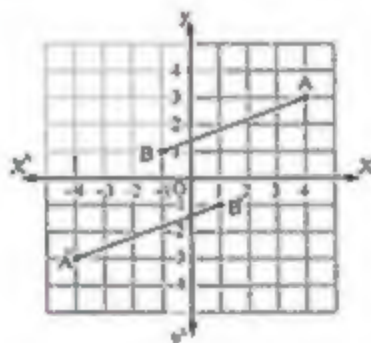
$$\therefore AC = \sqrt{25} = 5 \text{ cm.} \quad (\text{The req.})$$

[b]



4

[a]

[b]  $\therefore \overline{DC} \parallel \overline{EO}$ ,  $\overline{DE}$  is a transversal

$$\therefore m(\angle E) + m(\angle D) = 180^\circ$$

(interior angles on the same side of the transversal)

$$\therefore m(\angle E) = 180^\circ - 90^\circ = 90^\circ$$

$$\therefore m(\angle C) = 360^\circ - (120^\circ + 90^\circ + 90^\circ) = 60^\circ \quad (\text{The req.})$$

5

[a] In  $\triangle ABC$ : $\therefore$  D is the midpoint of  $\overline{AB}$  $\therefore$  E is the midpoint of  $\overline{AC}$ 

$$\therefore CB = 2ED = 16 \text{ cm.} \quad (\text{The req.})$$

[b]  $\therefore \angle ACD$  is an exterior angle of  $\triangle ABC$ 

$$\therefore m(\angle ACD) = 25^\circ + 25^\circ = 50^\circ \quad (\text{The req.})$$

## 13 Kafr El-Sheikh

1

- 1 c    2 b    3 b    4 c    5 d    6 b

2

- 1  $(1, 1)$     2 a square    3  $(1, -2)$   
4 half    5 28

3

[a] From the pentagon ABCDE

$$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D)$$

$$+ m(\angle DEA) = 540^\circ$$

$$\therefore m(\angle C) = 540^\circ - (110^\circ + 120^\circ + 90^\circ + 115^\circ) = 105^\circ \quad (\text{The req.})$$

[b]  $\therefore \overline{ED} \parallel \overline{BC}$ ,  $\overline{EC}$  is a transversal

$$\therefore m(\angle C) + m(\angle E) = 180^\circ$$

(interior angles on the same side of the transversal)

$$\therefore m(\angle C) = 180^\circ - 110^\circ = 70^\circ$$

 $\therefore \overline{FH} \parallel \overline{BC}$ ,  $\overline{FB}$  is a transversal

$$\therefore m(\angle B) + m(\angle F) = 180^\circ$$

(interior angles on the same side of the transversal)

$$\therefore m(\angle B) = 180^\circ - 130^\circ = 50^\circ$$

 $\therefore$  In  $\triangle ABC$ :

$$m(\angle BAC) = 180^\circ - (70^\circ + 50^\circ) = 60^\circ \quad (\text{The req.})$$

4

[a] In  $\triangle ABC$ :

$$\therefore m(\angle B) = 90^\circ$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2 = (7)^2 + (24)^2 = 625$$



## Geometry and Measurement

$$\therefore AC = \sqrt{625} = 25 \text{ cm.}$$

(First req.)

In  $\triangle ACD$ :

$$\because m(\angle D) = 90^\circ$$

$$\therefore (DC)^2 = (AC)^2 - (AD)^2 = (25)^2 - (15)^2 = 400$$

$$\therefore DC = \sqrt{400} = 20 \text{ cm.}$$

(Second req.)

[b] In  $\triangle ABC$ : $\because X$  is the midpoint of  $\overline{AB}$  $\because Y$  is the midpoint of  $\overline{AC}$ 

$$\therefore XY = \frac{1}{2} BC = 4 \text{ cm.}$$

 $\because X$  is the midpoint of  $\overline{AB}$  $\because Z$  is the midpoint of  $\overline{BC}$ 

$$\therefore XZ = \frac{1}{2} AC = 4 \text{ cm.}$$

 $\because Y$  is the midpoint of  $\overline{AC}$  $\because Z$  is the midpoint of  $\overline{BC}$ 

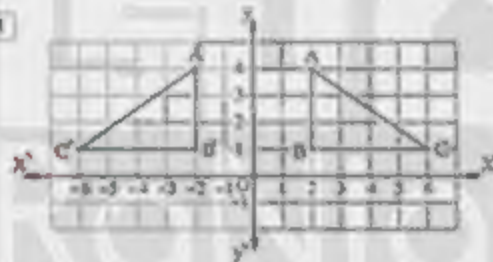
$$\therefore YZ = \frac{1}{2} AB = 3 \text{ cm.}$$

$$\therefore \text{The perimeter of } \triangle XYZ = 4 + 4 + 3 = 11 \text{ cm.}$$

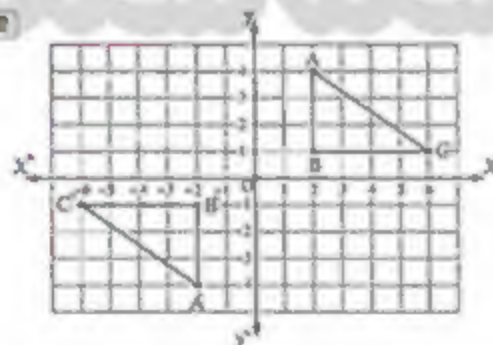
(The req.)

3

[a] 1



2

[b]  $\because E \in \overline{CD}$ 

$$\therefore m(\angle ADC) = 180^\circ - 60^\circ = 120^\circ$$

 $\because ABCD$  is a parallelogram

$$\therefore m(\angle B) = m(\angle ADC) = 120^\circ$$

(The req.)

## 14 Beni Suef

1

1 c

2 a

3 c

4 c

5 c

6 d

2

1 equal in length. 2 bisects the third side.

3 the sum of areas of the squares on the sides of the right angle.

4  $\triangle ZYC$ 

5 equal in measure.

3

$$[a] \because m(\angle EAC) + m(\angle BAC) + m(\angle EAB) = 360^\circ$$

$$\therefore m(\angle BAC) = 360^\circ - (130^\circ + 90^\circ) = 140^\circ$$

(First req.)

 $\because \overline{AB} \parallel \overline{CD}$ ,  $\overline{AC}$  is a transversal to them.

$$\therefore m(\angle C) + m(\angle CAB) = 180^\circ$$

(Two interior angles in the same side of the transversal)

$$\therefore m(\angle C) = 180^\circ - 140^\circ = 40^\circ$$

(Second req.)

[b]  $\because ABCD$  is a rhombus,  $\overline{AC}$  is a diagonal

$$\therefore m(\angle BCD) = 2 \times 32^\circ = 64^\circ$$

$$\therefore m(\angle D) = 180^\circ - 64^\circ = 116^\circ$$

(The req.)

4

[a] In  $\triangle ABC$ : $\because D$  is the midpoint of  $\overline{AB}$  $\because E$  is the midpoint of  $\overline{AC}$ 

$$\therefore BC = 2 DE = 2 \times 4 = 8 \text{ cm.}$$

$$\therefore BD = AD = 3 \text{ cm.}$$

$$\therefore CE = AE = 2 \text{ cm.}$$

 $\therefore$  The perimeter of  $DBCE$ 

$$= 3 + 8 + 2 + 4 = 17 \text{ cm.}$$

(The req.)

[b] In  $\triangle ADC$ :

$$\because m(\angle ADC) = 90^\circ$$

$$\therefore (AD)^2 = (AC)^2 - (CD)^2 \\ = (20)^2 - (16)^2 = 144$$

$$\therefore AD = \sqrt{144} = 12 \text{ cm.}$$

(First req.)

In  $\triangle ADB$ :  $\because m(\angle ADB) = 90^\circ$ 

$$\therefore (AB)^2 = (AD)^2 + (BD)^2 \\ = (12)^2 + (9)^2 = 225$$

$$\therefore AB = \sqrt{225} = 15 \text{ cm.}$$

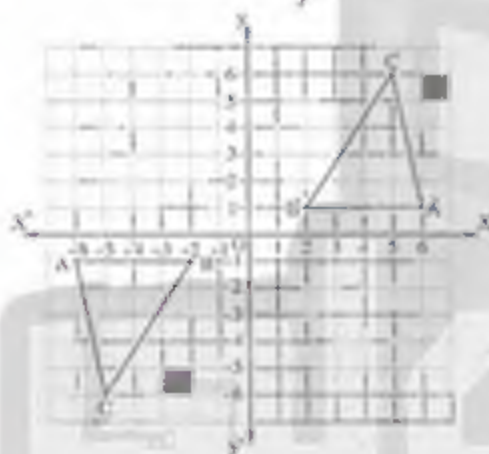
(Second req.)

## Answers of Final Examinations

1



2



15 Souhag

1 c

2 c

3 c

4 d

5 a

6 a

2

1 parallel to the third side &amp; equal to half of its length.

2 1.5

3  $(-2, -1)$ 

4 Y

5 bisects the third side.

3

[a] In  $\triangle ABC$ : $\therefore X$  is the midpoint of  $\overline{AB}$  $\therefore Y$  is the midpoint of  $\overline{AC}$  $\therefore XY = \frac{1}{2} BC = 4.5$  cm. $\therefore X$  is the midpoint of  $\overline{AB}$  $\therefore Z$  is the midpoint of  $\overline{BC}$  $\therefore XZ = \frac{1}{2} AC = 5$  cm. $\therefore Y$  is the midpoint of  $\overline{AC}$  $\therefore Z$  is the midpoint of  $\overline{BC}$  $\therefore YZ = \frac{1}{2} AB = 3.5$  cm. $\therefore$  The perimeter of  $\triangle XYZ = 4.5 + 5 + 3.5$   
 $= 13$  cm. (The req.)[b]  $\therefore \overline{DE} \parallel \overline{CB}$ ,  $\overline{BD}$  is a transversal $\therefore m(\angle B) = m(\angle D) = 60^\circ$  (alternate angles) $\therefore \angle DAC$  is an exterior angle of  $\triangle ABC$  $\therefore m(\angle DAC) = 50^\circ + 60^\circ = 110^\circ$  (The req.)

4

[a]  $\therefore \overline{DA} \parallel \overline{CB}$ ,  $\overline{AB}$  is a transversal $\therefore m(\angle B) = m(\angle EAB) = 120^\circ$   
(alternate angles) $\therefore m(\angle B) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$ 

and they are interior angles on the same side of the transversal

 $\therefore \overline{AB} \parallel \overline{CD}$  $\therefore \overline{AD} \parallel \overline{BC}$  $\therefore ABCD$  is a parallelogram (Q.E.D.)[b] In  $\triangle ABC$ : $\therefore m(\angle B) = 90^\circ$  $\therefore (AC)^2 = (AB)^2 + (BC)^2 = (3)^2 + (4)^2 = 25$  $\therefore AC = \sqrt{25} = 5$  cm. (First req.)In  $\triangle ACD$ : $\therefore m(\angle ACD) = 90^\circ$  $\therefore (CD)^2 = (AD)^2 - (AC)^2 = (13)^2 - (5)^2 = 144$  $\therefore CD = \sqrt{144} = 12$  cm. (Second req.)

5

[a]



[b] 1 CBM

2 MCD

3 CDM

4 DEM